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Variable Grouping Mechanisms and Transfer Strategies in Multi-objective Optimisation

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Variable Grouping Mechanisms and Transfer Strategies in Multi-objective Optimisation

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Abstract

For large-scale optimisation problems a well-known technique is the decomposition of a complex problem into smaller ones, to solve these independently. The key for an effective optimisation is the correct decomposition of the problem. The variables which are optimised together should interact with each other. In the single-objective case a lot of methods were developed to detect these interactions and create a grouping of variables. This grouping methods are used effectively in the single-objective environment, in the multi-objective environment not much effort was put in the grouping of variables. The idea of this work is to develop so called Transfer Strategies to apply single-objective grouping methods to the multi-objective case. Two trivial, two dynamic and one intelligent method, which analyses the variable interactions, were transferred from single- to the multi-objective case. The new methods were compared against existing ones on a theoretical and empirical basis. The theoretic evaluation analyses the found variable interactions of the approaches and compares them against the correct ones. To do this, the structure of the LSMOP benchmark problems is described in detail. In the empirical analysis the five new proposed methods outperform a state-of-the-art multi-objective grouping method and significantly improve the performance of the state-of-the-art algorithm LMEA.

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List of Acronyms

EA Evolutionary Algorithm

CC Cooperative Coevolution

SOP Single-objective Optimisation Problem

MOP Multi-objective Optimisation Problem

FE Function Evaluation

IGD Inverted Generational Distance

HV Hypervolume

LSMOP Large-Scale Multi and Many-objective Test Problem

WFG Walking Fish Group

MOEA/DVA Multi-Objective Evolutionary Algorithm based on Decision Variable Analysis

LMEA Large-Scale Many-Objective Evolutionary Algorithm

WOF Weighted Optimisation Framework

SMPSO Speed-constrained Multi-objective Particle Swarm Optimisation

GDE3 third version of the Generalised Differential Evolution Algorithm

CCGDE3 Cooperative Coevolutionary GDE3

NSGA-II Non-Dominated sorting Genetic Algorithm II

GM grouping method

PDG Position Distance Grouping

CVA Control Variable Analysis

VC Variable Clustering

T-ENS tree-based non-dominated sorting approach

IA Interdependence Analysis

CA Correlation Analysis

DG Differential Grouping

DG2 Differential Grouping 2

MRAND Multi-objective Random Grouping

MDELTA Multi-objective Delta Grouping

MDG2 Multi-objective Differential Grouping 2

TS Transfer Strategy

TSO Transfer Strategy Objective

TSV Transfer Strategy Variable

G1 1-Group

GN N-Group

MCS Maximal Complete Subgraphs

PlatEMO MATLAB Platform for Evolutionary Multi-objective Optimisation

IQR Inter Quartile Range

1. Introduction

We live in a digitalised world. The complexity and difficulty of problems with which companies and industries are confronted increases. Problems which cannot be solved analytically arise in numerous sectors, for example, chemistry, physics, insurance, automotive and banking sector.

Consider the following example: A battery of an arbitrary electric car has to be designed. It should fulfil several different criteria: it has to be as small and cheap as possible and should have the largest possible capacity at the same time. The question is what is the overall *best* battery regarding all criteria at the same time? We want to find the smallest, cheapest battery with the most capacity. One with a very big capacity is maybe bigger and more expensive, a small and cheap one has most probably not as much capacity as the first one. A battery which is superior in all three aspects at the same time against any other combination is most likely non existent. The criteria are contradictory in the way that improving one will most likely worsen another. A battery half the size of another will be cheaper and smaller but will only have half the capacity.

In the mathematical context, the task is an optimisation of several objectives. In the example we have three objectives which have to be optimised at the same time, namely the price, size and capacity. An optimal solution can often not be found, the idea is to find batteries which provide a good trade-off between all three aspects. The goal is not to find a single battery but several ones with a good values for the objectives from which decision makers can choose.

Mathematically this is called a multi-objective optimisation problem. There are three objectives; price, size and capacity, where the first two have to be minimised and the third maximised. For many real world applications it is nearly impossible to solve these problems in an analytical way. Instead meta-heuristics like Evolutionary Algorithms (EA) or Particle Swarm Optimisation, which are inspired by biology, are used effectively in this field. The idea behind

EAs is derived from the evolutionary theory. During evolution only individuals which are fit and adjusted to the environment survive to then pass on their genes to the next generation. EAs adapt this scheme and formulate a mathematical optimisation procedure. Individuals are the possible solutions which are described by a set of input parameters of the problem. These can be altered by operators called mutation and crossover. The fitness is measured with a predefined fitness function, in the mathematical context it is called a objective function.

For the car example input parameters are every aspect of the battery one can change when designing it, starting with the materials, cells, cables, connections, different electric components, cover panel and the list goes on. These input parameters are called decision variables in the mathematical context and can be altered and changed. Their *values* effect the three considered criteria, or objectives. Due to the complexity of actual problems, the number of these variables can be quite high. Usually Multi-objective problems with more than 100 decision variables are called large-scale.

With increasing number of variables the complexity of problems increases, known as *Curse of Dimensionality*. Most of the EAs developed in the last decade cannot perform well on these large-scale problems. In recent years the interest in such methods increases. Nearly all of them depend on the idea of a decomposition of the problem. This decomposition is done by combining several decision variables in groups and optimising these independently. The complex problem is divided into smaller sub-problems which can be solved independently.

The key for the right decomposition is to find the correct variable groups, and to find these groups the interactions between variables have do be determined correctly. Approaches to find these interactions are called grouping methods. The correct grouping of variables is an open research question. Existing multi-objective grouping methods need a high computational budget which is infeasible for most of the real world applications where a single evaluation of an individual can take a long time. Also it is not clear if they find the correct variable interactions. A lot of effort was put in the development of grouping methods for problems with only a single objective. Since the interaction of variables depend on the objective functions they cannot be applied to multi-objective approaches.

The general idea of this work is therefore to examine and research how this well-known single-objective grouping methods can be applied to the multi-objective case.

1.1. Goals of the Work

The overall topic of this work are variable grouping methods (GMs) in large-scale multi-objective optimisation problems. The main research objective of the work is the transfer of variable GMs from the single- to the multi-objective case. The goals of this work are as follows:

1. Develop strategies which transfer single-objective grouping methods to the multi-objective case

2. Examine the capabilities of existing and new approaches theoretically

3. Evaluate the performance of state-of-the-art grouping methods using the proposed Transfer Strategies

The first goal is the core of the work. The single-objective GMs have to be adapted to use them in an multi-objective environment. Methods for this transfer are developed and applied to an single-objective GM.

The second goal is based on a theoretical analysis of existing benchmark problems, state-of-the-art and new GMs. The found groups of several approaches are tested against the theoretical correct results. To do this, the structure of benchmark problems is analysed to derive the true variable interactions from the function definitions.

The third goal is the analysis of the proposed new approaches against each other and existing ones. Empirical results are compared against the theoretic ones. The influence of different GMs on the performance of algorithms is analysed. To give a good overview about the performance of the transferred approaches experiments with different numbers of decision variables and benchmark problems are executed and evaluated.

1.2. Organisation of the Document

The structure of this work is described in the following. In Chapter 2 the theoretical background and necessary definitions are explained. The general problem which is treated in this work is pointed out. The next Chapter 3 describes actual state-of-the-art approaches for evolutionary algorithms and single- and multi-objective GMs. An overview about existing approaches is given, the differences, advantages and disadvantages are discussed. Chapter 4 is the core of this thesis. Ideas and approaches to transfer single-objective methods are described. The transferred GMs are explained in detail. Chapter 5 contains the evaluation of this work. The own approaches and solutions are compared against existing ones. It is split in two parts, the theoretical and empirical analysis. The last Chapter 6 is the conclusion, the work which was done and the obtained results are summarised. Possible future research questions which came up during the work on the thesis are mentioned. The Appendices A to E contain all information and results which were obtained.

2. Theoretical Background

In this chapter the theoretical background of this work is described. The necessary information and definitions to understand the thesis are provided.

2.1. Multi-Objective Optimisation

This thesis is based on the optimisation of multiple objectives at the same time. Objective functions with several input values and one output value are considered. Unless otherwise stated, the function value has to be minimised (and not maximised). In an Single-objective Optimisation Problem (SOP) there is only one objective to optimise. A lot of work has been published in the field of single-objective optimisation [13]. Mathematically a multi-objective problem is defined as:

$$\min_{\vec{x}} \vec{f}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), ..., f_m(\vec{x})) \tag{2.1}$$

where $\vec{x} \in X$ is the input (or decision) vector $\vec{x} = (x_1, x_2, ..., x_n)$ defined in the decision space $X \subseteq \mathbb{R}^n$ [13].

Large-scale multi-objective optimisation covers problems which have a *large* number of decision variables n. Here, problems are considered as large-scale if they contain 100 or more decision variables. When there are a lot functions to optimise simultaneously (objective functions m) the problem type is called many-objective optimisation. According to this definition, the problem is considered as multi-objective if the number of objective functions is 2 or 3, as many-objective if $m > 3$ or higher.

2.2. Pareto-Optimality

In multi- and many-objective problems the conflicting functions have to be optimised simultaneously and an optimal solution (where all function values are minimised at the same time) cannot be found most in most cases. Because of this the goal is not to find one *best* solution but a set of *good* solutions. To compare solutions against each other the principle of *Pareto-Optimality* is used [24].

The solution of a Multi-objective Optimisation Problem (MOP) can be seen as a vector containing all function values $u = (f_1(\vec{x}), f_2(\vec{x}), ..., f_n(\vec{x}))$. The goal is to minimise these values. To compare the vectors their *Pareto-dominace* criterion is used: A vector u *dominates* another vector v if it is *partially less* than v, meaning that $\forall i \in \{1, ..., m\}, u_i \leq v_i \wedge \exists i \in \{1, ..., m\} : u_i < v_i$ [24]. In other words, one vector *dominates* another if one value of its values smaller and all other are smaller than or equal. A solution u is said to be *Pareto-optimal* and the corresponding vector *non-dominated* if and only if there exist no other vector which dominates u. The set of all *Pareto-optimal* solutions is called *Pareto-Set*.

Considering the example MOP with one decision variable x with only four instances $x = (x_a, x_b, x_c, x_d)$ and three objective-functions:

$$\begin{cases} f(x_a) = (2,3,4) \\ f(x_b) = (2,3,3) \\ f(x_c) = (1,3,3) \\ f(x_d) = (1,2,3) \end{cases} \tag{2.2}$$

The resulting vector of x_b and dominates the vector of x_a The vectors of x_c and x_d both dominate the vectors of x_a and x_b. $f(x_c)$ and $f(x_d)$ are also *non-dominated* and the corresponding solutions belong to the *Pareto-Set*. When this set is displayed in the solution space (by plotting the objective-function values against each other) the resulting visualisation is called *Pareto-front*.

As said before, the goal is to find not only one solution but a set of solutions. This set should have a great diversity. In the example are two solutions in the Pareto-set but further evidence like which one of them is *better* cannot be derived. The idea is to provide a set of solutions which are on one hand as close as possible to the Pareto-front and on the other hand have a great diversity. From this solutions another decision maker (for example a human) should choose the final solution, but this is no mathematical decision and therefore not relevant in this context.

The single solutions are compared against each other by their Pareto-dominance property. To compare several different sets of solutions, for example to compare the result quality of two algorithms against each other, performance indicators are used. They try to measure the distance of the found solutions to the Pareto-front and the diversity of the sets. Several performance indicators were developed to measure the quality of a solution set. In this work Inverted Generational Distance (IGD) [36] and Hypervolume (HV) [26] are used, they will be further described in the evaluation.

2.3. Evolutionary Algorithms

Evolutionary Algorithms (EAs) are used in this work to solve the multi-objective optimisation problems as described above. The general idea of an EA is inspired by the evolutionary process from biology [5]. When a problem is given, EAs generate a population of solutions (here: individuals) for the problem which are evaluated using a fitness function. The value of this function states how *fit* an individual is. In order to get better individuals mutation and crossover operators are used to recombine the individuals. A mutation is a random change of an individual, a crossover a combination of two or more parent individuals to a new child. Mutation and crossover depend on the formulation of the problem, they can be defined in lots of different ways [21, 15, 5, 37]. Applied to the multi-objective case, the individuals are possible decision vectors $\vec{x}$ and m different fitness (objective) functions are applied to check the result quality.

Figure 2.1 shows the general structure of an EA. During the initialisation the initial population of individuals is generated and fitness values are assigned in the evaluation step. When the termination criterion is fulfilled the actual population is returned as result of the algorithm. If not, a new evolutionary generation cycle starts. The parents for recombination are selected, crossover and mutation are performed and the new individuals are evaluated. After this,

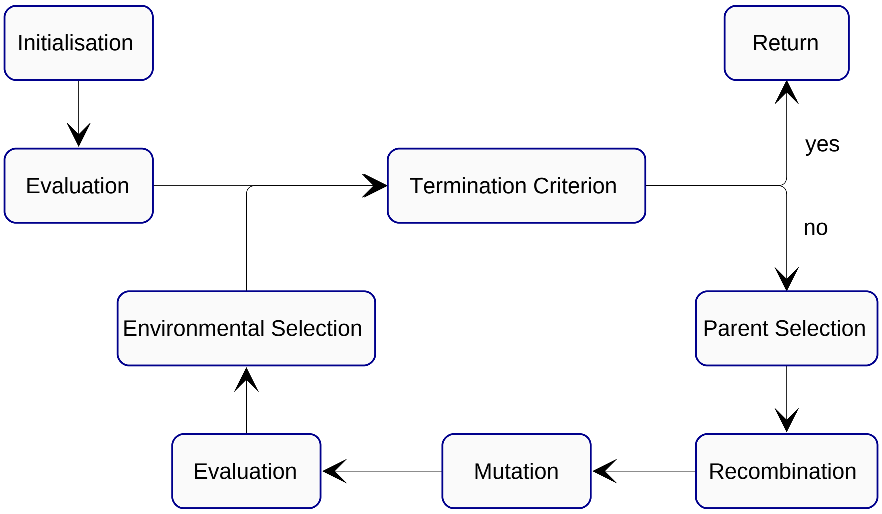


Figure 2.1.: The general structure of an EA

an Environmental Selection defines which individuals form the next population. After that the Termination Criterion is checked again.

The field of EAs is big, there exist a many different approaches and algorithms. This work is mainly based on two state-of-the-art EAs which will be described in Section 3.

2.4. Problem Decomposition

Multi or many-objective large scale problems lead to huge search spaces. In order to reduce them, approaches to decompose the problems into smaller ones and solve each one independently were developed. Two *kinds* of a decomposition are described below. Position Distance Grouping (PDG) is just described for completeness and a better understanding of the grouping, the main focus of this work lies on the interaction grouping. The considered state-of-the-art approaches in this work use the following scheme:

1. position-distance grouping
2. interaction grouping
3. optimisation

They first divide the overall set of decision variables in position and distance related ones, these properties will be described more detailed in Subsection 2.4.1. Position variables do not directly affect the convergence of a solution but the distribution, distance variables on the other hand are related to the convergence of the solution towards the Pareto-front of the problem. The algorithms handle these sets in a different way.

In the second step only the distance variables are considered and groups of *interacting* variables are formed. This step and the interaction criteria will be described in Subsection 2.4.2.

In the last step the optimisation process of the algorithms start. The obtained groups of position and distance variables and the interacting variable groups are used in a different way. In Subsection 2.4.3 the optimisation process based on the obtained groups is described.

2.4.1. Position Distance Grouping (PDG)

In a lot of problems not all of the variables have the same properties and relation to the functions which should be optimised. Altering specific variables does not have an influence (or only a small influence) on the convergence of the algorithm. These variables often only alter the positions of the solutions along the Pareto-front and affect therefore not the convergence but the diversity of the solutions. Because of this, they are called *divergence* or *position* variables.

The other ones which can be used to converge to the Pareto-front are called *convergence* or *distance* variables.

Considering the definition from [12], changing distance variables on its own must lead to dominating or non-dominating solutions, changing position variables on the other hand must not. When non of these conditions is fulfilled, the variable is distinguished as mixed.

Figure 2.2 provides a visualisation of a distance and position variable. The MOP considers the two objective functions $f_1(\vec{x})$ and $f_2(\vec{x})$ and the decision variables x_1 and x_2. The two objective functions are plotted against each other with varying values of x_1 (blue dots) and x_2 (red dots). The change of x_1 just affects the diversity or position of the solutions and not the convergence. Changing x_2 on the other hand leads to dominating solutions and affects the convergence or distance to the Pareto-optimum.

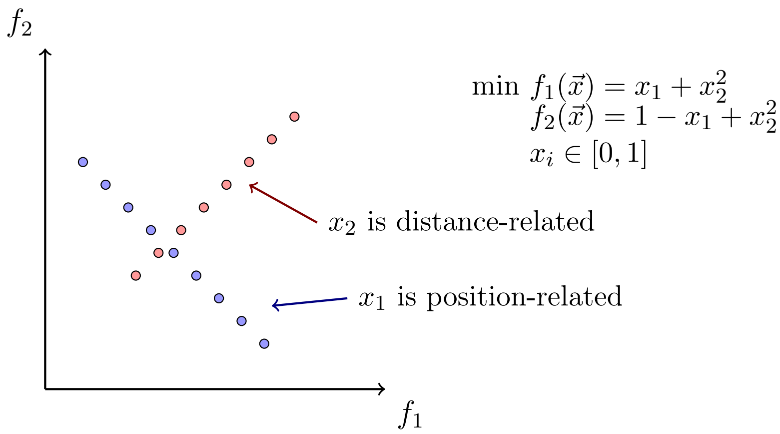


Figure 2.2.: Example for distance-related and position-related variables

To address the problem of identifying the type of the variables, the number of position variables can be specified for instance in the WFG test suite which is described later. With these test suite the performance of identifying the position and distance variables can be measured. But the main focus of this work lies on the interaction grouping, the position-distance grouping is only described because it is a necessary step before the interaction grouping can be executed. In this thesis a grouping describes the interaction grouping which is described in the following subsection (and not the position-distance grouping) when not stated different.

2.4.2. Interaction Grouping

Another approach of grouping variables does not depend on the convergence of the single variables but on the connection between them. The main idea is to form groups of *interacting* variables. These groups should be optimised independently.

There are different definitions of interacting variables [31, 25, 12, 34]. The definition from [12] and [34] is used in this thesis. Two variables are considered as non-interacting if the ordering of the values from the objective function $f_k(\vec{x})|_{x_i=a_1}$ and $f_k(\vec{x})|_{x_i=a_2}$ does not depend on the value for the variable x_j, using the notation $f_k(x)|_{x_i=a} = f_k(x_1, ..., x_{i-1}, a, ..., x_n)$. On the other hand, when there exist a value $x_j = b_2$ for which $f_k(\vec{x})|_{x_i=a_1}$ is smaller than $f_k(\vec{x})|_{x_i=a_2}$ and another $x_j = b_1$ which changes the proportion, so that $f_k(\vec{x})|_{x_i=a_1}$ is bigger than $f(\vec{x})|_{x_i=a_2}$, the two variables interact with each other.

Given a MOP defined in Equation 2.1, two decision variables x_i and x_j interact with each other in objective k, if there exist values a_1, a_2, b_1, b_2 so that both conditions of Equation 2.3 are fulfilled, using the notation $f_k(x)|_{x_i=a,x_j=b} = f_k(x_1, ..., x_{i-1}, a, ..., x_{j-1}, b, ..., x_n)$. A visualisation of the conditions is given in Figure 2.3.

$$f_k(x)|_{x_i=a_1,x_j=b_2} < f_k(x)|_{x_i=a_2,x_j=b_2} \tag{2.3a}$$

$$f_k(x)|_{x_i=a_1,x_j=b_1} > f_k(x)|_{x_i=a_2,x_j=b_1} \tag{2.3b}$$

When a change of x_j from b_2 (Equation 2.3a) to b_1 (Equation 2.3b) with fixed values for x_i changes the proportion of the objective values, x_i and x_j interact with each other. When such values do not exist, the two variables have no interaction.

The optimisation of two interacting variables depend on each other. This means we could not find the global optimum for x_i by only varying x_i, we have to consider also x_j. The other way round, when it is not possible to find four such values, we can optimize x_i on its own, the optimal value does not depend on x_j.

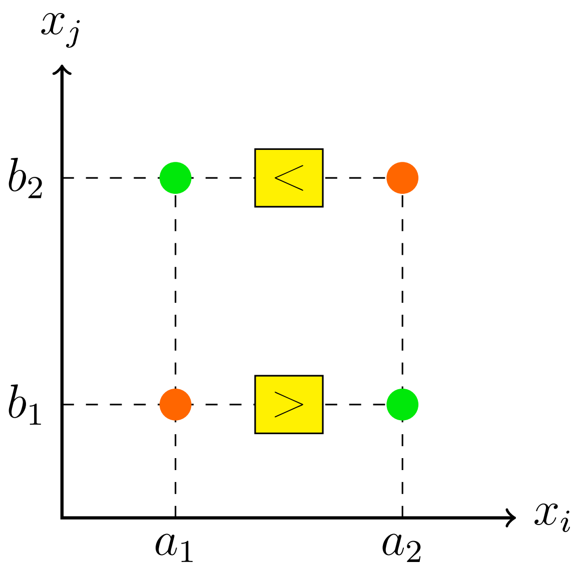


Figure 2.3.: Graphical representation of the interaction conditions from Equation 2.3

The definition of interacting variables leads to the definition of separable and non-separable functions, see Equation 2.4 [12]. Separable functions have no interactions between all of their variables. All decision variables can be optimised independently to find the global optimum. When there is at least one interaction between any of the decision variables, the function is called non-separable. A non-separable function is called fully non-separable if every variable interacts with all other decision variables.

$$\begin{aligned}\arg\min_{\vec{x}} f(\vec{x}) =& [\arg\min_{x_1} f(x_1, ..., x_i, ..., x_n), ..., \\ & \arg\min_{x_i} f(x_1, ..., x_i, ..., x_n), ..., \\ & \arg\min_{x_n} f(x_1, ..., x_i, ..., x_n))]\end{aligned} \tag{2.4}$$

Consider the following two example functions:

$$f_1(x_1, x_2) = x_1 + x_2 \tag{2.5}$$
$$f_2(x_1, x_2) = (x_1 - x_2)^2 \tag{2.6}$$

with $x_1, x_2 \in [0,1]$. The global optimum for the first function is $\arg\min_{x_1,x_2} f_1(x_1,x_2) = [0,0]$. Fixing the value for x_2 to any other does not affect the optimal value for x_1 which is still 0, therefore $\arg\min_{x_1} f(x_1,x_2) = 0$. The same holds when changing x_1, the optimal value for x_2 remains 0. So the first function can be separated into $\arg\min_{x_1,x_2} f(x_1,x_2) = [\arg\min_{x_1} f(x_1,x_2), \arg\min_{x_2} f(x_1,x_2)] = [0,0]$ and is therefore separable. f_2 on the other hand is not separable. The global optimum is $\arg\min_{x_1,x_2} f_2(x_1,x_2) = [a,a]$ with $a \in [0,1]$. One can clearly see that the variables cannot be optimised independently. When we fix $x_2 = a$ the optimal value for x_1 depends on the value of x_2. In this example the optimal value would be $x_1 = x_2 = a$. f_2 is therefore non-separable and because all two variables interact with each other also fully non-separable.

2.4.3. Group Optimising: Cooperative Coevolution

A lot of existing EAs in the single- and multi-objective case are based on a decomposition of the problem. The general idea is to split the problem in smaller sub-problems which can be optimised easier. In [5] one can see that a lot of algorithms use decomposition strategies, but less emphasis placed on the grouping or decomposition strategy than on the optimisation of the groups itself. Often some kind of random grouping strategies are used [13, 28]. Well known approaches are for instance a random grouping strategy with adaptive weighting (EACC-G) [28]. The following work uses a set of problem decomposers which are based on random grouping strategies with different group sizes [29].

However, groups of variables are often used to solve large-scale problems, a popular concept in this area is Cooperative Coevolution (CC) [20]. Separate populations of variables for each group are stored and optimised, or simply fixing the variables of all but one group and only changing interacting variables at the same time. The optimisation approach used in the algorithms in this work differs in one point from the CC scheme. In CC a solution is evaluated by combining the values of the actual subgroup with the best ones for the variables in all other subgroups. Here the groups are optimised one after another and the values for all other groups are fixed to their actual value. So an altered version of CC is used in this thesis because the optimisation processes from state-of-the-art algorithms are used.

The CC approach was first used in a lot of algorithms for single-objective optimisation [5, 13]. The first algorithm which applies CC to the multi-objective case was Cooperative Coevolutionary GDE3 (CCGDE3) [1]. It was an CC version of the former third version of the Generalised Differential Evolution Algorithm (GDE3) [10]. GDE3 was an the third iteration of an approach which used Differential Evolution to solve single and multi-objective problems and was also applicable for additional constraints. In order to handle multi-objective large-scale problems the CC approach was used to reduce the search space. The original work considered problems with 200 up to $5,000$ decision variables. The results were promising and the well known Non-Dominated sorting Genetic Algorithm II (NSGA-II) [7] and the previous version GDE3 were outperformed by [1]. A lot of the actual state-of-the-art algorithms like the ones considered in this work use a CC approach to decompose the problem [13, 12, 34].

3. State of Research

In this chapter the algorithms and strategies which are used as a basis of following work are explained. The state-of-the-art algorithms for large-scale optimisation MOEA/DVA and LMEA are described in general. Several single- and multi-objective grouping methods are described to give an overview about the previous work in this field.

3.1. State-of-the-Art Algorithms

State-of-the-art algorithms in multi- or many-objective optimisation are for instance the Multi-Objective Evolutionary Algorithm based on Decision Variable Analysis (MOEA/DVA) [12], Large-Scale Many-Objective Evolutionary Algorithm (LMEA) [34] and Weighted Optimisation Framework (WOF) [40, 38]. All of them somehow decompose the given problem in smaller ones and solve them independently. The first two are used in the concept and evaluation of this work, and are described in the following.

MOEA/DVA and LMEA both consist of the three parts described in Section 2.4. The first part is the Position Distance Grouping. In this step all decision variables are divided in two groups, the position and distance variables. The second step is the division of the distance variables in groups of interacting variables. This step is called interdependence or interaction analysis in the two algorithms. After this step the different groups of variables are optimised.

3.1.1. MOEA/DVA

MOEA/DVA is a state-of-the-art EA in the field of large-scale multi-objective optimisation [12]. The algorithm consists of the three parts described in Section 2.4: PDG, (interaction) grouping and optimisation. The main focus of the work was the new proposed method for the PDG called Control Variable Analysis (CVA). Algorithm 1 shows the general structure of MOEA/DVA.

Algorithm 1: MOEA/DVA from [12]

Input: NCA, NIA
Result: Pop
1 $[PV, DV] \leftarrow ControlVariableAnalysis(NCA)$;
2 $Pop \leftarrow fixPositionVariables(Pop, PV)$;
3 $[Pop, DVSet] \leftarrow InterdependenceAnalysis(Pop, DV, NIA)$;
4 $Neighbours \leftarrow computeNeighbors(Pop, PV)$;
5 $gen \leftarrow 0$;
6 $oldPop \leftarrow Pop$;
7 **while** $FEsavailable \land utility > thresold$ **do**
8   **for** $i = 1 : length(DVSet)$ **do**
9         $Pop \leftarrow SubcomponentOptimiser(DVSet\{i\}, Neighbours)$;
10   **end**
11   **if** $gen\%2 == 0$ **then**
12         $utility \leftarrow calculateUtility(Pop)$ $oldPop \leftarrow Pop$
13   **end**
14 **end**
15 $Pop \leftarrow uniformityOptimisation(Pop)$

CVA is executed in line 1, every variable is checked if it is most likely a distance or position variable. The description of position variables is given in Subsection 2.4.1. To figure this out a random individual is generated and only the considered variable is altered several times. The objective values of these points are evaluated. A variable is classified as distance (convergence) related if every generated solution dominates, or is dominated by any other solution. In other words: when we pick any two generated solutions one has to dominate the other. A variable is marked as position (divergence) related if no solution dominates any other generated solution. When none of these conditions is true, the variable is marked as *mixed*. A predefined parameter NCA describes how much solutions are generated per variable for the described test. Mixed variables are added to the set of distance variables for the rest of the optimisation. PV contains the position variables and DV the distance variables.

Position variables are fixed at the beginning of the algorithm and are not changed anymore, see line 2 of Algorithm 1. During the fix the first population *Pop* which contains the initial individuals is generated. After the CVA and the initialisation, the interaction grouping is executed with the population, the set of distance variables DV and the parameter NIA. It is called Interdependence Analysis (IA), uses Definition 2.3 for interacting variables and is described detailed in Subsection 3.4.1. The solutions of *Pop* are updated during the interaction check, because of this not only the variable groups $DVSet$ but also a new population is generated.

In line 4 the neighbours of variable groups are computed, they are latter used to select individuals for the recombination. The *gen* counts the actual generation, *oldPop* stores the population of the last generation.

During a generation every variable group in $DVSet$ is optimised independently in line 7 to 9. The used subcomponent optimiser is described in the following. After each optimisation step the values of the *neighbours* are updated. The optimiser uses these values for selecting the mating pool. An optimisation step for a given group of variables is executed for every individual in *Pop*. In every step two individuals of *neighboring* groups (or with a lower probability from a random group) are selected for Crossover using Differential Evolution described in [22].

Every second generation the *utility* is computed. This method measures the improvement which was made during the last optimisation step. It measures the convergence of the solutions of the actual population *Pop* in contrast to the one from the last generation *oldPop*. When the utility falls below a predefined threshold, MOEA/DVA quits the group based optimisation with the described subcomponent optimiser and uses a fallback EA, the $uniformityOptimisation$. In the original paper MOEA/D [32] was used, but other algorithms are also possible.

3.1.2. LMEA

LMEA is also a state-of-the-art EA in the field of large-scale multi- and many-objective optimisation. It follows the same structure as MOEA/DVA: PDG, interaction grouping and optimisation. Some parts of the algorithm are improvements of the preceding MOEA/DVA. The work was focused on two main aspects, first an improvement of the PDG with a new method called Variable Clustering (VC). The second was an improvement of the general and widely used Non-dominated sorting algorithm, namely: tree-based non-dominated sorting approach (T-ENS) [34]. After the VC, the interaction grouping Correlation Analysis (CA) is executed on the distance variables. CA is described detailed in Subsection 3.4.2.

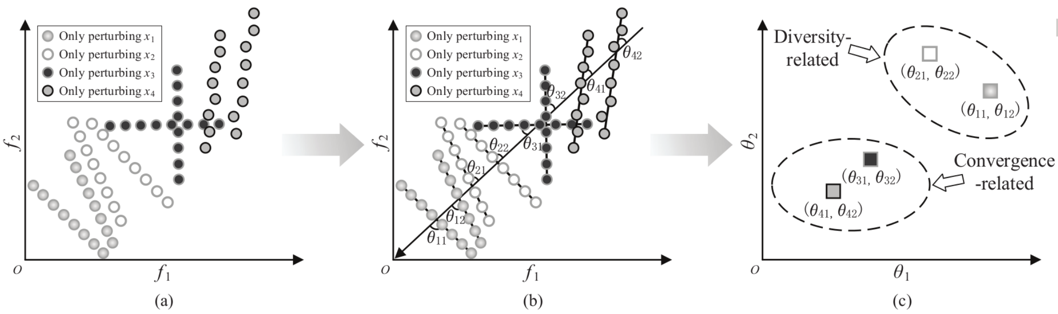


Figure 3.1.: The VC method from LMEA to detect the position and distance variables [34]

VC does not use the Definition of position and distance variables directly like CVA from MOEA/DVA. The structure of the methods is shown in Figure 3.1. The idea is to not directly use the definition because variables of some test suites and real world problems may not follow the exact rules. Considering a sample of 20 solutions and only one is dominating all other solutions but none of the others dominates any other solution. This variable would be considered as mixed, because neither the condition for position related, nor for distance related is fulfilled. But for the optimisation it would be probably better to consider it as position variable because only one solution of 20 improves the convergence.

Figure 3.1 shows the structure of VC. For a predefined number of $nSel$ solutions are randomly sampled. In the example two solutions are generated for every of the four considered decision variables x_1 to x_4. Every solution is perturbed $nPer$ times for the respective decision variable. In the example every solution is perturbed eight times. Lines are fitted for the perturbed solutions, see Figure 3.1 b). The angles between this lines and the normal vector of the origin are used as input values for a k-means algorithm.

Algorithm 2: LMEA from [34]

Input: $nSel, nCor, nPer$
Result: *Population*

```
1  Pop ← initializePop();
2  [PV, DV] ← VariableClustering(nSel, nPer);
3  DVSet ← CorrelationAnalysis(Pop, DV, nCor);
4  while FEs available do
5      for i = 1 : 10 do
6          Pop ← ConvergenceOptimisation(Pop, DVSet);
7      end
8      for i = 1 : M do
9          Pop ← DistributionOptimisation(Pop, PV);
10     end
11 end
```

Algorithm 2 shows the general structure of LMEA. The population *Pop* is initialised randomly in the first line. VC is executed with the parameters $nSel$ and $nPer$. CA creates the variable groups $DVSet$ and takes the actual population the distance variables DV and the parameter $nCor$ which describes the number of interaction checks. In every generation the distance variables are optimised with the *ConvergenceOptimisation* in line 6. In contrast to MOEA/DVA the position variables are also optimised with a special *DistributionOptimisation*. When no Function Evaluations (FEs) are available, the algorithm terminates.

The *ConvergenceOptimisation* takes the actual population *Pop* and the variable groups $DVSet$ and returns a new population, it is based on the proposed T-ENS. T-ENS is used to calculate the non-dominated front number of the solutions. The distance of the solutions to the axis origin is also computed. A single optimisation step looks as follows. This two criteria are used to select the parents for recombination . A binary tournament approach which is based on the previous two criteria selects two parents and two new offspring solutions are generated by only varying the variables from the actual group. The offspring solutions replace the parents in the population if and only if they have a smaller non-dominated front number. This optimisation step is executed for every group multiple times, the number of executions is identical to the number of variables in that specific group.

The improvement of the interaction grouping CA was not a key point of the original work, but it differs in some points from the IA of MOEA/DVA and is described detailed in Subsection 3.4.2.

3.1.3. Weighted Optimisation Framework

The WOF was first described in [40] and [38]. It is a state-of-the-art EA for solving large-scale multi-objective problems. It is also based on reducing the dimensionality of the problem by grouping variables together, but in a different way than the previous algorithms MOEA/DVA and LMEA. The grouping of WOF is not based on the interactions between the variables. The variables are divided into groups using linear grouping which will be further described in Subsection 3.3.1. Linear grouping just creates an arbitrary number γ of equal-sized groups. The ordering is based on the index of the variables.

The search space is reduced by changing all variables of a group at the same time and amount. Weight values are assigned to these groups and are changed in an own optimisation step. So-called transformation functions are used to alter the original values of the decision variables at once by applying the weight values to these functions. With this technique the dimensionality of the problem can be reduced from hundreds or thousands to γ. Internally WOF uses the Speed-constrained Multi-objective Particle Swarm Optimisation (SMPSO) [16] for optimisation purposes, but other metaheuristics can also be applied.

3.2. Performance Comparison

The three previous described algorithms MOEA/DVA, LMEA and WOF are all state-of-the-art EAs for solving large-scale multi- (and in the case of LMEA also many) objective optimisation problems. Performance results of this three can be found in their original papers [12, 34, 40, 38] and in a study which compares the three directly with each other [39].

The comparisons were based on multi and many-objective benchmark problems. The considered ones are the UF test suite which contains Unconstrained multi-objective test problems which were used on the congress of evolutionary computation in 2009 special session and competition [33]. The ZDT test suite contains several different multi-objective test problems [41]. The DTLZ suite contains scalable test problems for evolutionary multi-objective optimisation [8]. The test suites Walking Fish Group (WFG) [9] and a the new test suite which contains Large-Scale Multi and Many-objective Test Problems (LSMOPs) [4] are also used in this work and will be described detailed in 5.3.2.

MOEA/DVA was compared against NSGA-III [6], SMS-EMOA [2] and MOEA/D [32] on UF and WFG test suites [12]. NSGA-III is an extended version of the well-known NSGA-II which is designed to deal with many-objective problems. SMS-EMOA is mentioned as hypervolume-based S metric selection-based EA. MOEA/D is based on decomposition of the problem and was also used in MOEA/DVA as fallback method[32]. Test problems with up to 30 for WFG, and for UF problems up to 200 decision variables were used. So both *normal* and large-scale problems were considered. MOEA/DVA performs significantly better than the other three on most test instances. The other algorithms cannot catch up to the performance of MOEA/DVA.

LMEA was also compared against NSGA-III and MOEA/D and additionally against KnEA [35] and MOEA/DVA [12]. KnEA is a knee point driven EA which is explicitly designed for many-objectives [35]. For most of the experiments was the DTLZ test suite and some instances of the WFG and UF suites are used. The number of decision variables ranges from 100 to $1,000$ and the objectives from 5 to 10. The tested problems were large-scale many-objective problems. LMEA outperforms the four considered algorithms in most of the cases. KnEA and MOEA/DVA have in general better results than MOEA/D and NSGA-III.

The previous mentioned algorithm CCGDE3 cannot compete with the WOF neither in solution quality nor in convergence speed. This was analysed in the original WOF paper [38].

The comparison study from [39] is based on the LSMOP test suite which is also used in the evaluation of this work. The three algorithms were tested on the LSMOP test suite with up to $1,006$ decision variables and 2 and 3 objective functions. The tested problems are large-scale multi-objective problems. MOEA/DVA and WOF outperform LMEA in most of the considered test instances. Furthermore WOF performs significantly better than MOEA/DVA and LMEA in result quality and also in convergence speed. Often WOF just uses 0.1% to 10% of the computational budget and produces better results than the other two algorithms at the end of their optimisation.

3.3. Single-Objective Grouping -Methods

Several different grouping methods for single objective optimisation were summarised in [13]. The ones which are used in this work are described in the following.

3.3.1. Random and Linear Grouping

Random grouping is one of the simplest grouping methods and has been used often in the literature [28], [13]. Linear grouping was used in [40, 38] and has a similar approach. In both methods the number of groups must be specified before. Linear grouping just divides the overall group in n equal sized groups using the index of the variable as ordering. Random grouping creates also n equal sized groups but with randomly chosen variables. Most of the time the grouping is repeated at the end of a generation to assure that the interacting variables are together in one group with a certain probability [28]. Since groups can be obtained without any computational costs (in terms of function evaluations), these are fast and inexpensive methods. On the downside however, the effect can be quite limited when the problem at hand contains a lot of interacted variables.

3.3.2. Delta Grouping

Delta Grouping was introduced by Omidvar et al in [18]. The idea is also built on static group sizes and updating of the groups every generation, It is therefore a dynamic grouping method like random grouping. The group size must also be specified beforehand.

The general assumption is that the amount of change in decision variables helps to determine interacting ones. The idea is that variables with an similar amount of change during an generation are more likely to interact with each other. To measure the amount of change a *delta value* $\bar{\delta}$ is introduced. It measures the change of a decision variable between two generations. The change vector for all variables is given as $\Delta = \{\bar{\delta_1}, \bar{\delta_2}, ..., \bar{\delta_n}\}$. The delta values are computed with:

$$\bar{\delta_i} = \frac{\sum_{j=1}^{p} \delta_{i,j}}{p} \tag{3.1}$$

with population size p and $i \in \{1, 2, ..., n\}$. $\bar{\delta_{i,j}}$ is the delta value (absolute change) of the j-th individual of the population for the i-th decision variable. The decision variables are ordered according to their value in the change vector. The direction (ascending or descending) is not relevant. Groups are created according to the given group size and the obtained ordering. They are ordered and divided into equal sized groups using the given group size. The delta grouping approach does not use any function evaluations to obtain the grouping, besides the ones which are executed during the optimisation anyway.

3.3.3. Differential Grouping

The idea of Differential Grouping (DG) is to use Equation 2.3 directly for the detection of interacting variables [17]. For a combination of two variables x_i and x_j, DG checks Equation 2.3 to check the interaction criteria. In other words the question to answer is: When we change the value of a variable x_i, does the amount of change in the objective function $f(x)$ stays the same regardless of the value of x_j?. This is checked by comparing absolute fitness differences in response to changes in x_i and x_j.

Algorithm 3 shows the general structure of the DG approach. It uses the actual population of individuals and a threshold parameter e which states how much variation in the fitness value is necessary to count the interaction. *dims* are the considered decision variables, *seps* are used to temporarily store the separate variables (which does not have any interactions) and $DVSet$ (Distance Variable Set) hold the found groups.

Algorithm 3: The structure of Differential Grouping from [17]

```
Input: Pop, e
Result: DVSet
dims ← {1, ..., n};
seps ← {};
DVSet ← {};
for i ∈ dims do
    group ← {i};
    for j ∈ dims ∧ i ≠ j do
        if interactionCheck(i, j, Pop, e) then
            group ← group ∪ j;
        end
    end
    dims ← dims \ group;
    if |group| = 1 then
        seps ← seps ∪ group;
    else
        DVSet = DVSet ∪ {group};
    end
end
```

The interaction of the variables x_i and x_j is tested with the *interactionCheck*. This check generates the four values of Equation 2.3, visualised as dots in the previous Figure 2.3, considering the lower and upper bounds of the specified variables for the values a_1, a_2, b_1, b_2. The Equation is not checked directly, first the amount of change is computed:

$$\Delta_1 = f_k(x)|_{x_i=a_1,x_j=b_2} - f_k(x)|_{x_i=a_2,x_j=b_2} \tag{3.2}$$

$$\Delta_2 = f_k(x)|_{x_i=a_1,x_j=b_1} - f_k(x)|_{x_i=a_2,x_j=b_1} \tag{3.3}$$

If this change is bigger that the predefined threshold $|\Delta_1 - \Delta_2| > e$, the interaction check is fulfilled. The grouping approach is deterministic because the values of a_1, a_2, b_1, b_2 are the upper or lower bounds of the variables.

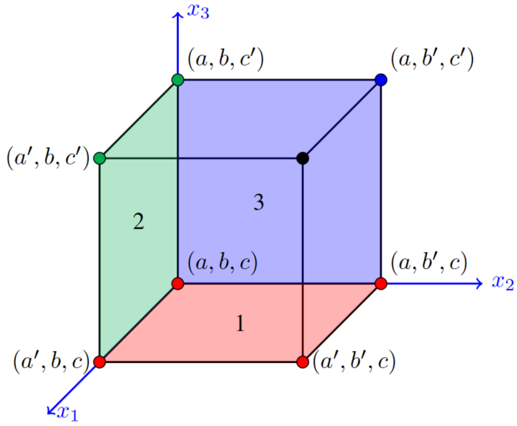
DG needs a huge computational budget for finding the interactions and build the groups, which rises quadratically with the number of decision variables. This is a problem when dealing with large-scale problems and too much FEs are used for the grouping process instead of the optimisation.

3.3.4. Differential Grouping 2

Differential Grouping 2 (DG2) [19] is an improvement of DG and reduces the number of used FEs. The main idea is to reuse previously evaluated FEs. In contrast to normal DG, DG2 only uses approximately half of the budget and determines not only the final groups but also the interactions between any two variables.

DG and DG2 only output the final groups, but DG2 computes a $n \times n$ interaction matrix which contains information about the interaction of any two of the n variables first. From this matrix the final groups are derived.

Algorithm 4 shows the pseudocode of DG2. The idea is to save and reuse previously evaluated solutions. The algorithm discovers the whole interaction matrix and do not *miss* any connections like normal DG. Figure 3.2 and the corresponding formulas visualise the detection of the interactions and help to understand the pseudocode. The example of the figure contains the objective function f with the three decision variables x_1, x_2 and x_3. These decision variables can take the corresponding values a, b and c, or the respective altered



$$x_1 \leftrightarrow x_2 : \Delta^{(1)} = f(a', b, c) - f(a, b, c), \Delta^{(2)} = f(a', b', c) - f(a, b', c)$$
$$x_1 \leftrightarrow x_3 : \Delta^{(1)} = f(a', b, c) - f(a, b, c), \Delta^{(2)} = f(a', b, c') - f(a, b, c')$$
$$x_2 \leftrightarrow x_3 : \Delta^{(1)} = f(a, b', c) - f(a, b, c), \Delta^{(2)} = f(a, b', c') - f(a, b, c')$$

Figure 3.2.: Visualisation of the interaction detection of DG2 directly taken from [19]

values values a', b' and c'. The goal is to detect the interactions between the three variables. To check the interaction of x_1 and x_2 the four red solutions have to be evaluated to compute Δ_1 and Δ_2, this is similar to DG, see Equation 3.2. To check the next interaction between x_1 and x_3 the two evaluations $f(a', b, c)$ and $f(a, b, c)$ can be reused. Considering the displayed cube the first interaction check computes all 4 red solutions, the red surface. The second one tries to compute the four solutions which spans the green surface. The two solutions which occur in both surfaces do not have to be evaluated again and can be reused. For the last interaction check only one solution has to be evaluated.

The structure of DG2 in Algorithm 4 is described in the following. It takes the actual population *Pop* and a parameter e similar as DG. The result of the algorithm is the interaction matrix λ. Line 1 to 7 initialises necessary containers to store several things. n is the number of decision variables, the start point f_{base} is initialised with the lower bounds of all variables ($f(a, b, c)$ in the preceding example). The vector $f_{n\times1}$ is used to save all objective values of solutions which differ only in *one* dimension from f_{base}. These would be the solutions $f(a', b, c)$, $f(a, b', c)$ and $f(a, b, c')$. f_i is the objective value of the lower bound of all variables but x_i being the midpoint of the possible values.

Algorithm 4: Differential Grouping 2 from cite

Input: $Population, e$

Result: λ

$n \leftarrow Pop.N$;
$\lambda \leftarrow zeros(n, n)$;
$F_{n\times n} \leftarrow NaNs(n, n)$;
$f_{n\times 1} \leftarrow NaNs(n, 1)$;
$x^1 \leftarrow lowBounds()$;
$f_{base} \leftarrow x^1$;
$m = 1/2(lowBounds() + upBounds())$;
for $i = 1 : (n-1)$ **do**
    if $isNaN(f_i)$ **then**
        $x^2 \leftarrow x^1$; $x^2_i \leftarrow m(i)$; $f_i \leftarrow obj(x^2)$;
    end
    for $j = i+1 : n$ **do**
        if $isNaN(f_j)$ **then**
            $x^3 \leftarrow x^1$; $x^3_j \leftarrow m(j)$; $f_j \leftarrow obj(x^3)$;
        end
        $x^4 \leftarrow x^1$; $x^4_i \leftarrow m(i)$; $x^4_j \leftarrow m(j)$;
        $F_{ij} \leftarrow obj(x^4)$;
        $\Delta_1 \leftarrow f_i - f_{base}$;
        $\Delta_2 \leftarrow F_{ij} - f_j$;
        $\lambda_{ij} \leftarrow |\Delta_1 - \Delta_2| > e$;
    end
end

The matrix $F_{n\times n}$ stores all objective values of solutions which differ in *two* dimensions from f_{base}. This would be the remaining three solutions from the example. The values of $f_{n\times 1}$ and $F_{n\times n}$ are evaluated one time and reused. The final groups are created by adding a variable to a group if it has an interaction with at least one variable of this group.

There are two major differences which lead to fewer FEs than normal DG. The first is that the base point f_{base} is the same in every interaction check, and it is not randomly generated but arbitrary set to the lower bounds of all variables. The second is the reuse of previously created solutions by varying them to a fixed value, the upper bound of every variable. DG2 uses $n(n+1)/2+1$ FEs to compute the whole interaction matrix. Even for high numbers of dimensions the number of used FEs seems passable.

3.4. Multi-Objective Grouping Methods

Two state-of-the-art approaches for grouping in the multi-objective case are described in the following. The two GMs are taken from the previously described MOEA/DVA and LMEA.

3.4.1. MOEA/DVA Grouping: Interdependence Analysis

The grouping of MOEA/DVA is based on the definition mentioned in Section 2.3. It is called Interdependence Analysis (IA) [12] and takes the actual population *Pop* and parameter NIA, see Algorithm 5. NIA indicates the number of interaction checks of two variables. In contrast to the single-objective methods DG and DG2, IA is probabilistic and not deterministic, because of this multiple interaction checks are executed with random values.

Algorithm 6 shows the general structure of IA. $DV = |n|$ contains all distance variables, this interaction grouping is only executed on these variables. Every variable i is checked against each other variable j in the two for loops in line 4 and 5. The interaction check is executed NIA times. For two variables i and j, Equation 2.3 must be fulfilled. An individual is randomly selected from the population and set as first reference point. The other three solutions are created with random values for a_2 and b_1, see Equation 2.3 and Figure 2.3. With these four solutions the interaction is checked. An interaction is counted

Algorithm 5: Structure of the interaction grouping Interdependence Analysis from MOEA/DVA [12]

Input: Pop, DV, NIA
Result: $[Pop, DVSet]$

```
n ← |DV|;
DVSet ← {};
interactions ← {};
for i = 1 : n − 1 do
    for j = i + 1 : n do
        checks ← 0;
        while checks < NIA do
            if DV(i) interacts with DV(j) on any objective then
                interactions ← interactions ∪ (i, j);
                break;
            end
            checks + +;
        end
    end
end
Pop ← updatePopulation();
DVSet ← maximalConnectedSubraphs(interactions);
```

if it exists in *any* objective function. The groups are created analogue to DG and DG2, a variable is added to a group if it has a connection to at least one variable of that group. The population is updated with the generated solutions during the interaction checks.

3.4.2. LMEA Grouping: Correlation Analysis

Algorithm 6: The interaction grouping of LMEA Correlation Analysis from [34]

```
Input: DV, nCor
Result: DVSet
sepGroups ← {};
DVSet ← {};
for v ∈ DV do
    actGroup ← {v};
    for group ∈ DV do
        for u ∈ group do
            for i = 1 : nCor do
                if v interacts with u then
                    flag ← true;
                    actGroup ← actGroup ∪ group;
                end
            end
        end
        if flag then
            break;
        end
    end
    if |actGroup| = 1 then
        DVSet ← DVSet ∪ actGroup;
    else
        DVSet = DVSet/{group};
        DVSet = DVSet ∪ {actGroup};
    end
end
```

The interaction grouping of LMEA is also based on the definition mentioned in 2.3 and quite similar to the one from MOEA/DVA. It is also a probabilistic approach called Correlation Analysis (CA) and takes the parameter $nCor$ as the number of interaction checks. Algorithm 6 shows the structure of CA. The interaction check itself (see line 7 to 12) is basically the same as in IA. It is also based on random values for Equation 2.3. Variables are regarded to interact if there exist an interaction in *any* of the objectives. Three function evaluations are used for one interaction check.

The difference lies in the scheme of the interaction checks. The idea is to iterate over all distance variables and add them to a group if they interact with at least one variable of this group. We iterate over all variables $v \in DV$ and check the interaction not against any other variable but against any other created group, see line 5. The interaction of the actual variable v is checked against every single variable u of the other *group*, see line 6. The interactions of variables v and u is checked $nCor$ times. If an interaction was detected, a new group consisting of v and the variables of *group* are created and added to the final groups $DVSet$, line 10 and 20/21. When no interactions were detected the variable v is added to $DVSet$ as single-group. The algorithm returns the groups as $DVSet$.

The problem with the multi-objective methods IA and CA is that they can miss some of the interactions because they are probabilistic. The problems are addressed more detailed in Subsection 4.1.

3.5. Classification of Grouping Methods

The previously described interaction grouping methods have different properties and can be classified as shown in Figure 3.3. Interaction grouping methods can be divided into three different categories: Trivial, Dynamic and Intelligent. Trivial groupings do not use any information from the problem to create the groups and fix these during the whole optimisation. Linear grouping which was used by WOF belongs to this category.

The second category is the dynamic grouping methods without an *intelligent* analysis of the variables. Random and Delta grouping fall into this category. They both change the grouping at the beginning of a every new generation. They are not considered as intelligent despite the fact that delta grouping uses more information than the trivial methods, but it does not analyse the interactions between the variables which is considered as *intelligent* in this context.

The third category contains the *intelligent* methods which analyse the interactions between the variables and use this information to create the groups. Both methods from MOEA/DVA and LMEA and the two versions of differential grouping DG and DG2 belong to this category.

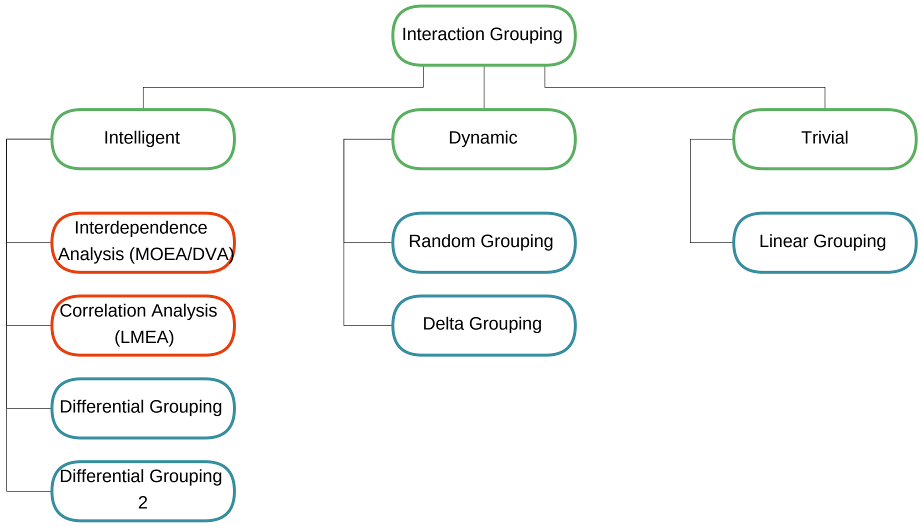


Figure 3.3.: Classification of interaction Grouping approaches

4. Concept

The first goal of the work is to develop strategies which transfer single-objective grouping methods to the multi-objective case, see Section 1.1. In the previous chapter several single and multi-objective grouping methods were described. Single objective optimisation was a research topic before the multi-objective case was considered, and therefore GMs are studied and developed for a longer time. There has not been very much work in grouping methods in the multi-objective case, algorithms simply use own methods like MOEA/DVA and LMEA. These two do not consider well known single-objective grouping methods like DG or DG2. Some information (like the formula of interacting variables) was used by the two algorithms, but only inefficient, probabilistic and expensive methods were used. Also only one objective is sufficient and the variables are put together in one group. In this thesis we have more objectives and just considering this one option might not be the best result.

This work tries to formalise groupings better and make them comparable to study different aspects of the grouping process in the multi-objective case. To do so, strategies are developed to transfer intelligent single-objective methods to the multi-objective case. These methods are called Transfer Strategy (TS). Dynamic methods like random and delta grouping are also applied to multiple objectives, but they do not consider variable interactions and therefore do not need the TS. In addition some trivial methods are developed.

First, the problems of the actual existing approaches are pointed out in Section 4.1. After that the core of this work, the TS are described in Section 4.2. The next Section 4.3 deals with other transferred dynamic and trivial approaches. The existing GMs and the new proposed ones are classified in Section 3.5 to get a better overview about what methods were proposed. In the last Section 4.5 *reference algorithms* are described which are used to evaluate the different GMs.

4.1. Problems of existing Methods

The problem with *overlapping components* or *shared variables* was described in some previous works which deal with grouping of variables [11, 19]. It describes a problem which occurs during the grouping of the variables. The *aim* of an interaction grouping is to form groups of variables that interact with each other, but how exactly the interactions in a single group has to look like, is not defined.

Consider an example interaction structure in Figure 4.1(a), variables are represented as vertices and the interaction between them by edges. The interactions are given but the question is what are the optimal groups. One option is to put all variables in one group which have a connection to a variable in that group. That would lead to one group containing all variables $\{x_1, x_2, x_3, x_4\}$. But x_3 and x_4 do not interact with each other. When we consider only groups where all variables have to interact with each other the two possible groups are $\{x_1, x_2, x_3\}\{x_4\}$ or $\{x_1, x_2, x_4\}\{x_3\}$. The variables which could occur in both groups are defined as *shared* variables. In the example this would be the orange marked x_1 and x_2.

Another problem is the schema of checking variable interactions in DG. A variable is only checked against the other variable which were not included in an group already. LMEA on the other hand checks a variable only against already existent variable groups. Both of the approaches can miss some variable interactions and lead to different groupings.

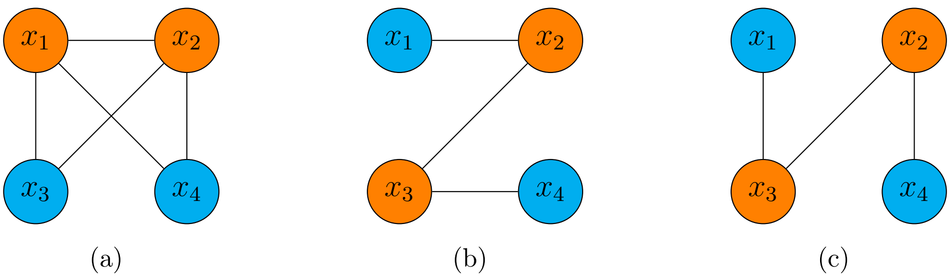


Figure 4.1.: Three example interaction graphs, vertices represent variables and edges interactions between them. Shared variables are marked orange.

In Figure 4.1(b) normal DG would not recognize the connection between x_2 and x_3, when checking the dimensions in lexicographical order. The dimension

x_1 would be checked against x_2, x_3 and x_4 respectively. x_1 and x_2 interact and will be put together in a group and all variables which are grouped together will be erased from the variable set which have to be checked, see 3 line 11. This means that in the next iteration we will not start with dimension x_2, because it is contained in a group already and erased from *dims*, we will start with x_3. x_3 will be just checked against x_4 and put together in a group. So the final two groups are $\{x_1, x_2\}$ and $\{x_3, x_4\}$. This leads to interaction between the subcomponents, because x_2 and x_3 interact with each other. The *right* solution would be just one group containing all variables. Where *right* in this context means that one connection between a variable is sufficient that they are put in the same group.

The checking scheme CA of LMEA is also analysed. In this one the actual variable is not checked against all others which have no group, it is checked only against the variables which are already contained in groups. This means that for the second interaction graph, Figure 4.1(c), in the first iteration x_1 would not be checked (because there are no groups yet) and will be put in a single group. x_2 is checked against x_1 and is added to the group. x_3 and x_4 will also be added. All connections are recognised and a group with all four variable is created. For the graph depicted in 4.1(c) the connection between x_2 and x_3 is missed. In the first steps x_1 and x_2 will be put in single groups because they are checked first and have no interaction, so we have the groups $\{x_1\}$ and $\{x_2\}$. x_3 is then checked against the first group containing x_1 and is added because they have interaction. The last variable x_4 will be added to $\{x_2\}$ which leads to the final groups $\{x_1, x_3\}$ and $\{x_2, x_4\}$. The interaction $x_2 - x_3$ is omitted.

The approaches have some problems and drawbacks. The proposed methods try to tackle this aspects.

4.2. Transfer Strategies (TS)

In this section the TS are introduced which are used to transfer single-objective grouping methods to the multi-objective case. Intelligent single-objective GMs like DG or DG2 use fitness values of the objective function to decide which variables are grouped together. The problem is that we have several objectives and variables might interact in one objective but not in the other. Single-objective methods can only make assumptions of the interaction of two variables in one *single* objective. The approaches have to be altered and could not be applied directly like the random or delta grouping approaches. One way of dealing with this problem would be to base the decision only on the first objective as done in [38]. With this approach valuable information about the interaction among other objectives is neglected. The previous described grouping methods from MOEA/DVA and LMEA consider two variables as interacting if they have interactions in any of the objective functions. Variables could be put in a group because they interact in only one objective but not in all others.

Another problem besides the multiple objectives is the general composition of variable groups. Recent multi-objective approaches from MOEA/DVA or LMEA put a variable in a group if it interacts with *any* of the variable in this group. This might lead to groups with very sparse interactions. Other possible strategies to fulfil this task should be considered. The computational costs to find the groups are very high with the multi-objective methods, because they are probabilistic approaches and therefore have to check an interaction multiple times. Single-objective methods like DG or DG2 on the other hand need a lot less FEs and are deterministic.

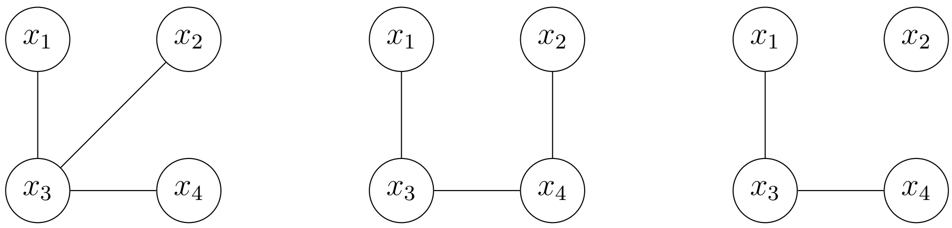
Another drawback of CA and IA is that they build the groups during the interaction check of the variables. The final groups depend not only on the interactions but on the order in which the interaction is checked. To tackle this problems, the check of interactions and the creation of the groups are separated from each other like in DG2.

The developed TS try to tackle these problems, they can be divided into two consecutive steps. The first one describes how to deal with the multiple objectives, more precise when the variables are considered to interact with each other. This part is called Transfer Strategy Objective (TSO). The second one describes different approaches to form groups from the found interactions, it is called Transfer Strategy Variable (TSV). Possible variants and combinations of these two steps are described.

4.2.1. Transfer Strategies for Objectives (TSO)

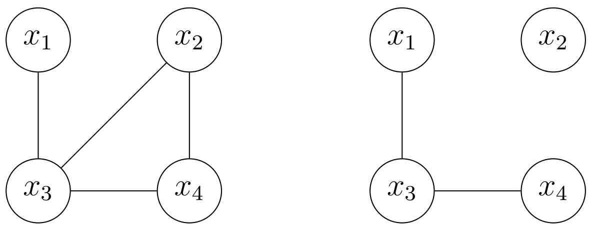
A mentioned above, single-objective grouping methods have only to deal with one objective function, but in MOPs there more. In most of the actual algorithms, a variable interacts if there is an interaction among any of the objective functions, for example in MOEA/DVA or LMEA. Another idea is to assure that the interaction occurs in every objective function to count as an interaction between two variables for the whole problem.

Figure 4.2 shows an example MOP with three objective functions f_m with $m \in \{1, 2, 3\}$ and four decision variables x_i with $i \in \{1, 2, 3, 4\}$. Vertices represent the four variables and and egde between two shows that they interact in this objective function. For example in the third objective function the variable x_3 interacts with x_1 and x_4.



(a) $f1$ (b) $f2$ (c) $f3$

Figure 4.2.: Interaction graphs of three objective functions. The Edges show interaction between two decision variables



(a) TSO_{single} (b) TSO_{all}

Figure 4.3.: Combined Interaction Graphs of the three functions from Figure 4.2 for the two variants of TSO

The goal of the TSO is to combine this conflicting information so that interactions are clear for the whole MOP and not only one objective function.

Two different strategies are proposed to combine the information: TSO_{single} and TSO_{all}. TSO_{single} regards two variables x_i and x_j as interacting for the whole MOP if the interaction occurs in *any* of the objective functions. This means that a *single* interaction in one objective is sufficient for this approach. The other method TSO_{all} regards two variables as interacting if they have interactions among *all* objective functions.

With these two methods the graphs of the problem can be combined to an interaction graph for the whole MOP which contains the interactions according to the respective strategy.

TSO_{single} leads to the *combined interaction graph* shown in Figure 4.3(a). A *single* egde (interaction) between two vertices (variables) x_i and x_j in any of the objective functions is sufficient to be be added to the combined graph. The interactions $x_1 - x_3$, $x_2 - x_3$ and $x_3 - x_4$ are added because of the the first objective. $x_2 - x_4$ is added because of f_2. The two connections of the third objective are already included. This leads to the combined interaction graph shown in 4.3(a).

TSO_{all} means that an egde has to be present in *all* objective functions to count for the combined graph. Only the interactions $x_1 - x_3$ and $x_3 - x_4$ are present in all objective functions, the resulting graph is depicted in Figure 4.3(b).

4.2.2. Transfer Strategies for Variables (TSV)

The TSV is the second step of the TS. TSV describes how the groups are formed based on the interaction of the variables. There are different ways to combine the interacting variables. As input the combined interaction graph from the preceding TSO is used, based on these connections the final variable groups are created. Similar to the previous approaches the two versions of the TSO are called TSV_{single} and TSV_{all}. TSV_{single} is the recently used standard method and was used for example in [3, 12, 13, 34]. A *single* connection between a variable an a group is sufficient and the variable is added to that group. More precise, a variable x_i is added to a group of variables if an edge exists between the x_i and any variable in the group considering the combined interaction graph. Each group of variables consists of a maximally connected subgraph of the combined interaction graph. For the graph in Figure 4.3(a) all variables would end up in one group because they are all connected by at least one edge. The graph created by TSO_{all}, see Figure 4.3(b), would lead to a group of the three connected variables $\{x_1, x_3, x_4\}$ and a group with the single variable x_2.

TSV_{all} on the other hand considers only Maximal Complete Subgraphs (MCS). An MCS is a fully connected subgraph to which no other vertex can be added so that the new subgraph remains complete. Since every variable can only be added to one group, the decomposition of the combined interaction graph into MCS is not unique, there exist overlapping MCS. This problem was mentioned as *shared variable problem* in [11, 19]. Shared variables are members of different MCS at the same time, they are *shared* by multiple MCS. Since one variable can only occur in one group, the decision which of the MCS is regarded as a final group is not well defined. In Figure 4.3(a) x_3 is a shared variable. It occurs in the MCS with the vertices x_1 and x_3 and the one containing x_2, x_3 and x_4. The two possible groupings are $\{x_1, x_3\}\{x_2, x_4\}$ and $\{x_1\}\{x_2, x_3, x_4\}$. The decision which is the final group is not well defined. In the implementation in this thesis the group which is found first by the algorithm is chosen. Because the variable are presented in natural or lexicographical order, the solution is deterministic but not unique. The combined interaction graph created by TSO_{single} 4.3(a) results in the first possible grouping $\{x_1, x_3\}\{x_2, x_4\}$. In the graph from TSO_{all} x_3 is again the a shared variable, it is present in the MCS with x_1, x_3 and x_3, x_4. TSV_{all} lead to the grouping $\{x_1, x_3\}\{x_2\}\{x_4\}$. The TSV is deterministic but it finds only one solution when there are maybe more.

4.2.3. Combinations of TSO and TSV

As mentioned above, TS consist of two consecutive steps TSO and TSV. The first one tackles the issue of multiple objectives, the second one creates the final groupings depending on the variable interactions. For both steps two alternatives were presented, the combinations of both lead to four different Transfer Strategies:

- OS+VS = $\{O_{single}, V_{single}\}$
- OS+VA = $\{O_{single}, V_{all}\}$
- OA+VS = $\{O_{all}, V_{single}\}$
- OA+VA = $\{O_{all}, V_{all}\}$

The TSO creates combined interaction graphs (Figure 4.3) which will then be used as input for the TSV. For the TSV again two alternatives TSV_{single} (VS) and TSV_{all} (VA) are used to create the following final variable groups:

- OS+VS: $\{x_1, x_2, x_3, x_4\}$
- OS+VA: $\{x_1, x_3\}\{x_2, x_4\}$ or $\{x_1\}\{x_2, x_3, x_4\}$
- OA+VS: $\{x_1, x_3, x_4\}\{x_2\}$
- OA+VA: $\{x_1, x_3\}\{x_2\}\{x_4\}$ or $\{x_3, x_4\}\{x_1\}\{x_2\}$

4.2.4. Relationship to existing Methods

When we compare the grouping mechanisms of MOEA/DVA and LMEA with the four TS a relation between them could be noticed. Both of them check Equation 2.3 with several randomly chosen input values to check if a variable has interaction with another. An interaction among a *single* objective is sufficient, this corresponds to the proposed TSO_{single}. To create the groups, maximally connected subgraphs are considered by both methods. A *single* connection is sufficient to put two variables in one group, a MCS is not necessary, this behaviour corresponds to the TSV TSV_{single}. So OS+VS can be regarded as the recently used standard approach, but the two methods do not exactly the same as OS+VS. They use a probabilistic approach and whether a connection is found depends on how often the interaction check is executed with different input values. OS+VS on the other hand is deterministic.

4.3. Transferred Grouping Methods

The approaches or algorithms which were transferred from single- to multi-objective problems are described here. The differences between the single- and multi-objective versions are described. First, the trivial methods Random and Delta grouping are explained, after that the approaches for transferring DG2 to the multi-objective case are mentioned.

4.3.1. Trivial Grouping Methods

The trivial grouping strategies are the simplest grouping strategies one can think of. 1-Group (G1) takes all decision variables and creates one big group which includes all of them. The variables are not divided into different groups, G1 represents the *normal* EA which does not use any grouping method. Comparisons to G1 can show if a grouping is better or worse than the normal approach without grouping. G1 is one extreme grouping with just one group, N-Group (GN) is the other extreme: every decision variables is put in a single group. For n decision variables GN will lead to n groups each containing one variable.

During the early stages of this thesis, some of these trivial groupings lead to promising results, because of this they are included in the evaluation.

4.3.2. Multi-objective Random & Delta Grouping

The only difference between Multi-objective Random Grouping (MRAND) and Multi-objective Delta Grouping (MDELTA) and their single-objective version is that the new method has to deal with more objective functions, but this does not affect the grouping because it is not based on the objective functions. For MRAND the grouping is *randomly* and therefore it is irrelevant how many objectives we consider. MDELTA also does not depend on the number of objectives. Only the values of the variables are analysed to find the groups, the objective function(s) are not necessary for this.

The disadvantage of these approaches is that they need the group size or the number of groups as a fixed parameter. For different problems and number of decision variables a fixed subcomponent size can lead to different groupings. To check which method performs best, a test with various parameters is done, before comparing MRAND and MDELTA to the other algorithms. Eight different versions of the two algorithms are described below.

One approach is to fix the group size of the algorithm, defined as SX with a group size of X. Four different group sizes are considered: $X = \{2, 5, 20, 50\}$. These approach leads to different numbers of subcomponents, depending on the number of decision variables, but fixes the groups sizes to the specified number. The second approach defines the *number* of subcomponents, and the

Table 4.1.: Number of groups and group size combinations of all SX and NX versions of MRAND and MDELTA

	S2	S5	S20	S50	N2	N5	N20	N50
100	50 x 2	20 x 5	5 x 20	2 x 50	2 x 50	5 x 20	20 x 5	50 x 2
200	100 x 2	40 x 5	10 x 20	4 x 50	2 x 100	5 x 40	20 x 10	50 x 4
500	250 x 2	100 x 5	25 x 20	10 x 50	2 x 250	5 x 100	20 x 25	50 x 10

group size of these will change with varying number of decision variables. It is denoted as NX with X being the number of subcomponent. The values for X are the same as in the SX approach: $X = \{2, 5, 20, 50\}$. This leads to eight overall versions of both of the two algorithms:

- SX = S2, S5, S20, S50
- NX = N2, N5, N20, N50

Table 4.1 shows for the three considered number of decision variables 100, 200 and 500 the number and size of the groups generated by the eight versions. The first value is number of groups, the second the groupsize. For example, the version S50 creates for 200 decision variables 4 groups with each a size of 50 variables.

4.3.3. Multi-objective Differential Grouping 2

Multi-objective Differential Grouping 2 (MDG2) is a multi-objective version of the DG2 described in Subsection 3.3.4. The depicted Algorithm 4 of the single-objective version was used as basis. The transfer to the multi-objective case is done by the described TS. MDG2 just executes the normal DG2 on all objective functions which leads to m different interaction matrices (or graphs), one for every objective. With this result the previously described TS are used to create a proper grouping of the variables. The result of MDG2 is the input for the described TS.

4.4. Classification of the Grouping Methods

Figure 4.4 shows the previous classification graph with the added approaches. The orange shaded nodes contain the new methods which were proposed and will be analysed in this work.

The new methods are namely G1 and GN which fall into the trivial category because they do not use information from the variables or the problem to obtain the grouping. The next ones are the two dynamic multi-objective versions of Random and Delta Grouping namely MRAND and MDELTA. Purple boxes illustrate in which versions the algorithms are available. The two approaches come with four SX and four NX with $X \in \{2, 5, 20, 50\}$ versions which try to find good group sizes or numbers of groups respectively. A multi-objective version of the DG2 is proposed and called MDG2. The four TS are used to create the final groups.

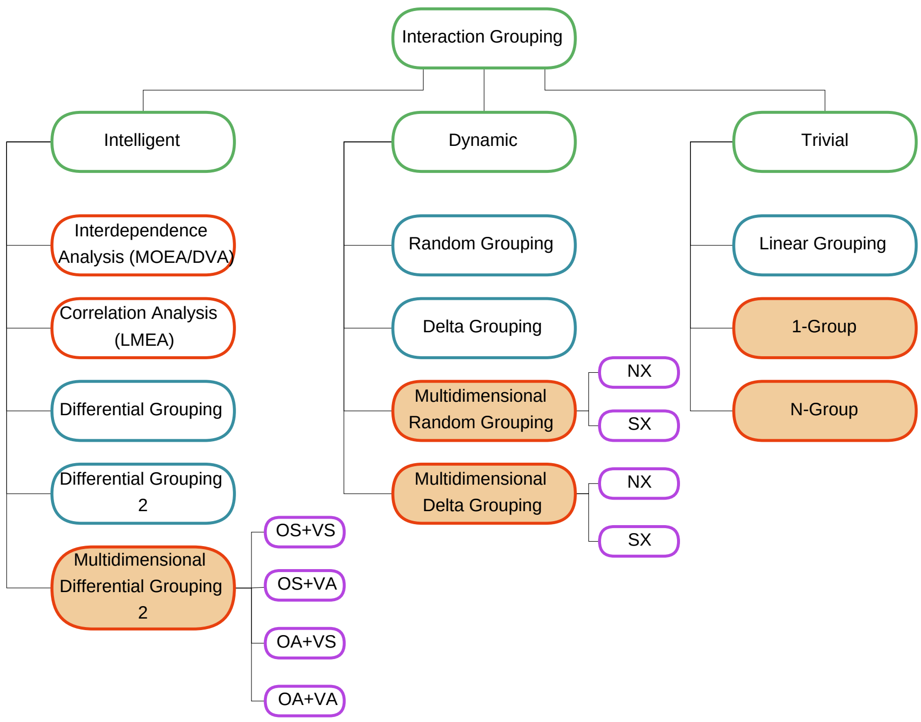


Figure 4.4.: Classification of existing and new Interaction Grouping approches

4.5. Reference Algorithms

To test how the grouping methods perform, empirical and theoretic tests are carried out. To check the impact of the different grouping methods on the results, two *reference algorithms* were used. The reason for this is that the quality of the grouping method should be measured, and not the optimisation performance of the used EA. High quality GMs should work not only with one algorithm, they should be used by several different EAs. Because of this the GMs are tested by inserting them in these reference algorithms which use different optimisation approaches for the given groups. GMs should work together with both of the described reference algorithms, only if the performance of both is high for one grouping, it should be general.

The PDG from LMEA, namely VC performs better than the corresponding CVA from MOEA/DVA [34]. Because of this and the fact that the analysis of the PDG is not in the scope of this work, VC is used in both reference algorithms.

Algorithm 7: Reference algorithm 1 (Ref_1) adapted LMEA

Input: $nSel, nPer, nCor, ...$
Result: Pop

$Pop \leftarrow initializePop()$;
$[PV, DV] \leftarrow VariableClustering(nSel, nPer)$;
$DVSet \leftarrow groupingMethod(DV, nCor, ...)$;
while *FEs available* **do**
    for $i = 1 : 10$ **do**
        $Population \leftarrow ConvergenceOptimisation(Pop, DVSet)$;
        $terminateIfNoFEsAvailable()$;
    end
    for $i = 1 : M$ **do**
        $Population \leftarrow DistributionOptimisation(Pop, PV)$;
        $terminateIfNoFEsAvailable()$;
    end
end

The first reference algorithm is the standard LMEA with some minor changes, the original LMEA was described in Subsection 3.1.2. Algorithm 7 shows the general structure. The *groupingMethod* is replaced by the actual tested one. In the used implementation the algorithms have to assure themselves that they do not use more FEs than allowed. After each execution of the convergence or distribution optimisation the number of remaining FEs is checked. The original LMEA implementation just checks if the maximum FEs are reached in the condition of the while loop. Inside of this loop the number of used FEs is not checked. Especially when dealing with fully separable functions, the convergence optimisation is very expensive. In those cases LMEA could use more FEs than allowed, number can grow up to approximately 1/3 more than allowed. The comparison against an algorithm which uses only the allowed FEs is maybe not fair. When $1,000,000$ FEs are allowed LMEA can use in some cases up to $1,300,000$. For a fair comparison against other algorithms it should be assured that all of them have the same computational budget. The functions $terminateIfNoFEsAvailable()$ checks the remaining FEs and terminates the algorithm if the maximal number was reached.

The second reference algorithm Ref_2 is a combination of parts of MOEA/DVA and LMEA. It is depicted in Algorithm 8. MOEA/DVA just fixes the position variables at the beginning. This static approach leads to a lack of distribution in the final population, because of this the distribution optimisation from LMEA is executed (line 10) after the subcomponent optimiser from MOEA/DVA (line7) which only optimises the distance variables.

Another reason why the original MOEA/DVA was not used is the utility function which was described in Subsection 3.1.1. It checks after every generation if the subcomponent optimisation of the found groups is still effective. When the solution quality does not increase anymore, a fallback EA is used instead of the group based optimisation. This idea is good for increasing the overall performance but the goal of the reference algorithms is to test the quality of the given GMs and not use a different optimisation approach which does not depend on the groups. It is possible that MOEA/DVA switches to the fallback EA after only one generation. The results are maybe better than the optimisation of the groups, but does not contain the wanted information which is the quality of the groups. Because of this the utility function was not used in Ref_2.

Algorithm 8: Reference algorithm 2 (Ref_2): combination of parts of MOEA/DVA and LMEA

```
Input: nSel, nPer, nCor, ...
Result: Pop
Pop ← initializePop();
[PV, DV] ← VariableClustering(nSel, nPer);
DVSet ← groupingMethod(DV, nCor, ...);
Neighbour ← calcNeighboursOfEachIndividual(Pop, PV);
while FEs available do
    for i = length(DVSet) do
        Pop ← SubcomponentOptimizer(Pop, Neighbour, DVSeti);
        terminateIfNoFEsAvailable();
    end
    Population ← DistributionOptimization(Pop, PV);
    terminateIfNoFEsAvailable();
end
```

5. Evaluation

The obtained results of the proposed grouping methods are described and analysed in this chapter. The evaluation can be divided in two categories: the theoretical (mathematical) and empirical analysis. Most often EAs are compared against each other based on an empirical analysis of particular designed benchmark problems. Additional to this a theoretical analysis is carried out in this work. The reason for this is the recently proposed test suite LSMOP. This test suite covers the researched interaction of variables directly. The interaction of variables is not a side effect like in other problem suites, it is wanted behaviour and can be manipulated with a predefined parameter.

First the goals of the evaluation are mentioned, they describe what we want to find out with the analysis and the experiments. After that the theoretical analysis is carried out in Section 5.2. The LSMOP test suite is analysed and the grouping results of MDG2 and the proposed TS is compared with existing approaches like CA from LMEA. After that in Section 5.4 the performance of the GMs are compared on the basis of large-scale benchmark problems. The last point of the evaluation is the Summary in Section 5.5. We take a look back at the obtained results and the goals of the work and evaluation.

5.1. Goals of the Evaluation

The goals of the work were mentioned at the beginning of this thesis:

1. Develop strategies which transfer single-objective grouping methods to the multi-objective case
2. Examine the capabilities of existing and new approaches theoretically
3. Evaluate the performance of state-of-the-art grouping methods using the proposed Transfer Strategies

The Transfer Strategies for grouping methods from the single- to the multi-objective case were developed in Chapter 4. The evaluation tackles the two remaining goals. To examine the capabilities of the existing and new approaches theoretically, a mathematical analysis is carried out, which is the first part of this evaluation. The empirical experiments tackle the third goal and compare the performance of the approaches.

The goal of the evaluation is not to compare the performance of the different algorithms against each other. The goal is to compare the proposed grouping methods with each other. The quality of the groups will be pointed out in the results, not the optimisation quality of the algorithms. We consider two reference algorithms which use different techniques to optimise the found groups to assure that the grouping of the variables is *good* in general and not only for a special algorithm. They were described in Section 4.5.

The theoretical and empirical test settings are chosen to fulfil the goal of this work. To do so, some scientific questions are tried to answer with the evaluation. The concrete questions can be abbreviated from the second and third goal of this work and are as follows:

1. Do the GMs find the theoretical correct groups?
2. Are the results constant for different optimisation algorithms, benchmark problems and number of variables?
3. How do the transferred GMs perform against existing multi-objective ones?
4. Are complex approaches better than simple ones?
5. Are theoretical and empirical results congruent?
6. What is more important, the grouping method itself or the how the groups are used in the optimisation process?

In the summary of the results in Section 5.5 the found answers to this questions are given.

5.2. Theoretical Analysis

In this section the GMs are analysed on a theoretical basis. It is not the performance of the algorithms that is considered but the grouping result directly to check if the GMs find the correct interacting variables. The results can be compared to the empirical ones.

To test this, the following three steps are necessary. First the mathematical basis of the LSMOP test suite is analysed with respect to the interacting variables in Subsection 5.2.1. The interacting variables are based on parameters for the LSMOP problems. The theoretical correct interactions are described in Subsection 5.2.2 for some arbitrary parameters. The number of decision variables and objective functions is kept small to give a clear visualisation of the results. In Subsection 5.2.3 the results of the MDG2 approach are analysed. The obtained interactions are compared against the theoretical correct ones. The found interactions are used as the starting point for the TS which are used to create the final groups in the last Subsection 5.2.4. The grouping results of MDG2 with the four TS are compared against the state-of-the-art method CA from LMEA.

5.2.1. Theoretic Grouping Description LSMOP

The Large-Scale Multi and Many-objective Test Problem (LSMOP) problems have specific characteristics which are mentioned in [4]. They were designed to specify subcomponents and therefore they fit perfectly to this evaluation. With information from the paper the variable groups can be determined. To better understand the groupings the theoretical basis and their instances are described in the following. Only the parts which are important for the grouping are mentioned, the other ones are omitted. All formulas were taken directly from the description of LSMOP [4].

Figure 5.1 shows the general structure of the LSMOP problems. It consists of four parts but the instances of the *linkage functions* $L(x^s)$ and the *shape functions* $H(x^f)$ are not described further because they do not affect the variable interactions. The instances of the *landscape functions* and the *correlation matrix* are described. With the combinations of the instances of these four parts, the nine final LSMOP problem instances were created [4].

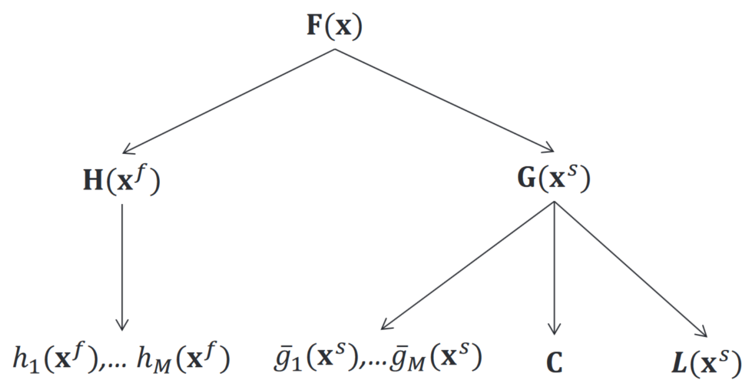


Figure 5.1.: general structure of all LSMOP problems. $H(x^f)$ is the *shape Matrix*, $G(x^s)$ the *landscape matrix* with the *landscape functions* $g_1(x^s), ...g_m(x^s)$, *correlation matrix* C and *linkage function* $L(x^s)$.The Figure was taken from [4]

The uniform design formulation of the problems is described as follows:

$$F(x) = H(x^f)(I + G(x^s)) \tag{5.1}$$

Where the objective functions are given as $F(x) = [f_1(x), ..., f_m(x)]$. $H(x) = [h_1(x^f), ..., h_m(x^f)]$ defines the shape of the Pareto-front and is called *shape matrix*. I is the identity matrix and $G(x) = diagonal(g(x^s), ..., g(x^s))$ is a diagonal matrix which defines the fitness landscape and is called *landscape matrix*. The *landscape matrix* and the *landscape functions* f_1 to f_m are most important for the grouping analysis. The concrete objective functions are:

$$f(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + g_1(x^s)) \\ f_2(x) = h_2(x^f)(1 + g_2(x^s)) \\ ... \\ f_m(x) = h_m(x^f)(1 + g_m(x^s)) \end{cases} \tag{5.2}$$

with the overall decision vector $\vec{x}$, the position variables $\vec{x}^f$ and the distance variables $\vec{x}^s$:

$$\vec{x} = (\vec{x}^f, \vec{x}^s) \tag{5.3}$$

The position variables are $x^f = (x_1, ..., x_{m-1})$ the distance variables are $x^s = (x_m, ..., x_n)$ with n overall decision variables. All nine LSMOP problems follow these structure and are created as a combination of predefined instances for $H(x^f)$ and $G(x^s)$. $G(x^s)$, which contains the *landscape functions*, is described detailed later in this subsection.

The distance variables are non-uniformly split into m *objective*-groups which are then further divided into nk subcomponents:

$$x^s = (x_1^s, ..., x_m^s) \tag{5.4}$$
$$x_i^s = (x_{i,1}^s, ..., x_{i,nk}^s) \tag{5.5}$$

The m groups are called *objective-groups*, as not to be confused with the subcomponents, which have the interactions that are considered in this work. The subcomponents describe the groups of interacting variables we want to find with the proposed grouping methods. Every distance variable has to occur once in one of the m objective-groups, see Equation 5.4. These m objective-groups consists of nk subcomponents, see Equation 5.5. To assure that the group sizes are equal for every independent run, a chaos based pseudo random number generator is used [4].

After these steps we have m overall objective-groups which consist each of nk subcomponents. But not every decision variable is correlated with every objective function. Matrix C describes the correlations between the m objective-groups and the m objective functions:

$$C(i,j) = \begin{pmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,m} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m,1} & c_{m,2} & \cdots & c_{m,m} \end{pmatrix} \tag{5.6}$$

A value of $C(i,j) = 1$ means that the objective function $f_i(x)$ is correlated with the decision variables in the objective-group x_j^s. A value of $C(i,j) = 0$ means that these two are not correlated, $C(i,j) = 1$ indicates correlation.

When x_j^s is not correlated to a *landscape function* g_i, the values of $g_i(x_j^s)$ will be ignored. More formally with the correlation matrix C and the *landscape functions* g_i, Equation 5.2 can be rewritten as:

$$f(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + \sum_{j=1}^{m} c_{1,j} \times g_1(x_j^s)) \\ ... \\ f_i(x) = h_i(x^f)(1 + \sum_{j=1}^{m} c_{i,j} \times g_i(x_j^s)) \\ ... \\ f_m(x) = h_m(x^f)(1 + \sum_{j=1}^{m} c_{m,j} \times g_m(x_j^s)) \end{cases} \tag{5.7}$$

$$p_n(d) = \sum_{\lfloor k=B^{n-1} \rfloor}^{B^n-1} log_B \left(1 + \frac{1}{k \cdot B + d}\right) \tag{5.8}$$

The generated landscape values of a objective-group x_j^s, $g_i(x_j^s)$ are only added to the overall objective value if and only if the objective i is correlated with the variable group j: $c_{i,j} = 1$. Otherwise the landscape value will not be considered, by multiplying it with $c_{i,j} = 0$.

Table 5.1.: Single-objective optimisation problems which were used to build the multi-objective problems for LSMOP [4]

Problem	Separability
η_1: Sphere function	Separable
η_2: Schwefel's problem	Non-Separable
η_3: Rosenbrock's function	Non-Separable
η_4: Rastrigin's function	Separable
η_5: Griewank's function	Non-Separable
η_6: Ackley's function	Separable

The *landscape functions* are described with a pool of six single-objective optimisation problems depicted in Table 5.1. In total there are three Separable and three Non-Separable functions. The LMSOP suite is scalable to any number of objectives: $g_i(x^s)$ with $1 < i < m$. The *landscape functions* are divided into two alternating groups:

$$\begin{cases} g^I(x_i^s) = \{g_{2k-1}(x_i^s)\} \\ g^{II}(x_i^s) = \{g_{2k}(x_i^s)\} \end{cases} \tag{5.9}$$

with $k = 1, ..., m/2$. Both $g^I(x_i^s)$ and $g^{II}(x_i^s)$ are assigned to one of the single-objective functions $\eta_1, ..., \eta_6$ of Table 5.1. Consider an example with $m = 4$, $g^I = \eta_2$ and $g^{II} = \eta_5$. $g_1(x^s)$ and $g_3(x^s)$ use $g^I(x_i^s) = \eta_2(x_i^s)$, $g_2(x^s)$ and $g_4(x^s)$ use $g^I(x_i^s) = \eta_5(x_i^s)$.

The *correlation matrix* has three instances: Separable, Overlapped and Full Correlations. The Separable Correlation matrix is just the identity matrix $C_1 = I$. Objective-group x_i^s is correlated with objective function f_i. For Overlapped Correlations all elements on the main diagonal are 1 (like I) and additional the right neighbours of the diagonal: $C_2(i,i) = 1$ and $C_2(i,i{+}1) = 1$. With C_2 an objective-group is connected to two objective functions. Full Correlations connect all variable groups with all objective functions. Here all elements are equal to 1: $C_3(i,j) = 1$.

Table 5.2.: Concrete instances of all nine LSMOP problems

Problem	g^I	g^{II}	C
LSMOP1:	η_1	η_1	C_1
LSMOP2:	η_5	η_2	C_1
LSMOP3:	η_4	η_3	C_1
LSMOP4:	η_6	η_5	C_1
LSMOP5:	η_1	η_1	C_2
LSMOP6:	η_3	η_2	C_2
LSMOP7:	η_6	η_3	C_2
LSMOP8:	η_5	η_1	C_2
LSMOP9:	η_1	η_6	C_3

The nine LSMOP instances are depicted in Table 5.2. The used *landscape functions* $H(x^f)$ and *linkage functions* $L(x)$ were omitted because they are not relevant in this context. The structure of the LSMOP test suite, *landscape functions* and the *correlation matrix* were described theoretically. In the next subsection the definitions are used to create concrete LSMOP instances.

5.2.2. Test LSMOP Instances

Here the interactions of variables will be described for a specific set of parameters for the LSMOP test suite. With formulas described above and the number of objectives m, the number of decision variables n and the number of subcomponents nk the correct interactions of the variables can be computed.

The number of decision variables is chosen small to visualize the grouping results and check if the different GMs find the correct groups or variable interactions, it is set to $n = 11$. The number of objectives $m = 2$ is the same as in the following empirical experiments to assure comparability and visualisation of the results. The parameter nk describes the number of subcomponents in each of the m parts of the distance variable vector (objective-groups) x_s^i with $i = 1, ..., m$. In the empirical experiments the standard value $nk = 5$ is used, here the number of subcomponents is reduced to $nk = 2$ because of the reduced number of decision variables. With these parameters and the formulas of the preceding subsection the nine LMSOP problems are instantiated and the variable interactions are calculated for each of them.

With the information that we have, two objective functions $m = 2$, the concrete correlation matrices and reduced problem structures can be described. With $m = 2$ the correlation matrices from Equation 5.6 have the following instances:

$$C_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5.10a}$$

$$C_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{5.10b}$$

$$C_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{5.10c}$$

Formula 5.9 can be simplified with the known number of objectives to:

$$f(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + \sum_{j=1}^{2} c_{1,j} \times g_1(x_j^s)) \\ f_2(x) = h_2(x^f)(1 + \sum_{j=1}^{2} c_{2,j} \times g_2(x_j^s)) \end{cases} \tag{5.11}$$

We have two conflicting objective functions $f_1(x)$ and $f_2(x)$. We can rewrite the sum formula in the following way:

$$\begin{cases} f_1(x) = h_1(x^f)(1 + c_{1,1} \times g_1(x_1^s) + c_{1,2} \times g_1(x_2^s)) \\ f_2(x) = h_2(x^f)(1 + c_{2,1} \times g_2(x_1^s) + c_{2,2} \times g_2(x_2^s)) \end{cases} \tag{5.12}$$

We can further replace $g_1(x)$ and $g_2(x)$ with the corresponding alternating functions $g^I(x)$ and $g^{II}(x)$ respectively:

$$\begin{cases} f_1(x) = h_1(x^f)(1 + c_{1,1} \times g^I(x_1^s) + c_{1,2} \times g^I(x_2^s)) \\ f_2(x) = h_2(x^f)(1 + c_{2,1} \times g^{II}(x_1^s) + c_{2,2} \times g^{II}(x_2^s)) \end{cases} \tag{5.13}$$

Until this step the instantiation for every of the nine problems is the same. To get the objective functions for every problem instance, we need to insert the problem specific values for $g^I(x)$, $g^{II}(x)$ and C. The three versions of the Correlation Matrix used to create the following three versions of the objective functions first. Formula 5.13 with the identity matrix C_1 can be rewritten as follows

$$f_{c1}(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + 1 \times g^I(x_1^s) + 0 \times g^I(x_2^s)) \\ f_2(x) = h_2(x^f)(1 + 0 \times g^{II}(x_1^s) + 1 \times g^{II}(x_2^s)) \end{cases} \tag{5.14}$$

further summarized to:

$$f_{c1}(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + g^I(x_1^s)) \\ f_2(x) = h_2(x^f)(1 + g^{II}(x_2^s)) \end{cases} \tag{5.15}$$

Inserting the other two Correlation Matrices into 5.13 lead to

$$f_{c2}(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + g^I(x_1^s) + g^I(x_2^s)) \\ f_2(x) = h_2(x^f)(1 + g^{II}(x_2^s)) \end{cases} \tag{5.16}$$

and

$$f_{c3}(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + g^I(x_1^s) + g^I(x_2^s)) \\ f_2(x) = h_2(x^f)(1 + g^{II}(x_1^s) + g^{II}(x_2^s)) \end{cases} \tag{5.17}$$

The problem instances can be created with one of the three previous objective functions and the single-objective functions from Table 5.1 for the two alternating g^I and g^{II}. For the first problems, we have $g^I(x) = g^{II}(x) = \eta_1$ and $C_1 = I.$, according to Table 5.2. When we insert these values in Equation 5.15 we get:

$$LSMOP1(x) = \begin{cases} f_1(x) = h_1(x^f)(1 + \eta_1(x_1^s)) \\ f_2(x) = h_2(x^f)(1 + \eta_1(x_2^s)) \end{cases} \tag{5.18}$$

The other eight LSMOP problems can be created in the same way, the complete formulas are given in the appendix.

Further information, more precise the number of decision variables n and the number of subcomponents nk, is used in the following to create the interacting variables.

The length and containing variables of each of the m objective-groups x_i^s is generated by a pseudo random number generator. It assures that for every independent run the sizes and variables in the m objective-groups remain the same.

The position and distance variables lead to the following allocation according to Equation 5.3 with $x^f = (x_1, ..., x_{m-1})$, $x^s = (x_m, ..., x_n)$ and $m = 2$:

$$x^f = (x_1, ..., x_{2-1}) = (x_1) \tag{5.19}$$

$$x^s = (x_m, ..., x_n) = (x_2, ..., x_{11}) \tag{5.20}$$

The overall decision variables $x = \{x_1, ..., x_{11}\}$ are divided into a group of position variables $\{x_1\}$ and distance variables $\{x_2, ..., x_{11}\}$.

The pseudo random number generator divides the 10 distance variables into two objective-groups according to $x^s = (x^s_1, ..., x^s_m)$ (Equation 5.4). The group sizes are: $|x^s_1| = 4$ and $|x^s_2| = 6$. The first objective-group x^s_1 consists of the first four distance variables: $x^s_1 = (x_2, x_3, x_4, x_5)$, the second one of the remaining six: $x^s_2 = (x_6, x_7, x_8, x_9, x_{10}, x_{11})$. Every objective-group x^s_i is equally divided into $nk = 2$ subcomponents ordered again by the indices of the variable, just like the allocation to the objective-groups. The subcomponents of the first objective-group x^s_1 are: $x^s_{1,1} = \{x_2, x_3\}$ and $x^s_{1,2} = \{x_4, x_5\}$ with $x^s_1 = (x^s_{1,1}, x^s_{1,2})$. The subcomponents of the second objective-group contain three variables each because $\frac{|x^s_2|}{nk} = \frac{6}{2} = 3$. The subcomponents are: $x^s_{2,1} = \{x_6, x_7, x_8\}$ and $x^s_{2,2} = \{x_9, x_{10}, x_{11}\}$ with $x^s_2 = (x^s_{2,1}, x^s_{2,2})$. The structure of the LSMOP suite with which was analysed in the preceding paragraphs for the chosen parameters can be summarised in Table 5.3.

Table 5.3.: Structure of the LSMOP test suite for $m = 2$, $n = 11$ and $nk = 2$

position/distance	x^f	x^s									
objective-groups		x^s_1				x^s_2					
subcomponents		$x^s_{1,1}$		$x^s_{1,2}$		$x^s_{2,1}$			$x^s_{2,2}$		
variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}

Table 5.4.: Variable interactions for the two objective functions f_1 and f_2 of LSMOP2, identical with the result of MDG2

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

Interactions according to Equation 2.3 occur among the variables in the subcomponents if the used single-objective function is Non-separable. For instance LSMOP1 uses η_1 for both of its objective functions. η_1 is separable and therefore no variable interactions occur in its objective-groups. LMSOP2 on the other hand uses η_5 and η_2 which are both non-separable, here the interactions become visible. The interaction matrix of LMSOP2 is shown in Table 5.4, "0" means no interaction, a "1" indicates interaction and " $-$ " is used on the main diagonal because a variable cannot interact with itself. Interactions are additionally marked with a green background. All interaction matrices are symmetric. The interactions are shown separate for objective functions f_1 and f_2. The vertical and diagonal lines help to distinguish the two objective-groups and their two subcomponents, compare Table 5.3. LSMOP2 uses the Correlation Matrix C_1, this leads to $f_{c1}(x)$, see Equation 5.15. According to $f_{c1}(x)$ the second objective-group x_2^s is not applied to the first objective function because $g^I(x_2^s)$ was multiplied with 0. Because of this, changes in variables of x_2^s does not affect the first objective function f_1 and can therefore not interact with each other among this function. The variables of x_2^s namely $\{x_6, ..., x_{11}\}$ have therefore only zeros in the depicted interaction matrix 5.4. The same applies to the first objective-group x_1^s for the second objective function $f_2(x)$. The other two combinations ($x_1^s \& f_1(x)$ and $x_2^s \& f_2(x)$) lead to the interacting variables in the subcomponents.

The other LSMOP problems can be constructed in the same way. In the next subsection the results of MDG2 are compared to the correct variable interactions.

5.2.3. Results of MDG2

The grouping quality of the proposed MDG2 and the four TS are tested theoretically. The created test instances of the 9 LSMOP problems are used to measure the results. Tables 5.5 and 5.6 show the results of MDG2 for LSMOP1 and LSMOP3-9. The results of LSMOP2 are depicted in the preceding subsection in Table 5.4. The created example test instance of the LSMOP test suite is considered with the three parameters $m = 2$, $nk = 2$ and $d = 11$. In all test problems x_1 is a position variable and is therefore not considered in the grouping. The interaction matrices for the two objective functions f_1 and f_2 are shown for the nine LSMOP problems. A "1" shows that MDG2 has noticed interaction between the two variables in the row and column. Analogous

Table 5.5.: Variable interactions detected by MDG2 for the two objective functions f_1 and f_2 of LSMOP1,3,4 and 5

LSMOP1

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

LSMOP3

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	0	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	0	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	0
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	0	1	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

LSMOP4

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

LSMOP5

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

Table 5.6.: Variable interactions detected by MDG2 for the two objective functions f_1 and f_2 of LSMOP6,7,8 and 9

LSMOP6

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

LSMOP7

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	0	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	0	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	0
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	0	1	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	0	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	0	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	0
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	0	1	-

LSMOP8

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

LSMOP9

f_2	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

f_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

a "0" shows that no interaction was noticed by MDG2. The correct found interactions are shown in green, false ones in red. More precise, the green ones represent true positives and the red ones false positives. True positives are correctly found interactions, false positives indicate where MDG2 shows interaction where no interaction exists. "0" defines the true negatives, no interaction exist and MDG2 also finds no interaction. False negatives, where no interaction exist but MDG2 finds interaction, do not appear in any of the results.

When looking on the results in Tables 5.5 and 5.6 a lot of things could be noticed. First of all that MDG2 finds in most cases the correct result, whether two variables have no interaction "0", or whether they have interaction, the green "1"s. Interesting is that the false results are in all cases false positives and no false negatives. So MDG2 can recognise interactions where there are no interactions, but on the other hand all existent interactions are found. A reason for this could be the parameter e of the DG2 and MDG2 which regulates the sensitivity of the interaction check.

Another interesting fact is that interactions are only existent in the previously described four subcomponents which contain the variables $\{x_2, x_3\}$,$\{x_4, x_5\}$,$\{x_6, x_7, x_8\}$ and $\{x_9, x_{10}, x_{11}\}$ respectively. This subcomponents are separated by vertical and horizontal lines in the two tables to give a better overview. It is interesting that not only the true but also the falsely detected interactions occur only inside these subcomponents. For instance in LSMOP9 no interactions should be noticed because f_1 and f_2 are both separable functions, but interactions are found not on random positions but in the second objective function, the second objective group inside their two subcomponents. This observation can also be made for the other false positives in LSMOP4 and 7. Again the falsely detected interactions occur only inside the two subcomponents of an objective group. The reason for the fact that the false values occur so structured could not be worked out. Possible reasons for this could be a mistake in the preceding theoretic analysis of the problem. Another reason could be that the interactions are present because of other dependencies of the LSMOP test suite which was not intended by the authors or simply not described in the paper. Since this is the first theoretical analysis of the interaction structure of LSMOP, this could be possible, but that are speculations. Functions of the well konwn ZDT and DTLZ test suite were considered as either separable or non-separable until non trivial variable interactions are detected during an analysis of the functions with a DG approach in [11]. So DG based methods were used before to identify interactions of functions which were considered separable, if this is the case again could not be determined.

When taking a closer look on the subcomponents itself there can be noticed that in some subcomponents every variable interacts with each other, or more precise they form an MCS, for instance in LSMOP6. In LSMOP3 the subcomponents have interactions too, but they form only a maximally connected subgraph and no maximally complete subgraph (MCS). The interactions $x_6 - x_7$ and $x_7 - x_8$ are present, but $x_6 - x_8$ is missing. Which version is *correct* could not be determined because it is not described by the LSMOP suite. Both variants are assumed to be correct because the kind of interactions inside a subcomponent is not explicitly defined by LSMOP [4].

In this subsection the grouping results of MDG2 which are two interaction matrices (one for every objective of the problem) were described and analysed. The theoretic analysis shows clearly that the proposed multi-objective version of DG2, namely MDG2 finds the correct variable interactions of the LSMOP test suite in most of the cases. All existing interactions were found by MDG2, in some cases it detects interactions where no interaction exists (false positives).

5.2.4. Comparison of TS and CA

In the following we will take a closer look on the four TS and CA from LMEA. We have noticed different interactions among different objective functions in the LSMOP test suite in the preceding Subsection 5.2.3. Because of this, the initial goal of the TS, to find the *correct groups of interacted variables*, is not directly possible. The *interacted variables* differ between the objective functions, so the *correct groups* cannot be determined. The variable interactions differ among the objective functions, but the variables have to be changed simultaneously for the multiple objectives. So the overall *correct groups of interacted variables* cannot be well defined.

The proposed TS are an approach to build groups from the found variable interactions. The key point is that a theoretical correct version of the TS cannot be found because the LSMOP problems describe only the correct interactions of the variables per objective not the correct groups which we want to find for the optimisation process of the EA. The LSMOP test suite describes the variable interactions which can differ between the objective functions. These variable interactions per objective are only the starting point of the TS which use them as input to create the groups. This means that the result of the TS can only be described but not mathematically verified.

1. find *correct* variable interactions
2. create the *correct* groups

So only the point 1 can by analysed on a mathematical basis, for the second one no final answer can be provided. The quality of the groups will be then by analysed in the following empirical evaluation.

Table 5.7.: Combined interaction matrices for the LSMOP2 problem and the two TSO strategies O_{single} (OS) and O_{all} (OA)

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0	x_2	-	0	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0	x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0	x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0	x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0	x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0	x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0	x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1	x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	1	-	1	x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	1	1	-	x_{11}	0	0	0	0	0	0	0	0	0	-

The TSO methods combine the interaction matrices of the multiple objective functions to one matrix. OS counts an interaction if it is present in at least one interaction matrix, it can be seen as a logical OR connection of boolean matrices with $1 = true$ and $0 = false$. In the resulting combined matrix all four subcomponents interact among each other. OA on the other hand counts an interaction only if it is present in all objective functions, it can be seen as AND connection of the matrices. This leads to no counted interactions because there are no overlaps in the two objective functions of LSMOP2, see Table 5.7. All other created interaction matrices are depicted in the Tables 5.8 and 5.9. The "1"s are marked in yellow for a better overview.

The first thing which attracts attention when considering the results is the strong difference between the two combined matrices. OS leads with the same

Table 5.8.: The combined interaction matrices of the two TSO methods OS and OA based on the detected interactions of two objective functions by MDG2 for LSMOP1,3,4 and 5

LSMOP1

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

LSMOP3

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	0	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	0	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	0
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	0	1	-

LSMOP4

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

LSMOP5

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

Table 5.9.: The combined interaction matrices of the two TSO methods OS and OA based on the detected interactions of two objective functions by MDG2 for LSMOP6,7,8 and 9

LSMOP6

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

LSMOP7

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	0	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	0	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	0
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	0	1	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	0	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	0	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	0
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	0	1	-

LSMOP8

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	1	0	0	0	0	0	0	0	0
x_3	1	-	0	0	0	0	0	0	0	0
x_4	0	0	-	1	0	0	0	0	0	0
x_5	0	0	1	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

LSMOP9

OA	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	0	0	0	0	0
x_7	0	0	0	0	0	-	0	0	0	0
x_8	0	0	0	0	0	0	-	0	0	0
x_9	0	0	0	0	0	0	0	-	0	0
x_{10}	0	0	0	0	0	0	0	0	-	0
x_{11}	0	0	0	0	0	0	0	0	0	-

OS	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	-	0	0	0	0	0	0	0	0	0
x_3	0	-	0	0	0	0	0	0	0	0
x_4	0	0	-	0	0	0	0	0	0	0
x_5	0	0	0	-	0	0	0	0	0	0
x_6	0	0	0	0	-	1	1	0	0	0
x_7	0	0	0	0	1	-	1	0	0	0
x_8	0	0	0	0	1	1	-	0	0	0
x_9	0	0	0	0	0	0	0	-	1	1
x_{10}	0	0	0	0	0	0	0	1	-	1
x_{11}	0	0	0	0	0	0	0	1	1	-

input to a very different matrix than OA. Because LSMOP problems have the same structure for a higher number of decision variables, the results of the two TSO methods will differ also in the empirical experiments and will maybe lead to varying results.

In the following we will take a closer look at the finally found groups and the used FEs. MDG2 uses for all considered LSMOP problems the same amount function evaluations because the number of input variables remains the same. For 10 distance variables MDG2 uses 56 FE to obtain the two interaction matrices. The considered state-of-the-art multi-objective GM is the CA from LMEA. This method is used to compare the new one against a state-of-the-art approach. CA is a probabilistic method and uses a different amounts of FEs for every problem and independent run. The used FEs for the considered example LSMOP2 range in 5 independent runs from 594 to 675 and are therefore approximately 10 times higher than the ones from MDG2. The results of CA are the groups of variables, the overall interaction matrix for the different objectives or the combination of them is not computed. The results of both methods will be compared by considering the found groups.

For LSMOP2 the two OS methods (OS+VS and OS+VA) lead to the following groups of variables: $\{x_2, x_3\}$, $\{x_4, x_5\}$, $\{x_6, x_7, x_8\}$ and $\{x_9, x_{10}, x_{11}\}$. These are the correct subcomponents of the LSMOP test suite (see Tables 5.3 and 5.7). Both OA methods lead to 10 groups each containing a single variable because the combined matrix of OA (see Table 5.7 right half) contains no interactions. These are quite different results. CA leads to the two groups $\{x_2, x_3\}$ and $\{x_4, x_5\}$, it puts every other variable in a single group. CA recognizes the interactions of the first objective but omits the one from the second one.

LSMOP1,5 and 9 are fully separable, the correct result would be a single group for every variable. All five methods (MDG2 with the four TS and CA) obtain the correct groups for LSMOP1 and 5. 10 groups each containing one variable were created. A big difference could be noticed in the computational budget. MDG2 needs again 56 FEs but CA uses 675 in every considered run. This is approximately 10 times higher, maybe this difference plays a role with a higher number of variables in the empirical analysis. The groups of LSMOP9 are more complicated. As mentioned above, MDG2 finds false variable interactions in LSMOP9, compare Table 5.6. The OA methods lead to a interaction matrix with only zeros, because the interactions occur only in f_2, therefore OA+VS and OA+VA lead to the correct grouping of 10 single groups. The two OS versions on the other hand consider the interactions and lead both to four single groups with x_2,x_3, x_24 and x_5 and the two groups $\{x_6, x_7, x_8\}$ and $\{x_9, x_{10}, x_{11}\}$. CA on the other hand creates in 5 distinct runs 3 times the correct result. In one run x_6 and x_8 are grouped together, in another run x_{10} and x_{11}. These interactions occur also in the preceding results of the OS versions. CA needs 663 to 675 FEs to obtain the results. It also finds some of the false positives which were also detected from MDG2.

LSMOP6 is a problem with a very high number of interactions, see Table 5.6. The two OS versions form the groups identical to the four subcomponents of LSMOP (like for LSMOP2). The two OS methods lead to the two groups $\{x_6, x_7, x_8\}$ and $\{x_9, x_{10}, x_{11}\}$ and four single groups which contain the remaining variables. CA on the other hand finds quite different groups. In four out of five runs the groups $\{x_2, x_3\}$ and $\{x_4, x_5\}$ are found, all other variables are in single groups. The last run only found the group $\{x_4, x_5\}$. The interactions of the second variable group ($x_6 - x_{11}$) are not detected although they are existent in both objective functions, compare LSMOP6 in Table 5.6.

Finally one can say that the OS and OA methods of the TS lead to very different results which are both comprehensible. The correctness cannot be mathematically proven because the LSMOP problems do not define correct groups, only variable interactions per objective. The performance has to be analysed in the following empirical analysis. MDG2 with the four TS were compared against the state-of-the-art grouping method CA from LMEA. MDG2 only needs 56 FEs whereas CA needs up to 675 which is approximately 10 times more to obtain the groups. The grouping results between the TS and CA differ. Which method is *right* cannot be proven. Sometimes MDG2 detects too many interactions, the four TS create different groupings from this input. CA seems to find not as much interactions as MDG2 and the TS, for example in LSMOP6. One can conclude that MDG2 finds more and CA less interactions, but MDG2 and the TS only use only 10 percent of the computational budget of CA. The following empirical analysis will show if the difference in the computational budget is a crucial factor for the optimisation of large-scale problems.

5.3. Experiment Specifications

The specifications of the empirical experiments are described in this section. With the information in this section the experiments of this work can be reproduced.

5.3.1. PlatEMO

The MATLAB Platform for Evolutionary Multi-objective Optimisation (PlatEMO) [23] in version 1.2 was used in this work to implement and compare the approaches. PlatEMO is a software suite based on the MATLAB platform from MathWorks [23]. It was published in January 2017 and contains therefore a lot of important and state-of-the-art algorithms for large-scale and multi- and many-objective optimisation. The main focus of the platform is to enable researchers to focus on their own research instead of implementing algorithms from other papers. Both described state-of-the-art algorithms MOEA/DVA and LMEA are also implemented in PlatEMO.

Test setups with several algorithms and well known problem suites can be executed in a user friendly way. The code of MOEA/DVA and LMEA was used and altered to answer the questions of this thesis.

The used PlatEMO implementation of LMEA was changed in one point. The algorithm checks only after every generation if the maximum number of function evaluations are reached. In one generation too much FEs can be used. It appears often that the algorithm uses 130.000 instead of 100.000 FEs while other algorithms only use up to 1000 more. Additional checks are inserted in LMEA just to terminate the algorithm when the computational budget is exhausted.

5.3.2. Test Problems

The two problem suites WFG [9] and LSMOP [4] are used to evaluate the results. They address different aspects of problem design. The WFG suite has a parameter to explicitly set the number of position variables [9]. With this feature the PDG could be tested. LSMOP is a relatively new test suite which contains more difficult problems than the WFG or other actual ones like DTLZ

[8] or ZDT [41]. It was explicitly designed for large-scale and many-objective optimisation and also tackles the interaction of the decision variables directly. The number of interacting groups can be set with a special parameter, this was used in the preceding theoretical analysis. Previous problems do not directly consider the dependence and independence of variables in their development process. Because of these characteristics it fits perfectly to the topic of this thesis. Both of them can be scaled with any number of decision variables and contain nine test instances. The groups of the WFG test suite were analysed in [12]. The interactions were described as follows:

Table 5.10.: Variable interactions of WFG according to [12]

Problems	Interaction Type
WFG1	no
WFG2	sparse
WFG3	sparse
WFG4	no
WFG5	no
WFG6	highly dependent
WFG7	highly dependent
WFG8	highly dependent
WFG9	highly dependent

WFG1, 4 and 5 are considered to have no interactions. The interactions of WFG2 and 3 are described as sparse, the rest as highly dependent interactions.

5.3.3. Quality Criterions

The goal of multi-objective optimisation is to obtain a set of Pareto-optimal solutions. A human expert can chose the best solutions from the given ones, depending on the problem. Two well known metrics to evaluate the results are considered in this thesis, One which uses the true Pareto-front as information and one without. The HV [26] and the IGD [36] were used to compare the results. Two quality criterions are considered to assure the validity of the results. HV calculates the multi-objective space between the solutions and and reference point [30]. For a two dimensional problem the union of all rectangular areas clamped between the solutions and the reference point is considered as HV. The IGD metric measures the distance of the solutions to the Pareto-front [30]:

$$IGD(X) = \frac{\sum v \in PF_{true} d(v, X)}{|PF_{true}|} \tag{5.21}$$

with the non-dominated solutions X and $d(v, X)$ defining the minimum euclidean distance of the solution v to the true Pareto-front. The standard implementation in the PlatEMO test suite was used.

5.3.4. Experiment Settings

The considered settings for the empirical experiments are described here. In general all nine instances of the benchmark suites LSMOP and WFG are considered. The number of *queried* decision variables are 100, 200 and 500 with $1,000,000$, $1,200,000$ and $6,800,000$ FEs respectively. *Queried* number of decision variables describe the number which is given to the test suites, the real number of decision variables differs from these. The LSMOP problems generate 6 variables more for each problem, so for 100, 200 and 500 *queried* ones 106, 206 and 506 are generated respectively. For the WFG problems the *queried* number is the same except for WFG2 and 3 which increase the number by 1. So the generated decision variables are 101, 201 and 501 for WFG2 and 3. The displayed number in tables describe the number of *queried* variables when not otherwise stated.

All experiments are executed with two objective functions. The population size for all considered EAs is set to 100. For the experiments with 100 and 200 variables 31 independent runs are executed for every algorithm and problem instance. For experiments with 500 variables 21 runs are executed. The tested GMs are inserted into the two reference algorithms Ref_1 and Ref_2. The subcomponent optimisers of both reference algorithms us Differential Evolution as evolutionary operator which is also the default value of the PlatEMO framework. The parameters for CA are set to $nSel = 2$, $nPer = 4$ and $nCor = 5$. The parameter nk of the LSMOP test suite is set to 5. The default parameter which indicates the number of position variables was set to its default value of 1 in the WFG suite. For all other parameters the default values are taken from PlatEMO.

5.3.5. Table Specifications

Two kinds of tables are considered in this work. The first one contains the IGD or HV values for several algorithms and the LSMOP or WFG test suite. The Wilcoxon ranksum test [27] (equivalent to the well known Mann-Whitney U-test [14]) is applied in every row against the respective best result of these row to test the statistical significance, it is assumed for a value of $p < 0.05$. The best result is shown in bold, the worst one in italic font. An "o" means that there is no statistical difference between this results and the best one. An "-" means that this result is statistically worse then the best one and "*" indicates that the Wilcoxon ranksum test was not executed because no best result could be found. The best value is shown with an "r" because it is the reference value for the significance test. A "- - -" is used when no value could be created. To get a better overview over the tables, colors are used in addition. The best result is shown with green background color, results with no statistical difference to the best one have yellow background, the others (which have statistically differences) are shown in red. For instance, when a table consists of a lot of yellow cells, the differences between the algorithms are not as big as in an table with more red cells. The colors give an overview about the relations of the tested algorithms at first glance.

The number of executed experiments is very high, the preceding described tables are depicted in the appendix. The interesting analysed parts are shown in the evaluation section. To analyse the information from these tables in the evaluation section, they are compressed in an *overall comparison table*. 9 problems from the complete table are compressed to one line in the overall comparison table. The shown number indicates how often the algorithm is statistically significant better than another one. The algorithm must have the best result, if this is the case, the solutions which are statistically worse are counted. A higher number indicates the superiority over other algorithms. A color range is applied to the table to increase the overview. The range goes from red to green, and from the lowest to the highest value of the corresponding table respectively.

Table 5.11 shows an example for a normal table which compares four algorithms A1 to A4 and a test suite with five problems P1 to P5. At the bottom the generated line for the overall comparison table is shown. A1 has two best values with each of them is statistically better than all other three algorithms which leads to a number of 6. A2 performs best in P2 but this result has

Table 5.11.: Example table with statistically significance check for four example algorithms A1 to A4 and five example problems P1 to P5. The generated line of an overall comparison table is appended at the bottom.

Problem	A1	A2	A3	A4
P1	**6.1458e-1**$^{(r)}$	4.9601e-1$^{(-)}$	3.6197e-1$^{(-)}$	*2.8939e-1*$^{(-)}$
P2	6.4665e-1$^{(-)}$	**6.5026e-1**$^{(r)}$	6.2659e-1$^{(-)}$	*6.0671e-1*$^{(-)}$
P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
P4	**6.1816e-1**$^{(r)}$	5.9883e-1$^{(-)}$	5.5128e-1$^{(-)}$	*5.2841e-1*$^{(-)}$
P5	*1.0920e-1*$^{(o)}$	1.0941e-1$^{(o)}$	**1.0992e-1**$^{(r)}$	1.0882e-1$^{(o)}$
overall:	6	2	1	0

statistically difference to only 2 other algorithms. A3 has one best result and statistical difference to one other solution (A4) which leads to a 1. The last one has no best result among the five example problems, this leads to a zero. With this scheme the comparison tables in the evaluation section are generated. With these tables an overview about a very wide range of different parameters can be given, clearly more than with the well known significance tables.

For every normal table with median HV and IGD values an additional table which contains the corresponding variation of the 31 or 21 independent runs is provided respectively. The Inter Quartile Range (IQR) is computed to show the distribution of the results.

For comparison of the different algorithms some different specifications of the benchmark problems are considered. The interesting parts are displayed and discussed in the result and evaluation section. The full tables with the median HV and IGD values to create the overall comparison tables and the IQR tables are depicted in the Appendices A to E.

5.4. Empirical Results

The empirical results are analysed in the following. First the dynamic methods MRAND and MDELTA are analysed among themselves to find appropriate versions of SX and NX. Several parameter tests are carried out to check which group sizes or numbers perform best. The results of MRAND are given in Subsection 5.4.1, the ones from MDELTA in Subsection 5.4.2. After that they are compared against each other in Subsection 5.4.3. The next Subsection 5.4.4 deals with the empirical results of MDG2 with the four TS which are compared among each other. In the last Subsection 5.4.5 an overall comparison of the GMs is given. The trivial methods G1 and GN are also considered in this comparison. When possible, the theoretical results which were determined before are connected to the empirical ones.

5.4.1. Multidimensional Random Grouping

This subsection is a comparison of the parameters for the multidimensional random grouping approach. Random approaches were used before an intelligent analysis of variable interactions was introduced. The main drawback of random approaches is that the group-size or the number of groups has to be specified beforehand.

To assure a fair comparison and to study the influence of group sizes, a lot of experiments were executed. An overview over a great number of problems is given to assure that the parameters are universally valid and not only on some specific problems. The complete used information is shown in the Appendix (Tables A.9 to A.16). The information of the eight tables is combined in the overall comparison Table 5.12. All eight versions (four SX and four NX) are compared against each other. Results are shown for both, 100 and 200 decision variables, Ref_1 and Ref_2, LSMOP and WFG, and both quality measures IGD and HV.

All nine test problems of the two test suites LSMOP and WFG are considered. The theoretical maximal best value and would be $3 * 9 = 27$, because one version can be statistically better than the other three in all nine test problems. For instance, the upper left value 19 shows that the MRAND version S2 was statistically better than 19 other results of the SX version group (S5, S20 and S50) for 100 decision variables, the Ref_1 algorithm on all nine instances of the LSMOP test suite considering the median IGD values. Mention that the four SX and NX versions are just compared against each other respectively, so an SX version was just compared against the three other SX counterparts, the same applies to the NX versions. A comparison of the best SX and NX versions against each other is given in Subsection 5.4.3.

Table 5.12.: Overall comparison of MRAND

				S2	S5	S20	S50	N2	N5	N20	N50
n=100	Ref_1	LSMOP	IGD	19	0	0	0	0	0	2	14
			HV	15	2	0	0	0	0	2	14
		WFG	IGD	0	3	5	7	1	1	8	2
			HV	0	3	6	9	1	1	7	3
	Ref_2	LSMOP	IGD	19	0	0	3	6	0	0	16
			HV	12	3	4	3	3	3	3	9
		WFG	IGD	0	3	5	7	7	7	3	0
			HV	0	3	6	9	13	4	2	0
n=200	Ref_1	LSMOP	IGD	18	2	0	0	0	0	3	18
			HV	17	0	0	2	0	0	0	18
		WFG	IGD	7	6	0	0	0	0	2	9
			HV	3	8	1	0	0	0	2	11
	Ref_2	LSMOP	IGD	12	7	0	2	0	5	3	14
			HV	12	6	2	2	3	5	3	12
		WFG	IGD	0	3	2	14	11	5	3	3
			HV	2	0	4	16	14	4	0	3

When comparing the results of the two quality criteria IGD and HV against each other, no notable differences can be noticed. However, when comparing the results of LSMOP against WFG in general, it is obvious that for the LSMOP problems often a superior SX and NX version can be determined. S2 and N50 are often superior against the other, this means that small groups are maybe better in general for the LSMOP problems because S2 and N50 lead to the smallest groups in their respective version group.

When we take a look at WFG, the results of the eight versions are in general more distributed than for LSMOP. For LSMOP there is often a superior version, for WFG there is also often a best one, but not as explicit as in LSMOP. For instance when comparing the first four rows and columns, it is clear that for LSMOP S2 is the best version with an IGD value of 19 and HV of 15. S5 achieves a HV value of 5, but all other five values are zero. In contrast for WFG, S50 seems to be the best version, but with an smaller

IGD of 7 and HV of 9. This schema can often be noticed when comparing the two test suites against each other. Small groups seem to be more efficient for LSMOP problems. S2 and N50, which produce the smallest groups, are often clearly the best ones. In WFG, bigger groups lead to better results, here S50 and N2 are best which led to bigger groups, except Ref_1 with $n = 200$ where smaller groups are better. Furthermore the results are more distributed than for LSMOP.

Major differences between 100 and 200 variables are can not be noticed. This indicates that all eight versions (the good and bad ones) are stable with varying number of decision variables. When directly comparing the two reference algorithms with each other, no big differences can be noticed. This shows that the found groups are the major reason for the performance and not the used optimisation procedure.

In general one can say that often extreme group sizes (very big or very small) lead to good results considering a random grouping approach. Versions which create moderate or non-extreme groups sizes like S5, S20, N5 and N20 are outperformed in most cases. LMSOP often works better with small groups, WFG with big ones. This illustrates the main problem of the random approach, that the group size have to be specified before. A general *good* working size cannot be determined and depends on the considered test problems. Due to the fact that new algorithms should not designed with respect to one specific set of problems but on an more general purpose, an universal statement which group size is better can not be made. This illustrates again the major drawback of random approaches, the predefined group size or number: we have to define the group size before we know which test suite and problems are considered to assure a fair comparison.

5.4.2. Multidimensional Delta Grouping

In this section an overall comparison of MDELTA is given. As can be seen in Table 5.13, the results are very similar to the ones from MRAND (compare Table 4.1). The reason for this might be the same structure of SX and NX versions. The size of the group has also had a major influence on the grouping result, like in MRAND. The results of the two tables are very similar, ignoring some outliers, so the influence of group sizes for a totally random approach and an approach which uses a little bit of information from the problem seems to be the same. The results say nothing about the actual performance (IGD and HV values) which is given in the next section.

Table 5.13.: Overall comparison of MDELTA

				S2	S5	S20	S50	N2	N5	N20	N50
n=100	Ref_1	LSMOP	IGD	16	0	0	0	0	0	0	17
			HV	15	2	0	0	0	0	0	17
		WFG	IGD	2	8	0	0	0	0	9	2
			HV	3	8	2	0	0	1	9	2
	Ref_2	LSMOP	IGD	20	0	0	1	2	0	0	17
			HV	12	4	5	0	2	5	3	10
		WFG	IGD	0	9	6	8	5	5	5	0
			HV	0	6	9	8	10	6	4	0
n=200	Ref_1	LSMOP	IGD	17	0	0	0	0	0	0	17
			HV	17	0	0	0	1	0	0	15
		WFG	IGD	6	6	0	0	0	0	0	12
			HV	5	6	2	0	0	0	3	12
	Ref_2	LSMOP	IGD	12	5	2	0	0	6	3	9
			HV	9	5	5	0	3	7	3	7
		WFG	IGD	0	4	1	12	10	5	4	2
			HV	0	3	3	15	14	3	2	2

Considering this table one aspect is more clear than in the results of MRAND in Table 5.12, that is the difference between the two reference algorithms. Ref_1 seems to work better with smaller groups for both of the problem suites. Comparing the reference algorithms by means of LSMOP, both perform better with smaller groups, but for Ref_2 the results are more wide-spread, while for Ref_1 the other methods have zeros in most of the cases.

More interesting than the previous point is the comparison by means of the WFG test suite. Ref_1 prefers smaller groups in general, but not the smallest. For $n = 100$ and Ref_1 the best methods are S5 and N20 and not the ones which lead to the smallest groups (S2 and N50). Ref_2 on the other hand produces better results with bigger groups, with $n = 200$ Ref_2s best variants are S50 and N2. This shows that different groupings work better together with specific optimisation methods. If a grouping (MDELTA+S2) works well together with an optimisation algorithm (Ref_1), this does not mean that the grouping has a *good* quality in general an can also be used with another optimisation algorithm (Ref_2). Good results in the first two lines of Table 5.13 show a good quality of a *combination* of an algorithm and a grouping, and not only the grouping.

Only considering one reference algorithm, for instance Ref_1, would have led to misleading results because smaller groups would be considered as generally better. This finding supports the choice of several different test specifications.

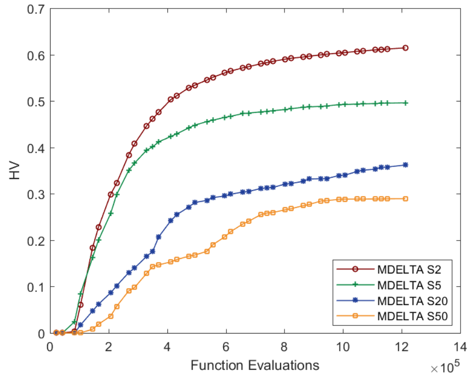
Table 5.14.: Median HV values of Ref_1 with MDELTA and the four SX methods on the LSMOP test suite with 200 decision variables and 1,200,000 FEs

Problem	MDELTA+S2	MDELTA+S5	MDELTA+S20	MDELTA+S50
LSMOP1	**6.1458e-1**$^{(r)}$	4.9601e-1$^{(-)}$	3.6197e-1$^{(-)}$	*2.8939e-1*$^{(-)}$
LSMOP2	**6.5026e-1**$^{(r)}$	6.4665e-1$^{(-)}$	6.2659e-1$^{(-)}$	*6.0671e-1*$^{(-)}$
LSMOP3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
LSMOP4	**6.1816e-1**$^{(r)}$	5.9883e-1$^{(-)}$	5.5128e-1$^{(-)}$	*5.2841e-1*$^{(-)}$
LSMOP5	*1.0920e-1*$^{(o)}$	1.0941e-1$^{(o)}$	**1.0992e-1**$^{(r)}$	1.0982e-1$^{(o)}$
LSMOP6	**2.1413e-3**$^{(r)}$	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$
LSMOP7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
LSMOP8	**2.4977e-1**$^{(r)}$	1.6710e-1$^{(-)}$	*1.0985e-1*$^{(-)}$	1.0988e-1$^{(-)}$
LSMOP9	**6.7764e-1**$^{(r)}$	6.4539e-1$^{(o)}$	5.3773e-1$^{(-)}$	*4.5550e-1*$^{(-)}$

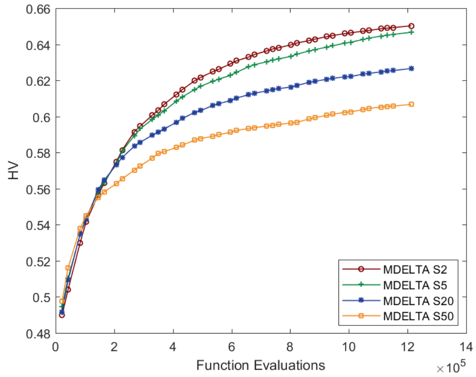
Table 5.14 shows median HV values of MDELTA+SX with Ref_1 on the LSMOP test suite with 200 variables (the first four values of row 10 in the overall comparison Table 5.13). The median values of these runs (except for LSMOP3,6 and 7 because HV was not achieved by all algorithms) are shown in Figure 5.2.

The HV values and convergence plots follow the same trend. First of all, the convergence plots show that MDELTA+S2 is clearly the best, but also that the others are not equally worse, which one can think by only looking at the overall comparison in Table 5.13. The overall table compares the best one against all other ones, but it does not compare the weaker ones against each other. LSMOP1,2,4,8 and 9 have the same structure, in all of them MDELTA+S2 is the best one, followed by S5, S20 and S50. The performance decreases with a growing group size. The trend becomes clear after approximately 100,000 or 200,000 function evaluations, and not only after the full 1,000,000 FEs. This trend is present in fully separable functions like LSMOP1 and 9, which is comprehensible, partially separable functions (LSMOP2) and mixed ones (LSMOP4 and 8). The small groups are better in this case regardless of the interactions between variables.

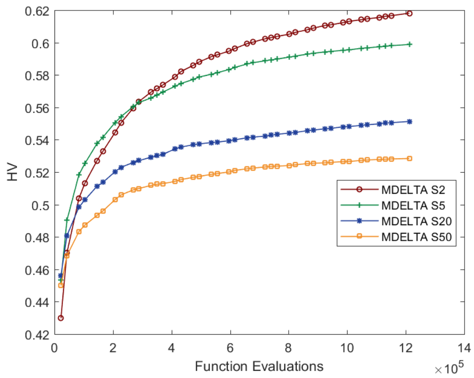
In LSMOP5, the approaches start relatively late to generate hypervolume, but reach the optimum fast after approximately 200,000 FEs. This leads to a



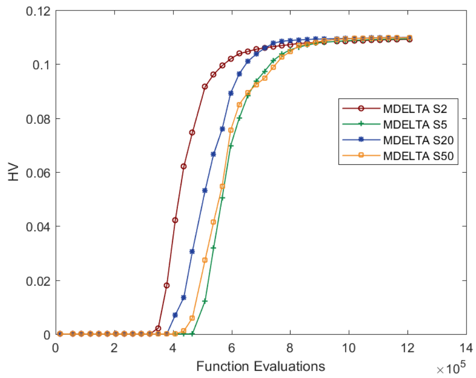
(a) LSMOP1



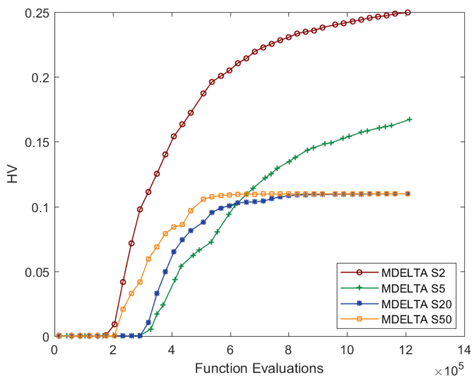
(b) LSMOP2



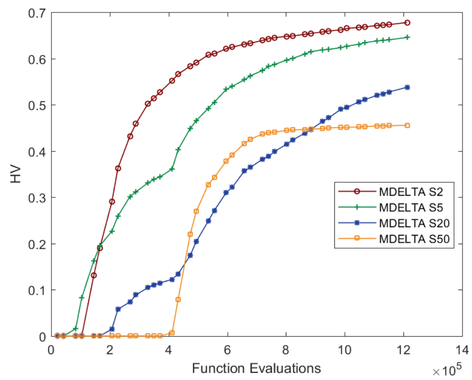
(c) LSMOP4



(d) LSMOP5



(e) LSMOP8



(f) LSMOP9

Figure 5.2.: Convergence plots of the median HV values of Ref_1 with MDELTA and the four SX methods on the LSMOP test suite with 200 decision variables and $1,200,000$ FEs, shown in Table 5.14

sigmoid shaped graph for all algorithms, see Subfigure 5.2(d). This plot can be observed often for LSMOP5 with different algorithms, grouping methods and parameters. LSMOP5 seems to have a different structure than the other LSMOP instances.

5.4.3. Comparison of MRAND and MDELTA

The two dynamic methods are compared against each other in this subsection. The best MRAND and best MDELTA version is determined to compare them against other methods in the overall comparison in Subsection 5.4.5. To get the best random and delta grouping, we will first compare the best SX against the best NX method respectively. After that a comparison of the best MRAND and MDELTA method is carried out. The results are compressed in Table 5.15. Due to the fact that IGD and HV values were not significantly different in the preceding tables 5.12 and 5.13, only the IGD value is given in the this table.

Table 5.15.: Overall comparison of MRAND and MDELTA considering only IGD values

			MRAND		MDELTA		MRAND	MDELTA	MRAND	MDELTA
			S2	N50	S2	N50	S2	S2	N50	N50
n=100	Ref_1	LSMOP	3	0	4	0	1	1	1	1
		WFG	2	0	0	0	1	2	0	3
	Ref_2	LSMOP	4	1	3	1	2	1	2	1
		WFG	0	0	1	0	0	1	0	0
n=200	Ref_1	LSMOP	6	0	7	0	1	1	1	1
		WFG	1	3	0	3	0	2	0	2
	Ref_2	LSMOP	4	2	3	3	2	1	3	0
		WFG	0	6	0	6	0	0	0	1

S2 and N50 lead not to the same group sizes as described in the concept in Subsection 4.3.2. 100 decision variables are *queried*, see Subsection 5.3.4. LSMOP problems lead to 106 decision variables, 105 distance related ones and one position variable. This means S2 creates 52 groups of 2 variables and one group with a single variable. N50 on the other hand creates a maximum of 50 groups. 105/50 = 2.1 which leads to a group-size of 3 to assure same group sizes within each of the eight overall strategies and a maximal number of 50 groups. The theoretical group sizes of Subsection 4.3.2 are not reached because this circumstance.

To compare MRAND against MDELTA, the best SX and NX methods are compared against each other, which are S2 and N50 for both approaches. Columns 5 and 6 compares MRAND and MDELTA both with a group-size of 2 the against each other (S2), columns 7 and 8 compares them with both 50 groups (N50). So the group sizes are the same, the only difference is the grouping approach, random and delta. The number of statistically significant differences is quite low with a maximum of 3 for 9 problems per test suite (and therefore also a maximal number of 9). The general performance of MRAND and MDELTA seems to be quite equal. For most of the problems no difference between the two approaches can be determined, but the differences occur on the same problem instances repeatedly. The pattern that MDELTA is better on LSMOP2 and MRAND is better on LSMOP4 occurs in 6 out of 8 cases. To check this behavior, Table 5.16 shows the results for $n = 200$, Ref_1 for LSMOP and the four algorithms. The first two and last two columns are tested against each other. The absolute differences between the results are not big but statistically significant and occur in 6 out of 8 cases in which MRAND is compared against MDELTA. Two of these cases are shown in the table, $n = 100$ with Ref_1 on LSMOP.

Table 5.16.: IGD values of Ref_1 with MRAND+S2 against MDELTA+S2 and MRAND+N50 against MDELTA+N50 on LMSOP with 100 decision variables and 1000000 function evaluations

Problem	MRAND+S2	MDELTA+S2	MRAND+N50	MDELTA+N50
LSMOP1	*3.5644e-2*$^{(o)}$	**3.5625e-2**$^{(r)}$	*6.2296e-2*$^{(o)}$	**5.7086e-2**$^{(r)}$
LSMOP2	*3.5003e-2*$^{(-)}$	**3.1553e-2**$^{(r)}$	*3.7551e-2*$^{(-)}$	**3.5458e-2**$^{(r)}$
LSMOP3	**9.6557e-1**$^{(r)}$	*1.0098e+0*$^{(o)}$	**1.0425e+0**$^{(r)}$	*1.0579e+0*$^{(o)}$
LSMOP4	**4.9708e-2**$^{(r)}$	*6.0935e-2*$^{(-)}$	**5.1230e-2**$^{(r)}$	*6.8732e-2*$^{(-)}$
LSMOP5	**4.7698e-1**$^{(r)}$	*6.2532e-1*$^{(o)}$	**7.3897e-1**$^{(r)}$	*7.4209e-1*$^{(o)}$
LSMOP6	*7.3005e-1*$^{(o)}$	**6.9817e-1**$^{(r)}$	*7.4366e-1*$^{(o)}$	**7.4347e-1**$^{(r)}$
LSMOP7	**1.3473e+0**$^{(r)}$	*1.3495e+0*$^{(o)}$	**1.3463e+0**$^{(r)}$	*1.3494e+0*$^{(o)}$
LSMOP8	*1.3515e-1*$^{(o)}$	**1.2810e-1**$^{(r)}$	**1.4120e-1**$^{(r)}$	*1.4697e-1*$^{(o)}$
LSMOP9	**4.8592e-1**$^{(r)}$	*5.1701e-1*$^{(o)}$	*5.5040e-1*$^{(o)}$	**5.0566e-1**$^{(r)}$

The differences on LSMOP2 and 4 are clear. That in all other problems the performance remains the same (regarding the statistically significance), show that MRAND and MDELTA seem to have small but commensurable differences. Something in the problem structure of LSMOP2 (partially separable) and LSMOP4 (mixed) causes the difference.

In general the considered multi-objective approaches for the random and delta grouping have very similar performance along the compared versions (SX and NX) and the different considered test cases. Both are dynamic methods which do not use FEs to create the grouping. MDELTA analyses the absolute changes in the considered decision variables and creates the groups depending on the size of the change. This additional information does not really help the algorithm to perform better than the corresponding random grouping. However, it does not degrade the result quality and does not use additional FEs. Therefore the approaches can be considered as equal regarding the performance. The repeated pattern for LSMOP2 and 4 seem to have special properties which indicate that there are differences in the approaches, but these differences are not present in any other problem and can be omitted when giving a general overview.

5.4.4. Transfer Strategies with MDG2

In this Subsection the four Transfer Strategy are compared against each other. The comprehensive results are compared against each other in the same way like in the preceding subsections and also for $n = 500$ decision variables. After that, some results are analysed in more detail.

Table 5.17 shows the results of MDG2 with the four different TS. The first aspect which attracts attention are the relatively low values compared to MRAND and MDELTA (Tables 5.12 and 5.13). The values from the overall comparison of MRAND and MDELTA scale between 0 and 20, in the actual table between 0 and 10. The theoretic maximal value would be $9 * 3 = 27$ for the nine considered problem instances per test suite and 3 other algorithms. The lower values indicate that the results of the four TS have less statistical different results than the four MRAND and MDELTA versions.

The reason for the lower values is that in the results, no clear best version could be determined. Also the differences between the results are more often not statistically relevant. In contrast to MRAND and MDELTA in which best versions (smallest and biggest groups) could be determined very clearly,

Table 5.17.: Overall comparison of DG2 and the four TS

				OS+VS	OS+VA	OA+VS	OA+VA
n=100	Ref_1	LSMOP	IGD	0	1	5	4
			HV	0	0	4	6
		WFG	IGD	0	3	0	2
			HV	0	3	0	1
	Ref_2	LSMOP	IGD	1	0	2	7
			HV	3	1	2	2
		WFG	IGD	1	5	0	2
			HV	2	4	0	2
$\sum_{100}$				7	17	13	26
n=200	Ref_1	LSMOP	IGD	1	4	2	6
			HV	1	0	2	6
		WFG	IGD	0	1	1	2
			HV	0	3	1	0
	Ref_2	LSMOP	IGD	3	3	2	2
			HV	8	0	2	2
		WFG	IGD	0	0	0	4
			HV	0	0	0	2
$\sum_{200}$				13	9	10	24
n=500	Ref_1	LSMOP	IGD	2	0	6	6
			HV	2	0	6	4
		WFG	IGD	0	2	1	2
			HV	0	0	1	4
	Ref_2	LSMOP	IGD	0	2	12	0
			HV	2	3	7	0
		WFG	IGD	0	0	0	2
			HV	1	0	0	3
$\sum_{500}$				7	7	31	17
$\sum_{100} + \sum_{200} + \sum_{500}$				27	33	54	67

here no such clear best versions can be traced. Comparing in general the four TS over all test instances, and counting the overall dominated other values, OA+VA is the best one with 67, followed by OA+VS with 54, OS+VA 33 and OS+VS with 27. The values give an overview about the performance when comparing the four algorithm with each other. It describes the average overall performance considering all executed experiments. The performance differs strongly between the considered test suites, reference algorithms and number of decision variables. For instance, for 200 decision variables the OS version OS+VS is with a value of 13 superior to OS+VA (9) and OA+VS with (10).

As depicted in Subsection **??**, the TS are ordered using their theoretical group size. OS+VS tends to bigger groups, and OA+VA to smaller ones. Small groups seem to work better in general, when considering the group-size. The TSO versions seem to have a greater impact on the results, the OS methods have overall values of 27 and 33, the OA methods 54 and 67 which equals a difference of only 6 and 13 respectively. This illustrates that the TSO methods have a bigger influence on the results than TSV, which is quite logical because TSO is the first step of the combined TS and creates the basis for the TSV methods. When comparing the two TSV methods VA lead to better results for both TSO methods with values of 33 against 27 in the OS case and 67 against 54 for OA.

This finding shows that the central question of this work (how to transfer grouping approaches to the multi-objective case) is not trivial and that the quality of the results differ. OA seems to work better in general but OS has also some cases in which it is superior to OA. A simple answer to the question how the information of several objectives should be combined cannot be given. In the following, the behaviour of the methods is analysed in detail.

When comparing the results of the two reference algorithms with each other, structural differences between them can be noticed. The results of Ref_1 are more stable along the differing number of decision variables like the ones from Ref_2. It produces differing results for varying number of variables. Ref_1 leads for LSMOP in general to good results for OA methods in contrast to OS. Considering WFG, the results are quite mixed where no clear *best* method can be determined from the compressed results in Table 5.17. Ref_2 on the other hand has also clear best methods for the LSMOP, but this changes with an altered number of decision variables, the results does not depend on TSO like for Ref_1. Also a difference between the HV and IGD values can be noticed which is maybe an indicator for unstable results. For $n = 100$ the two OA methods perform better, for $n = 200$ the results are quite mixed. However for $n = 500$ OA+VS is clearly the best method with a IGD indicator of 10 and HV of 7, but in contrast to the results of Ref_1, the other OA method performs worst of all four, with both zero dominated results. OS+VS has also very weak performance, the TSO and nor the TSV are therefore crucial for the results like in Ref_1, but a specific combination of the two steps. A deeper insight in this results are given in the following.

The differences between the two reference algorithms show that the result does not depend on the grouping alone but also on the optimisation process of the found groups. The results further indicate that for different optimisation processes, different groups are effective. There is no *best* grouping which gains superior results together only with the *best* optimisation process. Not every grouping seem to work together with every optimisation process, they have to stick together. Both of them should therefore be coordinated with each other. Just because a grouping works good with a specific algorithm, this does not mean it works together with another. The two parts should be developed together and not independent from each other to achieve better results.

To analyse in more detail, the differences between OA and OV, some LSMOP problems from the first reference algorithms are analysed further.

Table 5.18.: Median IGD values for Ref_1 with MDG2 and the four TS for the LSMOP test suite with 100 decision variables and $1,000,000$ FEs

Problem	OS+VS	OS+VA	OA+VS	OA+VA
LSMOP1	1.3749e-2$^{(o)}$	**1.3550e-2**$^{(r)}$	1.4594e-2$^{(o)}$	*1.5101e-2*$^{(o)}$
LSMOP2	4.2302e-2$^{(-)}$	*4.2361e-2*$^{(-)}$	**2.7792e-2**$^{(r)}$	2.8625e-2$^{(o)}$
LSMOP3	*9.5886e-1*$^{(-)}$	8.9326e-1$^{(-)}$	**5.9298e-1**$^{(r)}$	6.0404e-1$^{(o)}$
LSMOP4	*1.4990e-1*$^{(-)}$	1.4602e-1$^{(-)}$	5.7010e-2$^{(o)}$	**5.4147e-2**$^{(r)}$
LSMOP5	4.7878e-1$^{(o)}$	*6.5130e-1*$^{(-)}$	**4.3436e-1**$^{(r)}$	5.1016e-1$^{(o)}$
LSMOP6	*7.3982e-1*$^{(-)}$	**5.9514e-1**$^{(r)}$	6.8864e-1$^{(o)}$	7.2588e-1$^{(o)}$
LSMOP7	**1.2873e+0**$^{(r)}$	1.3063e+0$^{(o)}$	*1.5162e+0*$^{(o)}$	1.3345e+0$^{(o)}$
LSMOP8	9.3913e-2$^{(o)}$	**9.2113e-2**$^{(r)}$	*9.4772e-2*$^{(o)}$	9.3089e-2$^{(o)}$
LSMOP9	5.9746e-1$^{(-)}$	*6.3058e-1*$^{(-)}$	4.8033e-1$^{(o)}$	**4.7855e-1**$^{(r)}$

Table 5.18 shows the results for the LSMOP test suite. Here the finding of the preceding overall comparison is supported. TSO is crucial for the results. OA methods are significantly better than the OS methods. In 5 cases the OA

methods are significantly better OS methods. In the cases in which OS perform better (LSMOP1,6,7 and 8) there is no significance difference between the best results and the two from OA. This shows the general better performance of OA methods. This is interesting because in the state-of-the-art grouping methods from MOEA/DVA and LMEA an interaction in a single objective function is sufficient like in the OS methods. Maybe the results of these grouping methods can be improved by considering only interaction which occur in every objective function like OA.

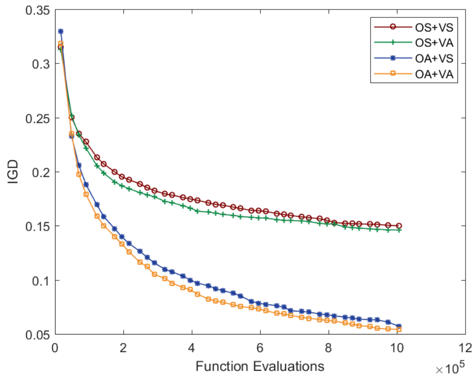


Figure 5.3.: Convergence plot of the median IGD values of Ref_1 with MDG2 and the four TS, 100 decision variables and $1,000,000$ FEs for LSMOP4, given in Table 5.18

Figure 5.3 shows the convergence plot of the IGD values for LSMOP4 of 5.18. The difference is small (IGD values range from 0.05 to 0.35) but clear. The OA methods (cyan and red) perform better than the OS methods (yellow and magenta). TSV seems not to make a big difference, the two OS and OA methods seem to stick together respectively. The graphic visualises the trend which was identified before.

In general one can say that OA methods lead to better results but the results are not very robust. The optimisation algorithms had great influence on the results. With Ref_2 the OS methods are sometimes better or equal to the OA methods. This supports the observation of MRAND and MDELTA, namely that the optimisation process and the grouping have to be coordinated to perform well. The results differ also considering with changing number of decision variables from 100 to 500.

5.4.5. General Comparison

In this subsection an overall comparison of the considered grouping methods is carried out. The considered GMs are namely Correlation Analysis (CA) from LMEA, MDG2 with the four TS, the best delta and random groupings (MRAND+S2 and MDELTA+S2), and the trivial methods GN and G1. Because the focus of this work lies on the TS, all four of them and not only the best one is compared against each other. This leads to 9 algorithms in total. Only the HV values are considered for the overall comparison, the IGD values are in most cases the same and do not deliver additional information.

Table 5.19.: General Comparison of the nine final grouping methods, only HV values

			MDG2				MRAND	MDELTA			
			OS		OA						
			VS	VA	VS	VA	S2	S2	GN	G1	CA
n=100	Ref_1	LSMOP	5	0	9	7	8	0	5	0	2
		WFG	5	1	5	4	8	0	12	8	0
	Ref_2	LSMOP	7	10	0	0	7	11	8	0	0
		WFG	0	12	0	3	0	0	8	32	6
$\sum_{100}$			17	23	14	14	23	11	33	40	8
n=200	Ref_1	LSMOP	4	0	0	6	5	0	13	8	0
		WFG	3	5	6	0	3	4	15	8	0
	Ref_2	LSMOP	23	0	0	9	8	0	0	7	2
		WFG	0	4	10	0	0	0	8	38	0
$\sum_{200}$			30	9	16	15	16	4	36	61	2
n=500	Ref_1	LSMOP	9	0	8	6	10	0	6	7	0
		WFG	0	3	6	9	0	6	6	13	0
	Ref_2	LSMOP	7	8	9	3	16	0	0	8	0
		WFG	0	5	5	0	0	0	8	42	0
$\sum_{300}$			16	16	28	18	26	6	20	70	0
$\sum_{100}+\sum_{200}+\sum_{300}$			63	48	58	47	65	21	89	171	10

Table 5.19 shows the HV values for the nine considered algorithms for both LSMOP and WFG and both reference algorithms, 100, 200 and 500 decision variables. When looking at the overall comparison, some patterns become visible. The first is the general distribution of the numbers. There is no

superior GM which is clearly the *best* one. The portion of zeros is lower than in the preceding tables, this shows that even the weaker algorithms perform sometimes statistically better than the others. What should be kept in mind is that the combination of Ref_1 and CA is equivalent to the original LMEA algorithm. The results of the new methods can therefore be compared to an actual state-of-the-art approach.

The first interesting fact that attracts attention is the bad performance of CA, with only 10 superior solutions in total, and the good performance of the trivial solutions GN and G1 with 93 and 152 superior results respectively. The main reason for this is the high computational budget used by CA, this becomes visible when considering the later described figures.

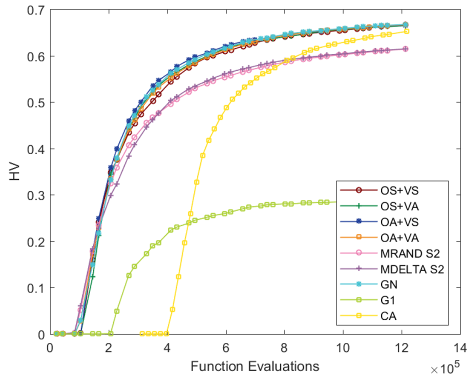
When comparing the two reference algorithms against each other, Ref_2 often has the best results with $G1$ (32, 38 and 30) which is a very high number of statistically dominated solutions. Ref_1 on the other hand performs better together with GN, which is the other extreme. The finding of the preceding subsections is supported, where Ref_1 works better with smaller groups and Ref_2 with bigger ones. With the two extreme versions G1 and GN, this effect gets even stronger.

Comparing the results of the two test suites, in general they assure similar results, except for G1 where the difference is very high 32 to 0 for $n = 100$ and Ref_2. In general one can say (when considering the trivial methods) that LSMOP works better with GN and WFG better with G1.

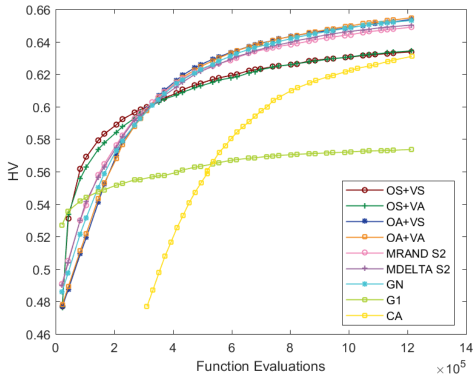
Table 5.20.: Median HV values for the nine algorithms for Ref_1, 200 decision variables and $1,200,000$ FEs for the LSMOP test suite

P	OS+VS	OS+VA	OA+VS	OA+VA
1	6.6684e-1$^{(o)}$	6.6465e-1$^{(-)}$	6.6580e-1$^{(o)}$	6.6653e-1$^{(o)}$
2	6.3375e-1$^{(-)}$	6.3435e-1$^{(-)}$	6.5377e-1$^{(o)}$	**6.5478e-1**$^{(r)}$
3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
4	5.3563e-1$^{(-)}$	5.3299e-1$^{(-)}$	6.2532e-1$^{(o)}$	6.2689e-1$^{(o)}$
5	1.0897e-1$^{(-)}$	1.0909e-1$^{(-)}$	1.0868e-1$^{(-)}$	1.0901e-1$^{(-)}$
6	—$^{(-)}$	—$^{(-)}$	3.2601e-3$^{(o)}$	—$^{(o)}$
7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
8	**3.1538e-1**$^{(r)}$	3.1448e-1$^{(o)}$	3.1351e-1$^{(o)}$	3.1470e-1$^{(o)}$
9	5.8259e-1$^{(-)}$	5.6038e-1$^{(-)}$	6.8503e-1$^{(o)}$	6.8915e-1$^{(o)}$

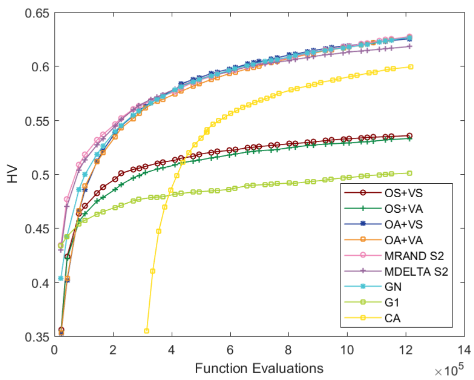
P	MRAND+S2	MDELTA+S2	GN	G1	CA
1	6.1394e-1$^{(-)}$	6.1458e-1$^{(-)}$	**6.6723e-1**$^{(r)}$	*2.8803e-1*$^{(-)}$	6.5224e-1$^{(-)}$
2	6.4892e-1$^{(-)}$	6.5026e-1$^{(-)}$	6.5304e-1$^{(o)}$	*5.7362e-1*$^{(-)}$	6.3097e-1$^{(-)}$
3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
4	**6.2738e-1**$^{(r)}$	6.1816e-1$^{(-)}$	6.2652e-1$^{(o)}$	*5.0098e-1*$^{(-)}$	5.9941e-1$^{(-)}$
5	1.0929e-1$^{(-)}$	1.0920e-1$^{(-)}$	1.0912e-1$^{(-)}$	**1.0998e-1**$^{(r)}$	*1.0853e-1*$^{(-)}$
6	1.1120e-2$^{(o)}$	2.1413e-3$^{(o)}$	**2.4416e-2**$^{(r)}$	—$^{(-)}$	—$^{(-)}$
7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
8	2.5841e-1$^{(-)}$	2.4977e-1$^{(-)}$	3.1335e-1$^{(o)}$	*1.1000e-1*$^{(-)}$	3.0907e-1$^{(-)}$
9	6.7104e-1$^{(o)}$	6.7764e-1$^{(o)}$	**6.9623e-1**$^{(r)}$	*2.4044e-1*$^{(-)}$	6.4859e-1$^{(-)}$



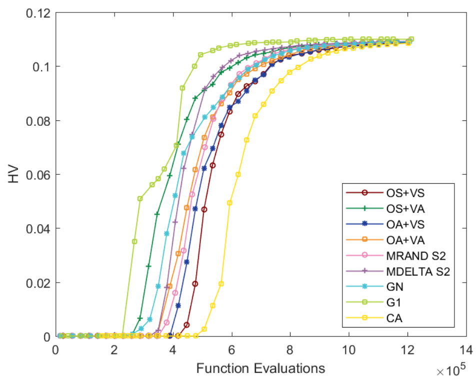
(a) LSMOP1



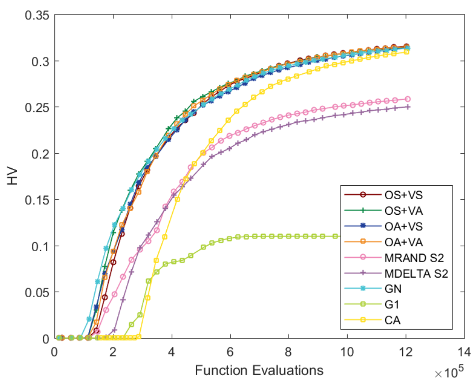
(b) LSMOP2



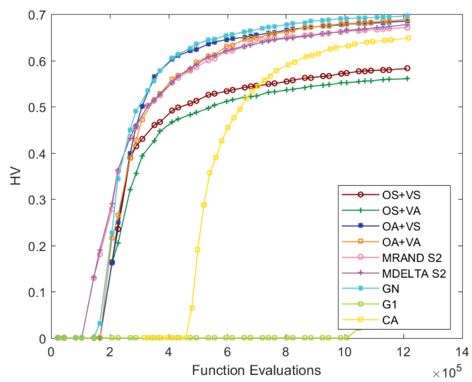
(c) LSMOP4



(d) LSMOP5



(e) LSMOP8



(f) LSMOP9

Figure 5.4.: Convergence plots for the Median HV values of the nine algorithms for Ref_1, 200 decision variables and $1,200,000$ FEs on the LSMOP test suite, shown in Table 5.20

To get a better insight in the results, Table 5.20 shows the median HV values for Ref_1, $n = 200$ and the LSMOP test suite. The Figures 5.4 show the convergence plots for the data in the table. The plots for LSMOP3 and 7 are omitted because no HV could be achieved by any of the algorithms, LSMOP6 is excluded because only three three algorithms achieve very low and no expressive HV values.

Table 5.20 was selected because it represents an average row of the overall comparison table. One can clearly see that Ref_1 works very good together with very small groups like GN and MDG2 with OA. An outlier is LSMOP5 where the other exteme grouping G1 leads to the best result and any other solutions are statistically worse. This is interesting because in the theoretical analysis it was found that LSMOP5 has no variable interactions and is fully separable.

Another interesting fact is that CA produces no best result and has in any case statistically difference to the best one. One reason for this becomes visible when considering the convergence plots for this results in Figure 5.4. In any figure one can clearly see that that CA starts very late with the optimisation process and uses a lot of FEs to obtain the variable groups. CA uses approximately 3 to 5 hundred thousand FEs to obtain the groups which is very high because there are only $1,200,000$ FEs available in total. The optimisation does not have enough FEs left to catch up to the performance of the other methods.

When looking at the graphs in general, some patterns become visible. First of all one can see that the optimisation of several methods stick together during the whole optimisation process. This shows that these methods maybe found the same or similar groups, because the groups are the only difference between the plots, the optimisation algorithm is with Ref_1 always the same. MRAND+S2 and MDELTA+S2 stick together in every of the six graphs which is comprehensible and supports the findings from Subsection 5.4.3, that the results of MRAND and MDELTA are very similar.

The convergence plots of CA does not have anything in common with the other plots. The same applies to G1 whose performance is in most cases even worse that the one from CA. The OS and OA methods from MDG2 seem to produce groups with a similar quality respectively. This can be seen best considering LSMOP4 and 9 where the OS methods stick together. GN does not stick to any other method, it is just in most cases the best one. In most cases, half of the available FEs are sufficient to show which grouping performs best.

Table 5.21.: Median HV values for the nine algorithms for Ref_1, 200 decision variables and $1,200,000$ FEs for the WFG test suite

P	OS+VS	OS+VA	OA+VS	OA+VA
1	6.0500e-1$^{(-)}$	6.2606e-1$^{(-)}$	6.5210e-1$^{(-)}$	6.4523e-1$^{(-)}$
2	6.1291e+0$^{(o)}$	**6.1292e+0**$^{(r)}$	6.1275e+0$^{(o)}$	6.1277e+0$^{(o)}$
3	5.6272e+0$^{(o)}$	5.6275e+0$^{(o)}$	**5.6283e+0**$^{(r)}$	5.6264e+0$^{(-)}$
4	3.3604e+0$^{(-)}$	3.3613e+0$^{(o)}$	3.3605e+0$^{(-)}$	3.3604e+0$^{(-)}$
5	**2.9833e+0**$^{(r)}$	2.9826e+0$^{(o)}$	2.9827e+0$^{(o)}$	2.9826e+0$^{(o)}$
6	3.3268e+0$^{(o)}$	3.3252e+0$^{(-)}$	3.3252e+0$^{(o)}$	3.3270e+0$^{(o)}$
7	2.4635e+0$^{(o)}$	2.4575e+0$^{(o)}$	2.4186e+0$^{(-)}$	2.4857e+0$^{(o)}$
8	3.0750e+0$^{(-)}$	3.1760e+0$^{(-)}$	3.0814e+0$^{(-)}$	3.1725e+0$^{(-)}$
9	1.9976e+0$^{(-)}$	2.0367e+0$^{(-)}$	1.8903e+0$^{(-)}$	2.0011e+0$^{(-)}$

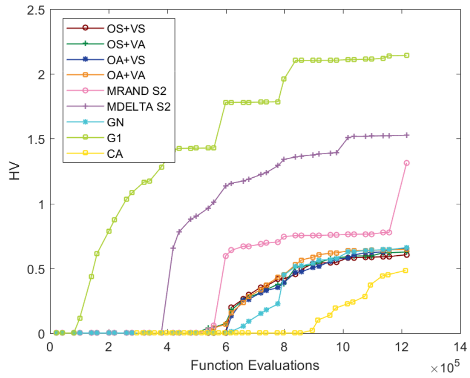
P	MRAND+S2	MDELTA+S2	GN	G1	CA
1	1.3132e+0$^{(-)}$	1.5267e+0$^{(-)}$	6.5965e-1$^{(-)}$	**2.1428e+0**$^{(r)}$	*4.8345e-1*$^{(-)}$
2	5.8528e+0$^{(-)}$	5.8955e+0$^{(-)}$	5.1615e+0$^{(-)}$	5.3884e+0$^{(-)}$	6.0764e+0$^{(-)}$
3	5.3696e+0$^{(-)}$	5.3971e+0$^{(-)}$	4.6879e+0$^{(-)}$	4.8996e+0$^{(-)}$	5.5792e+0$^{(-)}$
4	**3.3616e+0**$^{(r)}$	3.3612e+0$^{(o)}$	3.3610e+0$^{(o)}$	3.3613e+0$^{(o)}$	3.3612e+0$^{(o)}$
5	2.9801e+0$^{(-)}$	2.9812e+0$^{(o)}$	2.9830e+0$^{(o)}$	2.9798e+0$^{(-)}$	2.9808e+0$^{(-)}$
6	3.3268e+0$^{(o)}$	**3.3282e+0**$^{(r)}$	3.3275e+0$^{(o)}$	3.3267e+0$^{(o)}$	3.3254e+0$^{(-)}$
7	2.6022e+0$^{(o)}$	**2.6686e+0**$^{(r)}$	2.4639e+0$^{(-)}$	2.4911e+0$^{(o)}$	2.4343e+0$^{(o)}$
8	3.1166e+0$^{(-)}$	3.1231e+0$^{(-)}$	**3.2060e+0**$^{(r)}$	2.9180e+0$^{(-)}$	2.9277e+0$^{(-)}$
9	2.5635e+0$^{(-)}$	2.7313e+0$^{(o)}$	**2.8428e+0**$^{(r)}$	1.9892e+0$^{(-)}$	2.0643e+0$^{(-)}$

To compare the previous results for the WFG test suite, a table and figures for the same number of decision variables 200, reference algorithm Ref_1 and performance metric HV are given in Table 5.21 and the corresponding Figures 5.5 and 5.6.

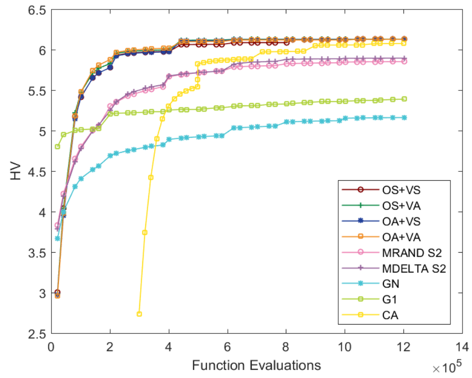
Despite the fact that the two lines for the LSMOP and WFG test suite seem to look quite similar in the overall comparison Table 5.19, the Tables 5.20 and 5.21 have notable differences.

The performance of the four MDG2 versions is relatively equal, either three or four of them have good results (WFG2,3,5,6 and 7) or all of them have worse results (WFG1,8 and 9). There is no split between TSO unlike LSMOP where OA methods perform better than their OS counterparts. This supports the finding that WFG prefers in general bigger groups than LSMOP.

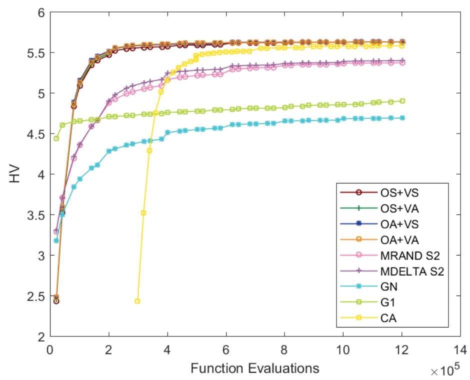
The trivial methods GN and G1 have similar performance like for LSMOP. CA also have similar performance, again no best result could be achieved. The convergence plots help to understand the emergence of the results. Comparing the figures for LSMOP against WFG, the first general difference which catches the eye is the fast convergence of the algorithms to their optimum. In Figures 5.4 the best algorithm was clear after half of the available FEs, but the HV value increases in the most of the plots continuously till the end (except for LSMOP5). In the the WFG test suite the algorithms reach their optimum



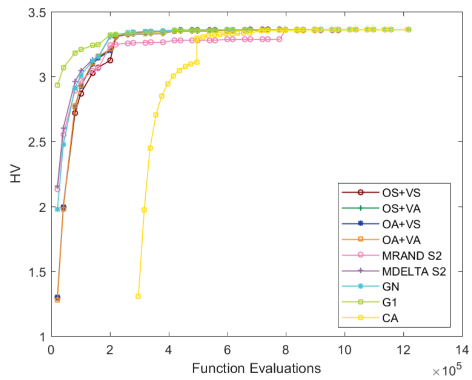
(a) WFG1



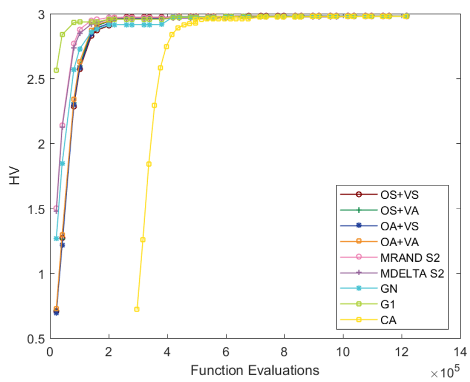
(b) WFG2



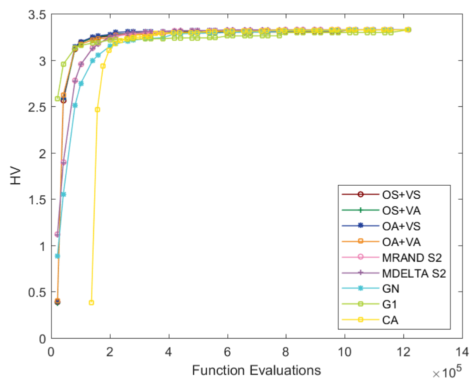
(c) WFG3



(d) WFG4

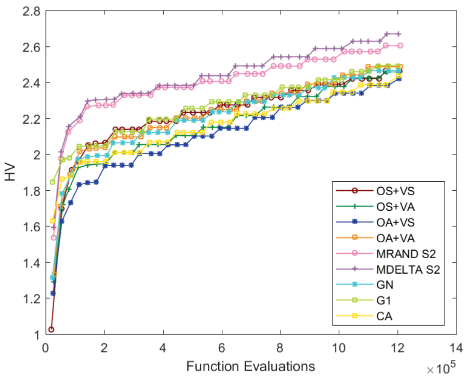


(e) WFG5

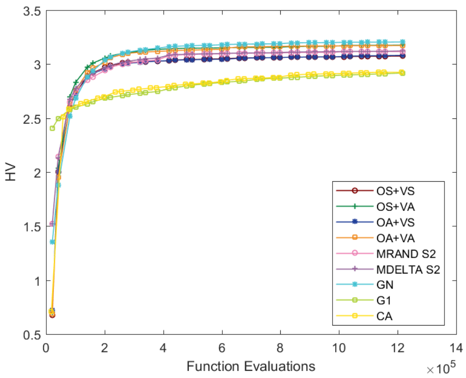


(f) WFG6

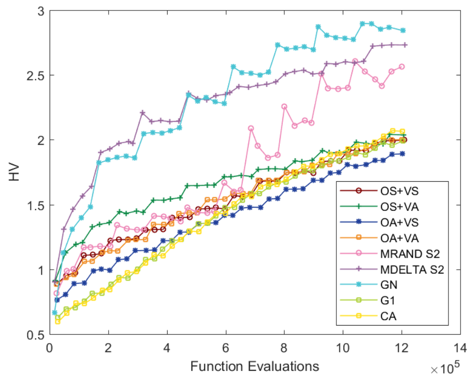
Figure 5.5.: Convergence plots for the median HV values considering the nine algorithms with $n = 200$, Ref_1 and WFG1 - 6, given in Table 5.21



(a) WFG7



(b) WFG8



(c) WFG9

Figure 5.6.: Convergence plots for the median HV values considering the nine algorithms with $n = 200$, Ref_1 and WFG7 - 9, given in Table 5.21

often after approximately $500,000$ FEs, see WFG2,3,4,5,6 and 8. After that, the HV values make no notable changes. The HV values are very good, with a median HV from approximately 3 to 6. This shows that the LSMOP test problems are, in general, more difficult than the ones from the WFG test suite.

In WFG4, 5 and 6 the plots are similar, except for CA whose optimisation starts later, but it can catch up to the others. WFG2 and 3 provide similar plots. One can see clearly that GN performs worst, followed by G1, the two dynamic methods MRAND+S2 and MDELTA+S2. CA and the four MDG2 versions perform best. It is interesting that WFG2 and 3 are the only ones from the test suite which have sparse interactions according to [12], see Subsection 5.3.2. This might be the reason why GN and G1 have had bad performance, because none of their grouping approaches represent sparse interactions, only fully (G1) or no interactions (GN).

WFG1, 4 and 5 have no interactions, the theoretical correct grouping would be n single groups, so GN creates the correct groups. But GN never achieves the best result considering these three problems, see Table 5.21. For WFG1 the counterpart G1 achieves the best results, followed by MDELTA+S2, MRAND+S2, the four MDG2 versions, GN and CA. The fact that G1, which is the other extreme than GN, lead to the best results, show that in this case, the theoretical correct groups are not useful for optimisation. In the plots of WFG4 and 5 all algorithms reach their optimum very fast, no similarities between these both and WFG1 can be seen.

The last category are WFG6,7,8 and 9 which have highly dependent interactions according to MOEA/DVA. MDELTA+S2 performs best for WFG6 and 7, but the majority of the other results are not statistically worse. For WFG8 and 9 GN performs best, in both cases only one result has no significant difference to the best one, which is the one from MRAND+S2 for WFG9. This can also be seen in the corresponding plot, see Figure 5.6 WFG9. All four problems have highly dependent interactions which should lead to bigger groups, but GMs which lead to small groups stiller perform better. Again the theoretical correct groups seem not to be useful for optimisation.

All the differences inside the three classes with no interactions, sparse and highly dependent interactions of the WFG suite show that the variable interactions may not influence the structure of the problem as strong as excepted. There are few connections between the interaction structure and the convergence and results of the problems. Often the exact opposite of the theoretical correct grouping lead to the best results.

In conclusion of the general comparison, one can say that the trivial GMs perform in a lot of cases very good. The other results are very mixed, sometimes a MDG2 version is better, sometimes a dynamic grouping approach. Only CA lead to constantly very bad results because of the great amount of FEs to create the variable groups. But also in the WFG test suite, where enough FEs available after the grouping phase, it can often not catch up to the results of the others.

The TS methods lead, in nearly all cases, to better results than the state-of-the-art GM CA from LMEA. The four methods have in general similar performance than MRAND+S2 when considering the overall comparison Table 5.19, but perform worse than the the trivial methods GN and G1.

When looking at the TS, the split between OA and OS methods is only noticed when considering the LSMOP test suite, it does not occur in the WFG problems. The TSO combine the interactions of both objective functions. The two methods lead for LSMOP to different results because the interactions differ among the objectives, compare the theoretic evaluation in Subsection 5.2.3. That there is no split between the TSO methods in WFG can indicate therefore that the interactions are the same for all two objective functions. This is an interesting result which shows some thing of the internal structure of the WFG test suite.

Another point that becomes clear during the whole evaluation is the relatively good performance of the trivial methods. An explanation for this might lie in the optimisation processes of Ref_1 and Ref_2 itself. Ref_1 works better with small groups like GN and Ref_2 better with big ones. One cannot say that a grouping is good, is has to be seen together with the optimisation process. Therefore no overall *best* groups can be determined which will work together with any other optimisation process.

The state-of-the-art algorithm LMEA, which is equal to Ref_1 with the CA, is outperformed by all other methods. The main reason for this is the high number of FEs used to create the groups, it cannot catch up to the other results in the later optimisation. But also in the WFG problems where enough FEs are available, is is outperformed in most of the cases by the other methods.

5.5. Result Summary

In this section the results, the most important findings are summarised. A theoretic and empirical evaluation was carried out to analyse the different GMs in the environments. The questions from the beginning of this chapter are answered if possible. These questions were defined in the goals of the evaluation, see Section 5.1.

1. Do the GMs find the theoretical correct groups?

This question is difficult to answer. The variable interactions of the LSMOP test suite are defined and were computed, but the interactions differ among the considered objective functions, because of this, no correct groups of variables could be determined. MDG2 detects, in most cases, if an interaction between the variables exist or not per objective. It uses approximately just 10 percent of the computational budget of the grouping method of Correlation Analysis (CA) from LMEA. Whether the final groups are correct could not be determined because this is not defined by the LSMOP test suite, see Subsection 5.2.4.

2. Are the results constant for different optimisation algorithms, benchmark problems and number of variables?

The different optimisation algorithms affect the empirical performance results. Two reference algorithms with subcomponent optimisers from state-of-the-art algorithms were used. A connection between the group-size and the reference algorithms could be noticed. Ref_1 works better with smaller groups, Ref_2 better with bigger variable groups. The different test suites LSMOP and WFG also lead to differing results of the grouping methods. LSMOP prefers smaller groups and WFG bigger ones. The detection of overall *good* grouping methods is difficult because of these dependencies. The decision to execute a very large amount of experiments with different specifications has paid off. With fewer experiments for example only with LSMOP and Ref_1, the false conclusion would be that smaller groups are in general better, or bigger ones if only WFG and Ref_2 were considered. The results are relatively stable for varying number of decision variables.

3. How do the transferred GMs perform against existing multi-objective ones?

The third question deals with the performance of the new approaches against existing ones. The new ones are MDG2 with the four TS, the two dynamic methods MRAND and MDELTA and the trivial ones G1 and GN. In nearly all executed experiments the state-of-the-art approach CA was outperformed by most of the new methods in performance quality and convergence speed. The combination of Ref_1 and CA is equal the state-of-the-art EA LMEA, which was also outperformed.

4. Are complex approaches better than simple ones?

The fourth question addresses the performance difference of simple approaches against complex ones. Complex approaches are MDG2 with the four TS and CA which analyse the interactions of the variables and create groups based on this interactions. Computational budget for optimisation is used for the grouping of the variables. The dynamic methods MRAND and MDELTA do not analyse the variable interactions, but they regroup the variables in every generation. They are not as complex as the intelligent ones. The trivial methods are the simplest ones and just put every variable in a single group (GN) or create one big group (GN). The result is that in a lot of cases the trivial methods are better than the dynamic or intelligent ones. They represent extreme groupings and do not use any FE to obtain the groupings. Fully separable functions lead to very good results for GN, when a lot of interactions occur G1 is often the best one. Contradictory results where GN performs well on non-separable functions and G1 on fully separable were also detected. This shows that in some cases the optimisation with the interacting variable groups might not be the best idea to achieve good results and simple approaches should be considered.

5. Are theoretical and empirical results congruent?

The theoretical results are taken into account in the empirical analysis. Whether the results are congruent can not finally be answered because the theoretically correct groups could not be determined for the LSMOP test suite. A congruent aspect is the analysis of the computational budget of the approaches. MDG2 and the four TS are deterministic and use only ten percent of the FEs of CA. This could also be noticed in the empirical results and was the main reason why CA had a very weak performance. In this thesis the amount of FEs which were used to obtain the grouping is also part of the GM.

The theoretic results of the trivial methods are congruent to the empirical ones in the way that in fully separable functions for example LSMOP1 and 9 GN performs best, which is also the theoretical correct grouping. For these two problems, G1 has a very bad performance which is also plausible because it is the other extreme than GN. However, there are also Contradictory results where a total different method lead to the best result, for example G1 for the fully separable problem LSMOP5.

6. What is more important, the grouping method itself or the how the groups are used in the optimisation process?

The analysis of what is more important, the grouping methods of the optimisation processes can be answered with a trade-off. An overall *good* grouping could not be found, different groups work together better with different problems suites or reference algorithms. The answer is that both are important, the grouping and the optimisation process, but they have to be considered as one unit. The grouping has to work together with the optimisation process and vice versa, they have to stick together. A superior GM which has superior performance in both of the reference algorithms could not be found. Smaller groups work better together with Ref_1, bigger ones with Ref_2.

6. Conclusion and Future Work

In this chapter the results of this work are summed up. The work which was done is summarised and the most important results and findings are presented. It is checked if the goals of the work were fulfilled. The last section gives an overview about future work in this field and new research questions which came up in this thesis.

6.1. Summary

The initial goals of this work were as follows:

1. Develop strategies which transfer single-objective grouping methods to the multi-objective case
2. Examine the capabilities of existing and new approaches theoretically
3. Evaluate the performance of state-of-the-art grouping methods using the proposed Transfer Strategies

In this thesis a lot of work was done. Existing state-of-the-art approaches for single- and multi-objective grouping were described and analysed. Evolutionary Algorithms based on decomposition are presented and used as basis for the comparison of the GMs. In total five new multi-objective GMs are proposed. The two trivial methods G1 and GN, the dynamic ones MRAND and MDELTA and the intelligent MDG2 with the for Transfer Strategies. All of them exist in several different versions. The intelligent ones which are based on the variable interactions are the core of the work. The TS describe several ways for the transfer to multiple objectives. The first goal of the work is fulfilled by this methods.

A theoretic and empirical evaluation was carried out. The theoretic evaluation addresses the second goal. The LSMOP test suite was analysed and the internal interaction structures of the variables are derived. The theoretical correct groups for the optimisation could not be determined because the interactions of variables differ between the multiple objectives. This structure was used and the state-of-the-art GM from LMEA, namely CA was compared against MDG2 in combination with the four TS. CA uses a higher computational budget and do not find as much correct interactions as the TS.

To tackle the third goal a lot of empirical experiments are executed. The GMs are tested against each other by inserting them into two reference algorithms which use different techniques to optimise the groups. A big amount of experiments were carried out to give a great overview over the performance of the GMs. Experiments with the two test suites LSMOP and WFG, the two reference algorithms Ref_1 and Ref_2 and different number of decision variables ranging from 100 to 500 were executed. First the dynamic methods MRAND and MDELTA are compared against each other, after that the TS, the last part was a general comparison of all considered approaches.

A lot of different things are obtained during this work, the main findings are as follows. All developed GMs outperform the state-of-the-art approach CA, the main reason for this is the high computational budget used by it. State-of-the-art EAs could be improved by using any of the five proposed GMs. The combination of CA and the first reference algorithm is identical to the state-of-the-art EA LMEA, which was also outperformed by all proposed methods.

The four TS have differences and in general one can say that the ones which produce smaller groups perform better. Interesting and surprising were the good performance of the trivial methods G1 and GN which just create one overall group or put each variable in a single group. This shows that in some cases the intelligent analysis of the interaction is not promising. No overall superior method could be identified, different methods prefer different reference algorithms, test suites and numbers of decision variables. For instance GMs which produce smaller groups work better together with the LSMOP test suite and the first reference algorithm, methods which produce bigger groups tend to work better with the WFG test suite and the second reference algorithm. This shows that no overall best GM could be identified and that the transfer to the multi-objective case is a complex question which should have more attention.

6.2. Future Work

Some research questions and ideas for the future work are given in this section. The ideas came up during the work of this thesis or after the analysis of the results.

The results show that trivial and dynamic GMs are often better than the intelligent analysis and do not need any FEs. A question is if other algorithms which use intelligent methods could also be improved by simpler grouping approaches? To test this the dynamic versions MRAND and MDELTA as well as the trivial ones GN and G1 could be applied to well-known algorithms which use a lot of FEs for their interaction analysis.

The next idea addresses overlapping subcomponents. All of the considered works on subcomponents consider sharp ones, a real decomposition of the problem. One variable can occur only in one group. In overlapping subcomponents one can soften this up. A variable is allowed to occur in more than one group, for instance the shared variables could be put in all of their groups. Nevertheless, the problem of finding all complete subgraphs is NP-complete and may lead to unacceptable runtime when considering large-scale problems.

The results show that different problems work together better with specific GMs. An idea would be to consider not only one grouping but iterate over a set of groups like the dynamic approaches. The TS and the dynamic and trivial methods do not use any FEs, so creating a pool of groups and iterate or chose one randomly every generation could be promising. A more ambitious approach would be to measure the convergence with the different groupings and select the best one during the optimisation. Such a method could select the actual best grouping and may be an interesting improvement of the GMs in this work.

The previous idea was based on a change of the grouping during an optimisation. Another idea would be the change of the optimisation procedure based on the grouping. Actual state-of-the-art algorithms use one optimisation procedure for fully separable functions which lead to a lot of single groups and non-separable functions which lead to one big group. That one optimisation method could work well with such different groups can be questioned. An idea could be to use different optimisers for different kinds of groups. For example use Ref_1 for smaller and Ref_2 for bigger ones.

Appendices

List of Appendix Tables

A. Multidimensional Random Grouping

The tables which are too big for the evaluation in the thesis are depicted in this appendices. The information of the tables are presented in the thesis in an abbreviated form. The table specifications are described in 5.3.5.

In this Section the tables to compare the different versions of the Multidimensional Random Grouping (MRAND) against each other are depicted. Each table compares four MRAND versions (SX or NX with $X = \{2, 5, 20, 50\}$ respectively) against each other. Each table contains the IGD and HV values for the four MRAND versions for 100 and 200 *queried* decision variables, this aspects of the tables remain the same. The varying parts of the tables are the reference algorithm, the version group and the problem suite. The combination of the two reference algorithms Ref_1 and Ref_2, the two test suites LSMOP and WFG and the two version groups SX and NX lead to a total of eight tables.

Table A.1.: Median IGD and HV values for Ref_1 with the four NX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	*2.7400e-1*$^{(-)}$	2.0626e-1$^{(-)}$	1.0977e-1$^{(-)}$	**6.2296e-2**$^{(r)}$
		P2	*8.4366e-2*$^{(-)}$	6.3185e-2$^{(-)}$	4.5343e-2$^{(-)}$	**3.7551e-2**$^{(r)}$
		P3	1.0729e+0$^{(*)}$	1.1096e+0$^{(*)}$	1.1014e+0$^{(*)}$	1.0425e+0$^{(*)}$
		P4	*1.9172e-1*$^{(-)}$	1.4242e-1$^{(-)}$	6.6279e-2$^{(-)}$	**5.1230e-2**$^{(r)}$
		P5	*7.4175e-1*$^{(o)}$	7.1810e-1$^{(o)}$	**5.2581e-1**$^{(r)}$	7.3897e-1$^{(o)}$
		P6	*1.0259e+0*$^{(-)}$	7.9916e-1$^{(-)}$	7.4410e-1$^{(o)}$	**7.4366e-1**$^{(r)}$
		P7	1.8561e+0$^{(*)}$	1.7525e+0$^{(*)}$	1.3596e+0$^{(*)}$	1.3463e+0$^{(*)}$
		P8	*3.6942e-1*$^{(-)}$	3.5153e-1$^{(-)}$	1.9519e-1$^{(-)}$	**1.4120e-1**$^{(r)}$
		P9	*6.9512e-1*$^{(-)}$	6.3336e-1$^{(-)}$	**5.2909e-1**$^{(r)}$	5.5040e-1$^{(o)}$
	HV	P1	*2.7807e-1*$^{(-)}$	3.9015e-1$^{(-)}$	5.3565e-1$^{(-)}$	**6.0019e-1**$^{(r)}$
		P2	*5.7477e-1*$^{(-)}$	6.0511e-1$^{(-)}$	6.3176e-1$^{(-)}$	**6.4287e-1**$^{(r)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	*4.2890e-1*$^{(-)}$	4.8456e-1$^{(-)}$	5.9120e-1$^{(-)}$	**6.1560e-1**$^{(r)}$
		P5	**1.1000e-1**$^{(r)}$	*1.0996e-1*$^{(o)}$	1.0997e-1$^{(o)}$	1.0998e-1$^{(o)}$
		P6	—$^{(-)}$	—$^{(-)}$	**9.4737e-2**$^{(r)}$	9.3188e-2$^{(o)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	*1.1000e-1*$^{(-)}$	1.1065e-1$^{(-)}$	2.0370e-1$^{(-)}$	**2.4004e-1**$^{(r)}$
		P9	*4.5541e-1*$^{(-)}$	5.1789e-1$^{(-)}$	6.0500e-1$^{(o)}$	**6.1810e-1**$^{(r)}$
n=200	IGD	P1	*3.6339e-1*$^{(-)}$	3.1462e-1$^{(-)}$	1.9768e-1$^{(-)}$	**1.5515e-1**$^{(r)}$
		P2	*7.3535e-2*$^{(-)}$	5.7939e-2$^{(-)}$	4.1273e-2$^{(-)}$	**3.7901e-2**$^{(r)}$
		P3	*1.0940e+0*$^{(-)}$	9.9980e-1$^{(o)}$	**9.5796e-1**$^{(r)}$	9.8849e-1$^{(o)}$
		P4	*1.2939e-1*$^{(-)}$	1.1310e-1$^{(-)}$	7.5867e-2$^{(-)}$	**5.4709e-2**$^{(r)}$
		P5	*7.2173e-1*$^{(-)}$	7.1093e-1$^{(-)}$	**6.0293e-1**$^{(r)}$	6.2002e-1$^{(o)}$
		P6	*1.5960e+0*$^{(-)}$	8.5246e-1$^{(-)}$	7.6716e-1$^{(-)}$	**7.2950e-1**$^{(r)}$
		P7	1.6558e+0$^{(*)}$	1.8312e+0$^{(*)}$	1.6872e+0$^{(*)}$	1.4343e+0$^{(*)}$
		P8	*3.5535e-1*$^{(-)}$	3.5468e-1$^{(-)}$	2.7779e-1$^{(-)}$	**1.5607e-1**$^{(r)}$
		P9	*8.1018e-1*$^{(-)}$	6.4452e-1$^{(-)}$	5.7270e-1$^{(-)}$	**5.2326e-1**$^{(r)}$
	HV	P1	*2.3169e-1*$^{(-)}$	2.8599e-1$^{(-)}$	4.1332e-1$^{(-)}$	**4.8203e-1**$^{(r)}$
		P2	*5.9207e-1*$^{(-)}$	6.1449e-1$^{(-)}$	6.3840e-1$^{(-)}$	**6.4354e-1**$^{(r)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	*5.1365e-1*$^{(-)}$	5.3555e-1$^{(-)}$	5.8232e-1$^{(-)}$	**6.1430e-1**$^{(r)}$
		P5	**1.0989e-1**$^{(r)}$	1.0961e-1$^{(o)}$	1.0983e-1$^{(o)}$	*1.0960e-1*$^{(o)}$
		P6	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$	**8.6111e-3**$^{(r)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	1.1000e-1$^{(-)}$	*1.0977e-1*$^{(-)}$	1.5183e-1$^{(-)}$	**2.2505e-1**$^{(r)}$
		P9	*3.1629e-1*$^{(-)}$	4.8611e-1$^{(-)}$	5.6250e-1$^{(-)}$	**6.2226e-1**$^{(r)}$

Table A.2.: Median IGD and HV values for Ref_1 with the four NX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	1.1037e+0$^{(*)}$	1.1140e+0$^{(*)}$	1.1098e+0$^{(*)}$	1.2005e+0$^{(*)}$
		P2	*4.8167e-2*$^{(-)}$	3.5871e-2$^{(-)}$	**2.1272e-2**$^{(r)}$	3.6181e-2$^{(-)}$
		P3	*5.1372e-2*$^{(-)}$	4.4456e-2$^{(-)}$	**2.1593e-2**$^{(r)}$	3.2809e-2$^{(-)}$
		P4	**1.3742e-2**$^{(r)}$	1.3769e-2$^{(o)}$	1.3919e-2$^{(o)}$	*1.3967e-2*$^{(o)}$
		P5	6.9798e-2$^{(o)}$	*6.9877e-2*$^{(-)}$	6.9841e-2$^{(-)}$	**6.9698e-2**$^{(r)}$
		P6	**1.9207e-2**$^{(r)}$	*1.9593e-2*$^{(o)}$	1.9422e-2$^{(o)}$	1.9477e-2$^{(o)}$
		P7	**7.8411e-2**$^{(r)}$	8.2875e-2$^{(o)}$	*8.6748e-2*$^{(o)}$	8.5730e-2$^{(-)}$
		P8	*5.7826e-2*$^{(-)}$	**5.3720e-2**$^{(r)}$	5.4088e-2$^{(o)}$	5.5039e-2$^{(o)}$
		P9	1.3921e-1$^{(-)}$	*1.4792e-1*$^{(-)}$	**9.6983e-2**$^{(r)}$	9.7740e-2$^{(o)}$
	HV	P1	2.6158e+0$^{(o)}$	**2.7082e+0**$^{(r)}$	2.6481e+0$^{(o)}$	1.7517e+0$^{(-)}$
		P2	5.8904e+0$^{(-)}$	5.9571e+0$^{(-)}$	**6.0566e+0**$^{(r)}$	5.9608e+0$^{(-)}$
		P3	5.3927e+0$^{(-)}$	5.4295e+0$^{(-)}$	**5.5567e+0**$^{(r)}$	5.4923e+0$^{(-)}$
		P4	3.3614e+0$^{(o)}$	3.3613e+0$^{(o)}$	**3.3622e+0**$^{(r)}$	3.3615e+0$^{(o)}$
		P5	2.9803e+0$^{(o)}$	2.9804e+0$^{(o)}$	2.9802e+0$^{(-)}$	**2.9809e+0**$^{(r)}$
		P6	3.2958e+0$^{(o)}$	3.2945e+0$^{(o)}$	**3.2963e+0**$^{(r)}$	3.2953e+0$^{(o)}$
		P7	**2.9888e+0**$^{(r)}$	2.9824e+0$^{(o)}$	2.9681e+0$^{(o)}$	2.9484e+0$^{(-)}$
		P8	3.0790e+0$^{(-)}$	3.1152e+0$^{(o)}$	**3.1226e+0**$^{(r)}$	3.1162e+0$^{(o)}$
		P9	2.6257e+0$^{(-)}$	2.6192e+0$^{(-)}$	2.8434e+0$^{(o)}$	**2.9121e+0**$^{(r)}$
n=200	IGD	P1	1.2078e+0$^{(*)}$	1.1594e+0$^{(*)}$	1.1408e+0$^{(*)}$	1.1804e+0$^{(*)}$
		P2	*1.1425e-1*$^{(-)}$	8.3900e-2$^{(-)}$	4.3363e-2$^{(-)}$	**2.6449e-2**$^{(r)}$
		P3	*1.2275e-1*$^{(-)}$	9.1094e-2$^{(-)}$	4.6129e-2$^{(-)}$	**2.4979e-2**$^{(r)}$
		P4	1.4882e-2$^{(o)}$	**1.4626e-2**$^{(r)}$	*1.5231e-2*$^{(o)}$	1.4996e-2$^{(o)}$
		P5	**6.9838e-2**$^{(r)}$	*6.9913e-2*$^{(o)}$	6.9899e-2$^{(o)}$	6.9889e-2$^{(o)}$
		P6	*1.5944e-2*$^{(o)}$	1.5814e-2$^{(o)}$	1.5930e-2$^{(o)}$	**1.5697e-2**$^{(r)}$
		P7	1.4898e-1$^{(o)}$	**1.3568e-1**$^{(r)}$	1.5692e-1$^{(o)}$	*1.8817e-1*$^{(o)}$
		P8	*6.2303e-2*$^{(-)}$	5.7707e-2$^{(-)}$	**5.1571e-2**$^{(r)}$	5.2341e-2$^{(o)}$
		P9	2.7451e-1$^{(-)}$	*2.9410e-1*$^{(-)}$	2.4588e-1$^{(-)}$	**1.1877e-1**$^{(r)}$
	HV	P1	2.1139e+0$^{(-)}$	2.3746e+0$^{(o)}$	**2.4725e+0**$^{(r)}$	2.2030e+0$^{(-)}$
		P2	5.5123e+0$^{(-)}$	5.6816e+0$^{(-)}$	5.9153e+0$^{(-)}$	**6.0286e+0**$^{(r)}$
		P3	5.0428e+0$^{(-)}$	5.1951e+0$^{(-)}$	5.4212e+0$^{(-)}$	**5.5349e+0**$^{(r)}$
		P4	**3.3616e+0**$^{(r)}$	3.3615e+0$^{(o)}$	3.3616e+0$^{(o)}$	3.3612e+0$^{(o)}$
		P5	**2.9799e+0**$^{(r)}$	2.9798e+0$^{(o)}$	2.9798e+0$^{(o)}$	2.9798e+0$^{(o)}$
		P6	3.3266e+0$^{(o)}$	3.3264e+0$^{(o)}$	3.3269e+0$^{(o)}$	**3.3275e+0**$^{(r)}$
		P7	2.6229e+0$^{(o)}$	**2.6874e+0**$^{(r)}$	2.5517e+0$^{(o)}$	2.3980e+0$^{(o)}$
		P8	3.0510e+0$^{(-)}$	3.0780e+0$^{(-)}$	3.1158e+0$^{(o)}$	**3.1226e+0**$^{(r)}$
		P9	1.9561e+0$^{(-)}$	1.8809e+0$^{(-)}$	2.0863e+0$^{(-)}$	**2.7248e+0**$^{(r)}$

Table A.3.: Median IGD and HV values for Ref_1 with the four SX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	**3.5644e-2**$^{(r)}$	9.8204e-2$^{(-)}$	2.0613e-1$^{(-)}$	*2.4558e-1*$^{(-)}$
		P2	**3.5003e-2**$^{(r)}$	4.2684e-2$^{(-)}$	6.1535e-2$^{(-)}$	*7.8274e-2*$^{(-)}$
		P3	**9.6557e-1**$^{(r)}$	*1.1139e+0*$^{(-)}$	1.0792e+0$^{(-)}$	1.0801e+0$^{(-)}$
		P4	**4.9708e-2**$^{(r)}$	6.1940e-2$^{(-)}$	1.3595e-1$^{(-)}$	*1.8783e-1*$^{(-)}$
		P5	**4.7698e-1**$^{(r)}$	6.5900e-1$^{(o)}$	6.9991e-1$^{(o)}$	*7.3566e-1*$^{(o)}$
		P6	**7.3005e-1**$^{(r)}$	7.4358e-1$^{(o)}$	7.5859e-1$^{(-)}$	*1.0256e+0*$^{(-)}$
		P7	1.3473e+0$^{(*)}$	1.2872e+0$^{(*)}$	1.6915e+0$^{(*)}$	1.8088e+0$^{(*)}$
		P8	**1.3515e-1**$^{(r)}$	2.7128e-1$^{(-)}$	3.4453e-1$^{(-)}$	*3.4525e-1*$^{(-)}$
		P9	**4.8592e-1**$^{(r)}$	4.9697e-1$^{(o)}$	*6.3023e-1*$^{(-)}$	5.9313e-1$^{(-)}$
	HV	P1	**6.4324e-1**$^{(r)}$	5.3201e-1$^{(-)}$	3.8188e-1$^{(-)}$	*3.4652e-1*$^{(-)}$
		P2	**6.4725e-1**$^{(r)}$	6.3525e-1$^{(-)}$	6.0686e-1$^{(-)}$	*5.8113e-1*$^{(-)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	**6.1720e-1**$^{(r)}$	5.9743e-1$^{(-)}$	4.9144e-1$^{(-)}$	*4.3690e-1*$^{(-)}$
		P5	*1.0997e-1*$^{(o)}$	**1.0999e-1**$^{(r)}$	1.0999e-1$^{(o)}$	1.0998e-1$^{(o)}$
		P6	8.9504e-2$^{(o)}$	**1.0033e-1**$^{(r)}$	—$^{(-)}$	—$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**2.5430e-1**$^{(r)}$	1.5744e-1$^{(-)}$	1.1916e-1$^{(-)}$	*1.1641e-1*$^{(-)}$
		P9	**6.8276e-1**$^{(r)}$	6.2691e-1$^{(-)}$	*4.8829e-1*$^{(-)}$	4.9905e-1$^{(-)}$
n=200	IGD	P1	**5.3233e-2**$^{(r)}$	1.3712e-1$^{(-)}$	2.4353e-1$^{(-)}$	*2.7995e-1*$^{(-)}$
		P2	**3.4332e-2**$^{(r)}$	3.7419e-2$^{(-)}$	4.7660e-2$^{(-)}$	*6.1403e-2*$^{(-)}$
		P3	9.6753e-1$^{(o)}$	**9.5958e-1**$^{(r)}$	*1.0969e+0*$^{(-)}$	1.0655e+0$^{(-)}$
		P4	**4.6468e-2**$^{(r)}$	5.6051e-2$^{(-)}$	9.4444e-2$^{(-)}$	*1.1626e-1*$^{(-)}$
		P5	**4.8913e-1**$^{(r)}$	5.3432e-1$^{(o)}$	6.3632e-1$^{(o)}$	*7.1424e-1*$^{(o)}$
		P6	**6.5753e-1**$^{(r)}$	7.1073e-1$^{(-)}$	8.2385e-1$^{(-)}$	*8.6140e-1*$^{(-)}$
		P7	1.4339e+0$^{(*)}$	1.4426e+0$^{(*)}$	1.8060e+0$^{(*)}$	1.7428e+0$^{(*)}$
		P8	**1.1585e-1**$^{(r)}$	2.4335e-1$^{(-)}$	3.4477e-1$^{(-)}$	*3.5913e-1*$^{(-)}$
		P9	**4.9482e-1**$^{(r)}$	5.5649e-1$^{(-)}$	6.0563e-1$^{(-)}$	*7.3364e-1*$^{(-)}$
	HV	P1	**6.1394e-1**$^{(r)}$	4.9490e-1$^{(-)}$	3.7374e-1$^{(-)}$	*2.9897e-1*$^{(-)}$
		P2	**6.4892e-1**$^{(r)}$	6.4440e-1$^{(-)}$	6.2903e-1$^{(-)}$	*6.0921e-1*$^{(-)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	**6.2738e-1**$^{(r)}$	6.1265e-1$^{(-)}$	5.5965e-1$^{(-)}$	*5.3174e-1*$^{(-)}$
		P5	*1.0929e-1*$^{(-)}$	1.0936e-1$^{(-)}$	1.0972e-1$^{(o)}$	**1.0980e-1**$^{(r)}$
		P6	**1.1120e-2**$^{(r)}$	—$^{(o)}$	—$^{(-)}$	—$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**2.5841e-1**$^{(r)}$	1.4409e-1$^{(-)}$	1.0980e-1$^{(-)}$	*1.0972e-1*$^{(-)}$
		P9	**6.7104e-1**$^{(r)}$	5.8603e-1$^{(-)}$	5.4315e-1$^{(-)}$	*4.0006e-1*$^{(-)}$

Table A.4.: Median IGD and HV values for Ref_1 with the four SX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	*4.8609e-1*$^{(-)}$	2.3759e-1$^{(-)}$	**1.2796e-1**$^{(r)}$	1.6691e-1$^{(-)}$
		P2	*1.0999e-1*$^{(-)}$	6.6879e-2$^{(-)}$	6.0075e-2$^{(o)}$	**5.8727e-2**$^{(r)}$
		P3	*1.1817e-1*$^{(-)}$	6.3779e-2$^{(o)}$	6.4820e-2$^{(o)}$	**5.6611e-2**$^{(r)}$
		P4	**1.3899e-2**$^{(r)}$	1.3925e-2$^{(o)}$	1.3899e-2$^{(o)}$	*1.3975e-2*$^{(o)}$
		P5	6.3712e-2$^{(o)}$	**6.3675e-2**$^{(r)}$	6.3725e-2$^{(o)}$	*6.3731e-2*$^{(o)}$
		P6	*1.9132e-2*$^{(-)}$	1.8911e-2$^{(-)}$	1.7566e-2$^{(-)}$	**1.6165e-2**$^{(r)}$
		P7	*1.4207e-2*$^{(-)}$	1.4153e-2$^{(-)}$	**1.3961e-2**$^{(r)}$	1.3997e-2$^{(o)}$
		P8	5.5198e-2$^{(-)}$	**5.0650e-2**$^{(r)}$	7.3437e-2$^{(-)}$	*8.8046e-2*$^{(-)}$
		P9	*3.4599e-2*$^{(-)}$	2.6560e-2$^{(o)}$	2.3468e-2$^{(o)}$	**2.3365e-2**$^{(r)}$
	HV	P1	4.2942e+0$^{(-)}$	5.6177e+0$^{(-)}$	**6.0504e+0**$^{(r)}$	5.8715e+0$^{(-)}$
		P2	5.5364e+0$^{(-)}$	5.7733e+0$^{(-)}$	5.8220e+0$^{(o)}$	**5.8245e+0**$^{(r)}$
		P3	5.0641e+0$^{(-)}$	5.3314e+0$^{(o)}$	5.3263e+0$^{(o)}$	**5.3676e+0**$^{(r)}$
		P4	3.3599e+0$^{(-)}$	3.3600e+0$^{(-)}$	3.3613e+0$^{(o)}$	**3.3618e+0**$^{(r)}$
		P5	3.0329e+0$^{(-)}$	3.0336e+0$^{(-)}$	**3.0342e+0**$^{(r)}$	3.0340e+0$^{(o)}$
		P6	3.2998e+0$^{(-)}$	3.3003e+0$^{(-)}$	3.3125e+0$^{(-)}$	**3.3235e+0**$^{(r)}$
		P7	3.3592e+0$^{(-)}$	3.3628e+0$^{(o)}$	3.3633e+0$^{(o)}$	**3.3633e+0**$^{(r)}$
		P8	3.1068e+0$^{(-)}$	**3.1255e+0**$^{(r)}$	2.9987e+0$^{(-)}$	2.9160e+0$^{(-)}$
		P9	3.1717e+0$^{(-)}$	3.2395e+0$^{(o)}$	**3.2538e+0**$^{(r)}$	3.2528e+0$^{(o)}$
n=200	IGD	P1	1.2919e+0$^{(*)}$	1.1901e+0$^{(*)}$	1.1756e+0$^{(*)}$	1.1838e+0$^{(*)}$
		P2	5.4538e-2$^{(-)}$	**2.6804e-2**$^{(r)}$	6.1810e-2$^{(-)}$	*8.7214e-2*$^{(-)}$
		P3	5.5815e-2$^{(-)}$	**2.7074e-2**$^{(r)}$	6.5481e-2$^{(-)}$	*9.6891e-2*$^{(-)}$
		P4	*1.5040e-2*$^{(o)}$	1.5027e-2$^{(o)}$	**1.4644e-2**$^{(r)}$	1.4695e-2$^{(o)}$
		P5	**6.9703e-2**$^{(r)}$	*6.9943e-2*$^{(-)}$	6.9891e-2$^{(-)}$	6.9937e-2$^{(o)}$
		P6	1.5865e-2$^{(o)}$	1.5842e-2$^{(o)}$	**1.5764e-2**$^{(r)}$	*1.5905e-2*$^{(o)}$
		P7	**1.4802e-1**$^{(r)}$	*1.7513e-1*$^{(o)}$	1.6669e-1$^{(o)}$	1.5880e-1$^{(o)}$
		P8	**5.1535e-2**$^{(r)}$	5.2159e-2$^{(o)}$	5.5517e-2$^{(-)}$	*5.9065e-2*$^{(-)}$
		P9	**1.5105e-1**$^{(r)}$	*2.8796e-1*$^{(-)}$	2.7340e-1$^{(-)}$	2.5673e-1$^{(-)}$
	HV	P1	1.3132e+0$^{(-)}$	2.2053e+0$^{(o)}$	**2.3133e+0**$^{(r)}$	2.1665e+0$^{(o)}$
		P2	5.8528e+0$^{(-)}$	**6.0201e+0**$^{(r)}$	5.8062e+0$^{(-)}$	5.6605e+0$^{(-)}$
		P3	5.3696e+0$^{(-)}$	**5.5233e+0**$^{(r)}$	5.3207e+0$^{(-)}$	5.1630e+0$^{(-)}$
		P4	**3.3616e+0**$^{(r)}$	3.3613e+0$^{(o)}$	3.3612e+0$^{(o)}$	3.3609e+0$^{(o)}$
		P5	**2.9801e+0**$^{(r)}$	2.9797e+0$^{(o)}$	2.9797e+0$^{(o)}$	2.9800e+0$^{(o)}$
		P6	3.3268e+0$^{(o)}$	**3.3275e+0**$^{(r)}$	3.3275e+0$^{(o)}$	3.3263e+0$^{(o)}$
		P7	**2.6022e+0**$^{(r)}$	2.4680e+0$^{(o)}$	2.5270e+0$^{(o)}$	2.5346e+0$^{(o)}$
		P8	3.1166e+0$^{(o)}$	**3.1207e+0**$^{(r)}$	3.0972e+0$^{(-)}$	3.0663e+0$^{(-)}$
		P9	**2.5635e+0**$^{(r)}$	1.8832e+0$^{(-)}$	1.9870e+0$^{(-)}$	2.0260e+0$^{(-)}$

Table A.5.: Median IGD and HV values for Ref_2 with the four NX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	**1.9324e-1**(r)	2.9548e-1(−)	*3.3660e-1*(−)	2.3010e-1(−)
		P2	*5.8268e-2*(−)	5.4114e-2(−)	3.4485e-2(−)	**2.6401e-2**(r)
		P3	**5.9055e-1**(r)	7.0706e-1(−)	7.0706e-1(−)	*7.0714e-1*(−)
		P4	*1.2451e-1*(−)	8.3085e-2(−)	3.5243e-2(−)	**2.7863e-2**(r)
		P5	*7.4208e-1*(−)	3.4242e-1(−)	3.4241e-1(−)	**3.4134e-1**(r)
		P6	*7.4313e-1*(−)	7.4260e-1(−)	7.4275e-1(−)	**7.4225e-1**(r)
		P7	1.2724e+0(*)	1.2693e+0(*)	1.2800e+0(*)	1.3443e+0(*)
		P8	*3.8830e-1*(−)	3.8298e-1(−)	3.7518e-1(o)	**3.7439e-1**(r)
		P9	*8.1004e-1*(−)	8.1004e-1(−)	2.5425e-1(o)	**1.9003e-1**(r)
	HV	P1	**4.2627e-1**(r)	2.3862e-1(−)	*2.3317e-1*(−)	3.8431e-1(−)
		P2	*6.2040e-1*(−)	6.2649e-1(−)	6.5444e-1(−)	**6.6535e-1**(r)
		P3	1.0997e-1(o)	**1.0999e-1**(r)	1.0999e-1(o)	*1.0992e-1*(−)
		P4	*5.1295e-1*(−)	5.6243e-1(−)	6.4590e-1(−)	**6.5861e-1**(r)
		P5	*1.1000e-1*(−)	2.0999e-1(−)	**2.1000e-1**(r)	2.0928e-1(−)
		P6	*1.0688e-1*(−)	**1.0835e-1**(r)	1.0780e-1(−)	1.0789e-1(o)
		P7	—(*)	—(*)	—(*)	—(*)
		P8	1.1000e-1(o)	**1.1000e-1**(r)	1.1000e-1(o)	*1.1000e-1*(o)
		P9	*3.7814e-1*(−)	3.7814e-1(−)	6.6734e-1(−)	**7.2415e-1**(r)
n=200	IGD	P1	*3.5920e-1*(−)	**2.9159e-1**(r)	3.4004e-1(−)	3.3773e-1(−)
		P2	*8.1021e-2*(−)	4.4233e-2(−)	3.8483e-2(−)	**3.1986e-2**(r)
		P3	*9.9549e-1*(−)	**7.0713e-1**(r)	7.0719e-1(o)	7.0856e-1(−)
		P4	*8.3604e-2*(−)	7.2321e-2(−)	2.9305e-2(−)	**2.1436e-2**(r)
		P5	*7.4205e-1*(−)	7.4205e-1(−)	**3.4199e-1**(r)	4.6949e-1(−)
		P6	7.5542e-1(o)	7.5410e-1(−)	*7.5727e-1*(−)	**7.5073e-1**(r)
		P7	1.6727e+0(*)	1.7166e+0(*)	1.7767e+0(*)	2.1140e+0(*)
		P8	*3.8453e-1*(−)	3.6512e-1(−)	3.6388e-1(−)	**3.6345e-1**(r)
		P9	8.1004e-1(−)	8.1004e-1(−)	*8.1004e-1*(−)	**2.1315e-1**(r)
	HV	P1	**3.3833e-1**(r)	2.7954e-1(−)	*2.2473e-1*(−)	2.3005e-1(−)
		P2	*5.8283e-1*(−)	6.4147e-1(−)	6.5008e-1(−)	**6.5918e-1**(r)
		P3	—(−)	**1.0980e-1**(r)	1.0973e-1(o)	1.0774e-1(−)
		P4	*5.7729e-1*(−)	5.9039e-1(−)	6.5831e-1(−)	**6.7075e-1**(r)
		P5	1.1000e-1(−)	1.1000e-1(−)	**2.0969e-1**(r)	*1.0892e-1*(−)
		P6	*5.3863e-2*(−)	7.1126e-2(−)	7.1651e-2(−)	**8.5659e-2**(r)
		P7	—(*)	—(*)	—(*)	—(*)
		P8	1.1000e-1(−)	**1.1000e-1**(r)	1.1000e-1(−)	*1.1000e-1*(−)
		P9	3.7814e-1(−)	3.7814e-1(−)	*3.7814e-1*(−)	**7.0092e-1**(r)

Table A.6.: Median IGD and HV values for Ref_2 with the four NX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	1.7029e-1(−)	**1.4960e-1**(r)	2.3391e-1(−)	*4.9699e-1*(−)
		P2	**5.7117e-2**(r)	5.9771e-2(o)	7.1163e-2(−)	*1.0970e-1*(−)
		P3	5.9801e-2(o)	**5.2489e-2**(r)	6.0490e-2(o)	*1.1851e-1*(−)
		P4	1.3943e-2(−)	**1.3811e-2**(r)	*1.3963e-2*(−)	1.3867e-2(o)
		P5	*6.3746e-2*(o)	6.3740e-2(o)	**6.3661e-2**(r)	6.3731e-2(o)
		P6	**1.6337e-2**(r)	1.8209e-2(−)	1.9004e-2(−)	*1.9287e-2*(−)
		P7	1.4174e-2(o)	**1.4170e-2**(r)	1.4261e-2(o)	*1.4385e-2*(−)
		P8	*8.7137e-2*(−)	7.3483e-2(−)	**5.1138e-2**(r)	5.4206e-2(−)
		P9	**2.2077e-2**(r)	2.3535e-2(o)	3.4664e-2(−)	*3.4939e-2*(−)
	HV	P1	5.8543e+0(−)	**5.9331e+0**(r)	5.6514e+0(−)	4.3094e+0(−)
		P2	**5.8391e+0**(r)	5.8209e+0(o)	5.7516e+0(−)	5.5377e+0(−)
		P3	5.3513e+0(o)	**5.3883e+0**(r)	5.3488e+0(o)	5.0627e+0(−)
		P4	**3.3620e+0**(r)	3.3616e+0(o)	3.3603e+0(−)	3.3594e+0(−)
		P5	**3.0345e+0**(r)	3.0343e+0(o)	3.0340e+0(o)	3.0327e+0(−)
		P6	**3.3231e+0**(r)	3.3099e+0(−)	3.3012e+0(−)	3.2958e+0(−)
		P7	**3.3639e+0**(r)	3.3631e+0(−)	3.3605e+0(−)	3.3566e+0(−)
		P8	2.9183e+0(−)	3.0096e+0(−)	**3.1303e+0**(r)	3.1109e+0(o)
		P9	**3.2588e+0**(r)	3.2555e+0(o)	3.1717e+0(−)	3.1708e+0(−)
n=200	IGD	P1	1.8070e-1(−)	**1.4557e-1**(r)	2.6345e-1(−)	*4.9224e-1*(−)
		P2	**1.1475e-1**(r)	1.2279e-1(−)	1.3084e-1(−)	*1.4078e-1*(−)
		P3	**1.2414e-1**(r)	1.3009e-1(o)	1.3051e-1(o)	*1.4268e-1*(−)
		P4	1.4105e-2(o)	**1.4055e-2**(r)	1.4171e-2(−)	*1.4277e-2*(−)
		P5	6.3753e-2(−)	6.3749e-2(−)	**6.3659e-2**(r)	*6.3791e-2*(−)
		P6	**1.4573e-2**(r)	1.4906e-2(−)	1.5477e-2(−)	*1.5554e-2*(−)
		P7	**1.4073e-2**(r)	1.4091e-2(o)	1.4277e-2(−)	*1.5187e-2*(−)
		P8	*1.0267e-1*(−)	8.3953e-2(−)	6.1440e-2(−)	**5.3328e-2**(r)
		P9	**2.1187e-2**(r)	2.2612e-2(o)	2.8304e-2(−)	*3.1725e-2*(−)
	HV	P1	5.8037e+0(−)	**5.9749e+0**(r)	5.4440e+0(−)	4.2666e+0(−)
		P2	**5.5105e+0**(r)	5.4677e+0(−)	5.4211e+0(−)	5.3704e+0(−)
		P3	**5.0356e+0**(r)	5.0076e+0(o)	5.0047e+0(o)	4.9481e+0(−)
		P4	**3.3578e+0**(r)	3.3572e+0(−)	3.3526e+0(−)	3.3508e+0(−)
		P5	3.0343e+0(o)	**3.0343e+0**(r)	3.0340e+0(o)	3.0326e+0(−)
		P6	**3.3428e+0**(r)	3.3378e+0(−)	3.3314e+0(−)	3.3306e+0(−)
		P7	**3.3634e+0**(r)	3.3629e+0(o)	3.3544e+0(−)	3.3420e+0(−)
		P8	2.8295e+0(−)	2.9317e+0(−)	3.0593e+0(−)	**3.1126e+0**(r)
		P9	**3.2657e+0**(r)	3.2642e+0(o)	3.2235e+0(−)	3.2014e+0(−)

Table A.7.: Median IGD and HV values for Ref_2 with the four SX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	**1.4588e-1**$^{(r)}$	*3.3148e-1*$^{(-)}$	3.2855e-1$^{(-)}$	1.9843e-1$^{(-)}$
		P2	**2.3937e-2**$^{(r)}$	3.1735e-2$^{(-)}$	*4.9413e-2*$^{(-)}$	4.2527e-2$^{(-)}$
		P3	*7.0735e-1*$^{(-)}$	7.0711e-1$^{(-)}$	7.0707e-1$^{(-)}$	**4.8494e-1**$^{(r)}$
		P4	**2.4374e-2**$^{(r)}$	3.4823e-2$^{(-)}$	5.7727e-2$^{(-)}$	*6.0555e-2*$^{(-)}$
		P5	**3.4110e-1**$^{(r)}$	3.4242e-1$^{(-)}$	7.4054e-1$^{(-)}$	*7.4208e-1*$^{(-)}$
		P6	**6.3628e-1**$^{(r)}$	7.4246e-1$^{(o)}$	7.4263e-1$^{(o)}$	*7.4293e-1*$^{(-)}$
		P7	1.3645e+0$^{(*)}$	1.3388e+0$^{(*)}$	1.3033e+0$^{(*)}$	1.2748e+0$^{(*)}$
		P8	**3.7289e-1**$^{(r)}$	3.8191e-1$^{(-)}$	*3.8537e-1*$^{(-)}$	3.8494e-1$^{(-)}$
		P9	**1.5321e-1**$^{(r)}$	2.6699e-1$^{(-)}$	*8.1004e-1*$^{(-)}$	8.1004e-1$^{(-)}$
	HV	P1	**5.0182e-1**$^{(r)}$	2.3334e-1$^{(-)}$	*2.3299e-1*$^{(-)}$	4.2125e-1$^{(-)}$
		P2	**6.6844e-1**$^{(r)}$	6.5892e-1$^{(-)}$	*6.3319e-1*$^{(-)}$	6.4280e-1$^{(-)}$
		P3	*1.0962e-1*$^{(-)}$	1.0999e-1$^{(-)}$	1.0999e-1$^{(-)}$	**1.2784e-1**$^{(r)}$
		P4	**6.6468e-1**$^{(r)}$	6.4692e-1$^{(-)}$	6.0581e-1$^{(-)}$	*6.0052e-1*$^{(-)}$
		P5	2.0730e-1$^{(-)}$	**2.1000e-1**$^{(r)}$	1.1000e-1$^{(-)}$	*1.1000e-1*$^{(-)}$
		P6	*1.0528e-1*$^{(-)}$	1.0792e-1$^{(o)}$	**1.0830e-1**$^{(r)}$	1.0727e-1$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	1.1000e-1$^{(o)}$	1.1000e-1$^{(-)}$	**1.1000e-1**$^{(r)}$	*1.1000e-1*$^{(-)}$
		P9	**7.6316e-1**$^{(r)}$	6.3370e-1$^{(-)}$	*3.7814e-1*$^{(-)}$	3.7814e-1$^{(-)}$
n=200	IGD	P1	**1.3970e-1**$^{(r)}$	3.3701e-1$^{(-)}$	*3.4012e-1*$^{(-)}$	2.6815e-1$^{(-)}$
		P2	3.2586e-2$^{(o)}$	**3.2556e-2**$^{(r)}$	*3.9253e-2*$^{(-)}$	3.7577e-2$^{(-)}$
		P3	*9.5222e-1*$^{(-)}$	7.0819e-1$^{(-)}$	7.0715e-1$^{(o)}$	**7.0701e-1**$^{(r)}$
		P4	**1.7460e-2**$^{(r)}$	2.1987e-2$^{(-)}$	4.0423e-2$^{(-)}$	*4.4764e-2*$^{(-)}$
		P5	*7.4235e-1*$^{(-)}$	**3.4094e-1**$^{(r)}$	3.4243e-1$^{(-)}$	7.4206e-1$^{(-)}$
		P6	7.5622e-1$^{(o)}$	**7.5160e-1**$^{(r)}$	7.5536e-1$^{(-)}$	*7.5676e-1*$^{(-)}$
		P7	2.1555e+0$^{(*)}$	1.8455e+0$^{(*)}$	1.7532e+0$^{(*)}$	1.4404e+0$^{(*)}$
		P8	**3.4422e-1**$^{(r)}$	3.6351e-1$^{(-)}$	3.6493e-1$^{(-)}$	*3.6671e-1*$^{(-)}$
		P9	**6.5119e-2**$^{(r)}$	1.7877e-1$^{(-)}$	*8.1004e-1*$^{(-)}$	8.1004e-1$^{(-)}$
	HV	P1	**5.1006e-1**$^{(r)}$	2.2951e-1$^{(-)}$	*2.2559e-1*$^{(-)}$	3.2515e-1$^{(-)}$
		P2	6.5570e-1$^{(-)}$	**6.5829e-1**$^{(r)}$	*6.4893e-1*$^{(-)}$	6.5098e-1$^{(-)}$
		P3	—$^{(-)}$	1.0832e-1$^{(-)}$	1.0989e-1$^{(o)}$	**1.0990e-1**$^{(r)}$
		P4	**6.7795e-1**$^{(r)}$	6.6952e-1$^{(-)}$	6.3929e-1$^{(-)}$	*6.3195e-1*$^{(-)}$
		P5	*1.0918e-1*$^{(-)}$	1.9561e-1$^{(o)}$	**2.0988e-1**$^{(r)}$	1.1000e-1$^{(-)}$
		P6	*6.5976e-2*$^{(-)}$	**8.3676e-2**$^{(r)}$	7.3607e-2$^{(-)}$	7.2152e-2$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**1.3437e-1**$^{(r)}$	*1.1000e-1*$^{(-)}$	1.1000e-1$^{(-)}$	1.1000e-1$^{(-)}$
		P9	**8.8806e-1**$^{(r)}$	7.1166e-1$^{(-)}$	*3.7814e-1*$^{(-)}$	3.7814e-1$^{(-)}$

Table A.8.: Median IGD and HV values for Ref_2 with the four SX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	*4.8609e-1*$^{(-)}$	2.3759e-1$^{(-)}$	**1.2796e-1**$^{(r)}$	1.6691e-1$^{(-)}$
		P2	*1.0999e-1*$^{(-)}$	6.6879e-2$^{(-)}$	6.0075e-2$^{(o)}$	**5.8727e-2**$^{(r)}$
		P3	*1.1817e-1*$^{(-)}$	6.3779e-2$^{(o)}$	6.4820e-2$^{(o)}$	**5.6611e-2**$^{(r)}$
		P4	**1.3899e-2**$^{(r)}$	1.3925e-2$^{(o)}$	1.3899e-2$^{(o)}$	*1.3975e-2*$^{(o)}$
		P5	6.3712e-2$^{(o)}$	**6.3675e-2**$^{(r)}$	6.3725e-2$^{(o)}$	*6.3731e-2*$^{(o)}$
		P6	*1.9132e-2*$^{(-)}$	1.8911e-2$^{(-)}$	1.7566e-2$^{(-)}$	**1.6165e-2**$^{(r)}$
		P7	*1.4207e-2*$^{(-)}$	1.4153e-2$^{(-)}$	**1.3961e-2**$^{(r)}$	1.3997e-2$^{(o)}$
		P8	5.5198e-2$^{(-)}$	**5.0650e-2**$^{(r)}$	7.3437e-2$^{(-)}$	*8.8046e-2*$^{(-)}$
		P9	*3.4599e-2*$^{(-)}$	2.6560e-2$^{(o)}$	2.3468e-2$^{(o)}$	**2.3365e-2**$^{(r)}$
	HV	P1	4.2942e+0$^{(-)}$	5.6177e+0$^{(-)}$	**6.0504e+0**$^{(r)}$	5.8715e+0$^{(-)}$
		P2	5.5364e+0$^{(-)}$	5.7733e+0$^{(-)}$	5.8220e+0$^{(o)}$	**5.8245e+0**$^{(r)}$
		P3	5.0641e+0$^{(-)}$	5.3314e+0$^{(o)}$	5.3263e+0$^{(o)}$	**5.3676e+0**$^{(r)}$
		P4	3.3599e+0$^{(-)}$	3.3600e+0$^{(-)}$	3.3613e+0$^{(o)}$	**3.3618e+0**$^{(r)}$
		P5	3.0329e+0$^{(-)}$	3.0336e+0$^{(-)}$	**3.0342e+0**$^{(r)}$	3.0340e+0$^{(o)}$
		P6	3.2998e+0$^{(-)}$	3.3003e+0$^{(-)}$	3.3125e+0$^{(-)}$	**3.3235e+0**$^{(r)}$
		P7	3.3592e+0$^{(-)}$	3.3628e+0$^{(o)}$	3.3633e+0$^{(o)}$	**3.3633e+0**$^{(r)}$
		P8	3.1068e+0$^{(-)}$	**3.1255e+0**$^{(r)}$	2.9987e+0$^{(-)}$	2.9160e+0$^{(-)}$
		P9	3.1717e+0$^{(-)}$	3.2395e+0$^{(o)}$	**3.2538e+0**$^{(r)}$	3.2528e+0$^{(o)}$
n=200	IGD	P1	*8.2091e-1*$^{(-)}$	4.3161e-1$^{(-)}$	1.6656e-1$^{(-)}$	**1.4500e-1**$^{(r)}$
		P2	*1.7438e-1*$^{(-)}$	1.3564e-1$^{(-)}$	1.3356e-1$^{(-)}$	**1.1885e-1**$^{(r)}$
		P3	*1.8317e-1*$^{(-)}$	1.3330e-1$^{(o)}$	1.3143e-1$^{(o)}$	**1.2874e-1**$^{(r)}$
		P4	1.4190e-2$^{(-)}$	*1.4241e-2*$^{(-)}$	1.4178e-2$^{(o)}$	**1.4088e-2**$^{(r)}$
		P5	*6.4049e-2*$^{(-)}$	**6.3679e-2**$^{(r)}$	6.3680e-2$^{(o)}$	6.3792e-2$^{(o)}$
		P6	*1.5716e-2*$^{(-)}$	1.5563e-2$^{(-)}$	1.5094e-2$^{(-)}$	**1.4820e-2**$^{(r)}$
		P7	*1.7377e-2*$^{(-)}$	1.4358e-2$^{(-)}$	1.4181e-2$^{(o)}$	**1.4050e-2**$^{(r)}$
		P8	5.4534e-2$^{(o)}$	**5.4110e-2**$^{(r)}$	7.6358e-2$^{(-)}$	*8.5594e-2*$^{(-)}$
		P9	*2.8217e-2*$^{(-)}$	2.5182e-2$^{(-)}$	**2.1576e-2**$^{(r)}$	2.3494e-2$^{(o)}$
	HV	P1	2.8247e+0$^{(-)}$	4.5447e+0$^{(-)}$	5.8747e+0$^{(-)}$	**5.9736e+0**$^{(r)}$
		P2	5.1906e+0$^{(-)}$	5.3985e+0$^{(-)}$	5.4091e+0$^{(-)}$	**5.4894e+0**$^{(r)}$
		P3	4.7534e+0$^{(-)}$	4.9928e+0$^{(o)}$	5.0008e+0$^{(o)}$	**5.0144e+0**$^{(r)}$
		P4	3.3522e+0$^{(-)}$	3.3516e+0$^{(-)}$	3.3551e+0$^{(-)}$	**3.3577e+0**$^{(r)}$
		P5	3.0292e+0$^{(-)}$	3.0331e+0$^{(-)}$	**3.0343e+0**$^{(r)}$	3.0343e+0$^{(o)}$
		P6	3.3285e+0$^{(-)}$	3.3298e+0$^{(-)}$	3.3344e+0$^{(-)}$	**3.3387e+0**$^{(r)}$
		P7	3.3214e+0$^{(-)}$	3.3545e+0$^{(-)}$	3.3628e+0$^{(-)}$	**3.3632e+0**$^{(r)}$
		P8	**3.1003e+0**$^{(r)}$	3.0994e+0$^{(o)}$	2.9753e+0$^{(-)}$	2.9216e+0$^{(-)}$
		P9	3.2218e+0$^{(-)}$	3.2339e+0$^{(-)}$	**3.2676e+0**$^{(r)}$	3.2506e+0$^{(o)}$

Table A.9.: IQR values of IGD and HV for Ref_1 with the four NX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	1.4534e-1	8.7236e-2	4.2530e-2	2.1337e-2
		P2	7.7564e-3	3.7977e-3	2.5522e-3	3.2769e-3
		P3	1.4636e-1	1.3293e-1	1.4368e-1	1.0505e-1
		P4	1.0925e-2	1.0880e-2	1.2536e-2	7.3696e-3
		P5	3.0351e-1	3.0169e-1	3.8077e-1	5.5094e-1
		P6	1.0005e+0	2.7913e-1	8.1984e-3	1.3678e-2
		P7	8.5760e-1	9.0783e-1	8.3369e-2	5.4307e-2
		P8	3.2588e-2	1.1455e-1	6.5169e-2	4.6418e-2
		P9	2.1205e-1	1.6666e-1	3.9369e-1	4.3774e-1
	HV	P1	1.5725e-1	1.1334e-1	5.4537e-2	3.6171e-2
		P2	1.2123e-2	5.3766e-3	3.8047e-3	4.8549e-3
		P3	—	—	—	—
		P4	1.5912e-2	1.2966e-2	2.2351e-2	1.2321e-2
		P5	4.5402e-4	5.3953e-4	8.5599e-2	1.1259e-1
		P6	—	1.3432e-2	3.1334e-2	3.4421e-2
		P7	—	—	—	—
		P8	1.9484e-2	5.3739e-2	4.8126e-2	3.3702e-2
		P9	1.5282e-1	1.2298e-1	1.5731e-1	2.0470e-1
n=200	IGD	P1	2.0175e-1	1.2986e-1	6.4722e-2	7.8382e-2
		P2	4.9425e-3	2.7435e-3	1.6382e-3	1.9652e-3
		P3	3.6702e-1	2.2764e-1	1.6215e-1	1.2789e-1
		P4	4.7842e-3	5.6879e-3	6.8299e-3	5.6656e-3
		P5	1.9438e-1	1.4565e-1	3.6332e-1	2.6690e-1
		P6	8.5246e-1	9.0832e-1	9.8496e-2	7.4140e-2
		P7	8.7896e-1	6.2579e-1	4.0030e-1	3.5518e-2
		P8	5.1960e-2	3.9289e-2	1.4829e-1	7.2970e-2
		P9	3.0234e-1	1.2107e-1	2.8852e-1	3.7041e-1
	HV	P1	1.6787e-1	1.3955e-1	6.7538e-2	6.7193e-2
		P2	7.4949e-3	3.9299e-3	3.3870e-3	2.3619e-3
		P3	—	—	—	—
		P4	7.2231e-3	7.9986e-3	9.2434e-3	7.6390e-3
		P5	9.7150e-4	1.0172e-3	7.6987e-2	1.2414e-3
		P6	—	—	—	2.4446e-2
		P7	—	—	—	—
		P8	3.2930e-2	4.2112e-3	8.6232e-2	7.4489e-2
		P9	3.2099e-1	1.4422e-1	1.0572e-1	1.0246e-1

Table A.10.: IQR values of IGD and HV for Ref_1 with the four NX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	6.3364e-2	4.7499e-2	7.0433e-2	7.5956e-2
		P2	1.2032e-2	6.7114e-3	4.9471e-3	7.8408e-3
		P3	1.1245e-2	1.1731e-2	2.9339e-3	6.5380e-3
		P4	8.3975e-4	6.3412e-4	5.0768e-4	6.2367e-4
		P5	3.7451e-4	1.6778e-4	2.3054e-4	4.2602e-4
		P6	6.9376e-4	4.9488e-4	4.5042e-4	4.5650e-4
		P7	2.5263e-2	2.2527e-2	3.4494e-2	1.4167e-2
		P8	6.0576e-3	8.2333e-3	6.0485e-3	3.6500e-3
		P9	4.3246e-2	4.1748e-2	7.7577e-2	8.0405e-2
	HV	P1	4.7696e-1	4.3154e-1	5.5560e-1	4.5225e-1
		P2	7.8021e-2	4.3407e-2	2.8986e-2	3.4379e-2
		P3	5.7424e-2	6.1245e-2	1.7755e-2	3.5938e-2
		P4	1.8157e-3	1.6241e-3	1.0286e-3	2.6018e-3
		P5	1.8688e-3	1.1924e-3	1.9852e-3	1.7398e-3
		P6	7.0788e-3	5.1820e-3	1.9474e-3	2.3347e-3
		P7	1.4124e-1	1.2244e-1	1.7565e-1	8.1666e-2
		P8	2.9576e-2	3.7714e-2	3.7759e-2	2.8894e-2
		P9	2.2352e-1	2.3397e-1	4.1152e-1	4.6305e-1
n=200	IGD	P1	1.1889e-1	8.3790e-2	1.0528e-1	1.0381e-1
		P2	1.2246e-2	1.3180e-2	5.7341e-3	3.7991e-3
		P3	1.5551e-2	1.1523e-2	8.6531e-3	2.3248e-3
		P4	2.0366e-3	2.3824e-3	3.8715e-3	3.3452e-3
		P5	3.7536e-4	2.2798e-4	2.5163e-4	2.0705e-4
		P6	4.8470e-4	6.7132e-4	5.9226e-4	5.6711e-4
		P7	7.9104e-2	8.6052e-2	8.9059e-2	7.2370e-2
		P8	8.7749e-3	6.1108e-3	5.6439e-3	3.6723e-3
		P9	6.3578e-2	4.3298e-2	1.1897e-1	1.4301e-1
	HV	P1	4.5694e-1	4.2324e-1	7.0105e-1	6.1008e-1
		P2	5.8785e-2	6.0268e-2	4.3581e-2	1.5969e-2
		P3	7.6057e-2	5.7115e-2	4.5276e-2	1.2898e-2
		P4	3.4887e-3	2.1342e-3	1.5180e-3	3.7730e-3
		P5	1.8452e-3	2.4071e-3	1.0604e-3	2.0671e-3
		P6	4.8199e-3	2.0286e-3	3.1071e-3	2.7827e-3
		P7	4.2586e-1	4.2211e-1	4.5716e-1	3.7401e-1
		P8	4.8564e-2	3.1660e-2	2.3374e-2	2.2402e-2
		P9	2.9122e-1	1.8191e-1	5.4988e-1	8.1819e-1

Table A.11.: IQR values of IGD and HV for Ref_1 with the four SX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

n	Indicator	Problem	S2	S5	S20	S50
n=100	IGD	P1	1.1298e-2	5.1888e-2	8.4135e-2	1.3155e-1
		P2	3.4569e-3	2.8991e-3	4.4792e-3	1.0272e-2
		P3	1.1979e-1	1.6972e-1	1.6900e-1	1.7951e-1
		P4	8.0815e-3	8.4211e-3	1.1344e-2	9.9938e-3
		P5	5.6419e-1	4.0282e-1	3.9840e-1	4.0059e-1
		P6	2.2477e-1	6.1307e-3	3.4716e-1	1.6295e+0
		P7	4.9723e-2	1.3745e-1	8.3136e-1	7.1155e-1
		P8	4.6373e-2	1.3170e-1	8.0742e-2	1.1021e-1
		P9	4.5985e-1	3.5731e-1	1.9726e-1	1.2262e-1
	HV	P1	2.0338e-2	6.2240e-2	8.7362e-2	1.5574e-1
		P2	6.1579e-3	4.0318e-3	6.2928e-3	1.4057e-2
		P3	—	—	—	—
		P4	1.3618e-2	1.3677e-2	1.2851e-2	8.6270e-3
		P5	1.6250e-1	9.9149e-2	8.3936e-2	7.9482e-2
		P6	6.4742e-2	4.4791e-2	3.0497e-2	—
		P7	—	—	—	—
		P8	2.8838e-2	8.7878e-2	7.4537e-2	6.3838e-2
		P9	1.9061e-1	1.1174e-1	1.9871e-1	1.9428e-1
n=200	IGD	P1	1.0519e-2	4.6459e-2	6.8665e-2	1.2881e-1
		P2	2.5660e-3	1.8522e-3	2.5492e-3	5.2024e-3
		P3	1.2787e-1	1.5599e-1	3.8433e-1	3.8858e-1
		P4	5.5735e-3	4.4659e-3	4.9343e-3	5.8990e-3
		P5	3.3469e-1	2.4687e-1	3.1269e-1	1.2020e-1
		P6	2.2826e-1	1.1194e-1	8.3616e-1	7.9483e-1
		P7	5.1441e-2	1.1938e-1	8.5237e-1	6.3514e-1
		P8	3.7113e-2	1.5043e-1	2.1748e-2	9.0818e-3
		P9	4.0808e-1	1.4193e-1	8.8959e-2	1.4902e-1
	HV	P1	1.4437e-2	4.8821e-2	6.5143e-2	1.7662e-1
		P2	4.2992e-3	2.7589e-3	4.0228e-3	8.3997e-3
		P3	—	—	—	—
		P4	8.2085e-3	7.4394e-3	5.7220e-3	7.7252e-3
		P5	2.1203e-3	1.2629e-3	1.3698e-3	4.7854e-4
		P6	2.9039e-2	1.6614e-2	—	—
		P7	—	—	—	—
		P8	3.5613e-2	8.1223e-2	1.9065e-2	3.9684e-3
		P9	1.5883e-1	9.2848e-2	9.0853e-2	1.1376e-1

Table A.12.: IQR values of IGD and HV for Ref_1 with the four SX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

n	Indicator	Problem	S2	S5	S20	S50
n=100	IGD	P1	8.0765e-2	8.9542e-2	6.6613e-2	9.3558e-2
		P2	6.1030e-3	3.1787e-3	7.5896e-3	1.5159e-2
		P3	5.8151e-3	2.8503e-3	1.0425e-2	1.2222e-2
		P4	8.8021e-4	8.7530e-4	4.0585e-4	6.2682e-4
		P5	1.0172e-3	2.7285e-4	2.6701e-3	2.1541e-4
		P6	4.8900e-4	4.4935e-4	6.2523e-4	5.9612e-4
		P7	3.7697e-2	2.1320e-2	4.8941e-2	3.4288e-2
		P8	4.5300e-3	5.9630e-3	7.9655e-3	8.0652e-3
		P9	6.3710e-2	8.0988e-2	4.7839e-2	4.6126e-2
	HV	P1	3.9577e-1	7.3969e-1	4.1438e-1	6.7494e-1
		P2	3.8215e-2	2.2737e-2	4.9288e-2	8.8449e-2
		P3	3.3412e-2	1.8770e-2	5.1659e-2	6.2438e-2
		P4	1.5616e-3	1.7198e-3	1.1258e-3	1.9285e-3
		P5	5.4331e-3	2.4941e-3	1.7163e-2	1.9202e-3
		P6	2.3123e-3	2.5830e-3	4.5562e-3	4.0898e-3
		P7	2.3757e-1	1.0761e-1	2.7261e-1	1.7263e-1
		P8	1.4208e-2	2.7144e-2	4.8390e-2	4.6148e-2
		P9	3.7514e-1	4.4658e-1	2.5237e-1	1.8423e-1
n=200	IGD	P1	1.2501e-1	1.1840e-1	1.4590e-1	1.1080e-1
		P2	5.2635e-3	4.3706e-3	1.1135e-2	7.7788e-3
		P3	4.4613e-3	3.0414e-3	8.2968e-3	1.4983e-2
		P4	1.5756e-3	2.1057e-3	1.8056e-3	1.2682e-3
		P5	6.7631e-4	3.7650e-4	3.5100e-4	3.1729e-4
		P6	5.2930e-4	6.0034e-4	4.3707e-4	6.2714e-4
		P7	8.8726e-2	6.3499e-2	7.6762e-2	7.2632e-2
		P8	5.1147e-3	5.1306e-3	6.4037e-3	5.8010e-3
		P9	1.4474e-1	1.4918e-1	5.7699e-2	9.8298e-2
	HV	P1	6.6897e-1	5.7508e-1	6.2205e-1	4.7565e-1
		P2	3.2893e-2	2.0070e-2	3.6658e-2	4.1550e-2
		P3	2.3674e-2	1.7232e-2	4.2199e-2	7.6693e-2
		P4	1.8192e-3	1.2762e-3	2.0343e-3	2.6321e-3
		P5	3.3059e-3	1.8748e-3	1.7801e-3	1.8580e-3
		P6	2.0258e-3	2.2219e-3	2.0249e-3	3.3565e-3
		P7	4.7514e-1	3.2078e-1	4.0014e-1	3.6628e-1
		P8	1.7798e-2	1.8844e-2	3.7148e-2	3.6198e-2
		P9	7.5701e-1	7.2238e-1	2.9792e-1	4.9199e-1

Table A.13.: IQR values of IGD and HV for Ref_2 with the four NX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	5.0989e-2	1.0604e-1	3.4436e-2	6.1787e-2
		P2	3.2715e-3	5.7076e-3	2.5399e-3	3.3169e-3
		P3	3.2070e-1	1.0130e-4	1.9472e-4	2.7893e-3
		P4	6.8224e-3	8.5515e-3	3.5826e-3	2.2021e-3
		P5	3.9966e-1	3.9967e-1	1.3053e-5	2.7010e-3
		P6	3.0786e-4	2.3965e-4	3.5738e-4	1.8556e-1
		P7	1.7161e-1	1.7427e-1	7.4096e-1	7.5575e-1
		P8	2.1493e-2	8.8649e-3	2.7955e-2	3.1138e-2
		P9	2.5535e-15	1.7764e-15	3.5946e-1	3.4095e-1
	HV	P1	6.6213e-2	1.5146e-1	4.8373e-2	8.6628e-2
		P2	4.8540e-3	8.6855e-3	3.5249e-3	5.0892e-3
		P3	2.6233e-1	2.6425e-5	2.5804e-5	1.0902e-4
		P4	1.0358e-2	1.2687e-2	5.2823e-3	4.0312e-3
		P5	8.3676e-2	1.0000e-1	2.7153e-7	4.5101e-3
		P6	1.0932e-3	5.0530e-4	9.3499e-4	1.9442e-3
		P7	—	—	—	—
		P8	3.8486e-8	2.3730e-8	2.3870e-2	1.8187e-2
		P9	2.2760e-15	2.2760e-15	1.6265e-1	1.0043e-1
n=200	IGD	P1	1.0055e-1	8.2513e-2	1.2828e-2	2.7845e-2
		P2	6.2516e-3	2.6804e-3	1.7783e-3	2.4083e-3
		P3	4.8398e-1	5.2314e-3	3.8058e-2	1.2828e-2
		P4	2.5838e-3	4.0317e-3	2.3913e-3	1.9245e-3
		P5	3.9237e-5	3.9963e-1	1.1368e-3	1.2520e-1
		P6	3.2211e-2	9.7413e-3	5.7559e-3	7.2446e-3
		P7	4.1881e-1	4.1812e-1	7.2993e-1	5.6490e-1
		P8	2.0547e-1	1.9471e-3	1.1692e-3	2.8216e-3
		P9	7.6379e-10	6.1675e-10	1.8755e-1	1.4502e-1
	HV	P1	3.6613e-2	1.1273e-1	6.6800e-3	3.7643e-2
		P2	8.8805e-3	3.8620e-3	2.5960e-3	3.4741e-3
		P3	5.5137e-3	2.3908e-4	1.4398e-4	1.9509e-2
		P4	3.3175e-3	4.1790e-3	4.1800e-3	3.7114e-3
		P5	7.4168e-7	9.9822e-2	2.5132e-3	7.9197e-4
		P6	3.5179e-2	1.6740e-2	1.2536e-2	1.2135e-2
		P7	—	—	—	—
		P8	1.8479e-7	1.2208e-7	1.7669e-7	1.5064e-3
		P9	1.5425e-9	1.1379e-9	1.2592e-1	1.0395e-1

Table A.14.: IQR values for IGD and HV for Ref_2 with the four NX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	6.6058e-2	6.1644e-2	6.5733e-2	8.5604e-2
		P2	1.2122e-2	2.3392e-2	2.4359e-2	1.8793e-2
		P3	1.4599e-2	2.5226e-2	2.5836e-2	2.4784e-2
		P4	2.1183e-4	2.7887e-4	2.3098e-4	2.5789e-4
		P5	1.6721e-4	1.7953e-4	2.5217e-4	2.5950e-4
		P6	6.9153e-4	1.0944e-3	1.1135e-3	1.1207e-3
		P7	2.8026e-4	3.7779e-4	3.1649e-4	9.4944e-4
		P8	7.3128e-3	9.9626e-3	4.9375e-3	9.2090e-3
		P9	9.8897e-3	1.2562e-2	7.5368e-4	2.1355e-3
	HV	P1	3.2948e-1	3.0161e-1	4.1668e-1	3.7634e-1
		P2	7.5926e-2	1.2981e-1	1.3953e-1	8.7877e-2
		P3	7.4489e-2	1.2797e-1	1.3032e-1	1.2025e-1
		P4	7.2847e-4	1.2508e-3	1.4753e-3	9.7645e-4
		P5	1.1403e-3	1.1209e-3	1.8595e-3	3.6482e-3
		P6	4.6497e-3	8.5381e-3	8.5010e-3	8.1145e-3
		P7	1.2174e-3	1.9754e-3	3.8935e-3	1.6927e-2
		P8	4.2595e-2	4.9934e-2	2.8701e-2	3.6420e-2
		P9	8.1556e-2	9.9165e-2	2.6301e-3	5.9276e-3
n=200	IGD	P1	3.5447e-2	3.6985e-2	6.3585e-2	8.3980e-2
		P2	1.2728e-2	2.2933e-2	2.0237e-2	2.3090e-2
		P3	1.9357e-2	2.0077e-2	2.1307e-2	2.1905e-2
		P4	2.0844e-4	2.5833e-4	3.0752e-4	3.3870e-4
		P5	1.9213e-4	1.6641e-4	1.4043e-4	3.2042e-4
		P6	4.0823e-4	3.1885e-4	5.4411e-4	5.7614e-4
		P7	2.9892e-4	3.1991e-4	3.9978e-4	3.8787e-3
		P8	1.3767e-2	7.5856e-3	6.2606e-3	4.8565e-3
		P9	5.7649e-3	3.3063e-3	1.0910e-2	2.4532e-2
	HV	P1	1.6514e-1	2.0124e-1	2.1261e-1	2.9985e-1
		P2	7.3735e-2	1.2495e-1	1.0971e-1	1.2494e-1
		P3	9.3660e-2	9.6176e-2	1.0258e-1	1.0501e-1
		P4	1.1078e-3	1.6952e-3	1.8677e-3	1.3738e-3
		P5	1.0640e-3	9.9072e-4	9.4395e-4	3.9912e-3
		P6	2.1974e-3	3.2760e-3	4.7956e-3	3.7591e-3
		P7	1.6479e-3	1.0336e-3	1.0101e-2	4.0452e-2
		P8	6.8907e-2	3.5043e-2	3.3302e-2	2.9398e-2
		P9	6.8719e-2	2.8374e-2	3.8438e-2	1.0692e-1

Table A.15.: IQR values of IGD and HV for Ref_2 with the four SX versions of MRAND as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	5.0217e-2	3.0721e-2	4.0547e-2	6.0448e-2
		P2	1.9681e-3	2.4442e-3	4.6888e-3	4.1310e-3
		P3	2.0579e-2	9.0029e-5	8.3272e-5	3.2434e-1
		P4	2.7503e-3	3.0092e-3	7.7557e-3	5.1924e-3
		P5	4.3463e-3	2.0261e-6	3.9967e-1	3.9967e-1
		P6	2.5053e-1	2.0870e-2	3.7299e-4	3.2213e-4
		P7	7.2990e-1	5.3351e-1	1.7373e-1	1.7237e-1
		P8	4.6940e-2	1.4727e-2	5.7067e-3	2.0471e-2
		P9	4.3017e-2	3.5601e-1	1.4433e-15	2.7756e-15
	HV	P1	7.1223e-2	4.2956e-2	5.9144e-2	8.2435e-2
		P2	3.8812e-3	3.4340e-3	6.4822e-3	6.2356e-3
		P3	3.0899e-2	1.8592e-5	1.7049e-5	1.5772e-1
		P4	4.4806e-3	4.8440e-3	1.1489e-2	1.1720e-2
		P5	1.1440e-2	2.4553e-7	9.9960e-2	9.9836e-2
		P6	3.2733e-3	1.4294e-3	8.4657e-4	8.9223e-4
		P7	—	—	—	—
		P8	4.6898e-2	2.7259e-8	1.7790e-8	5.2179e-8
		P9	5.8805e-2	2.1954e-1	1.6653e-15	2.4980e-15
n=200	IGD	P1	5.2049e-2	7.5072e-3	2.9463e-2	1.0760e-1
		P2	3.3349e-3	1.2127e-3	2.7111e-3	2.0875e-3
		P3	2.1605e-1	1.0135e-3	1.2840e-3	7.6251e-3
		P4	1.9469e-3	2.0652e-3	3.2790e-3	2.6663e-3
		P5	1.0635e-1	5.9758e-3	3.9859e-1	2.5534e-1
		P6	1.2482e-1	5.3890e-3	3.5298e-3	9.5321e-3
		P7	6.7505e-1	5.5443e-1	3.2016e-1	3.9631e-1
		P8	1.5845e-1	2.1785e-3	1.2028e-3	1.3690e-2
		P9	2.3575e-2	1.4896e-1	5.7959e-8	2.5910e-9
	HV	P1	7.0608e-2	1.1229e-2	4.3107e-3	1.5534e-1
		P2	5.1677e-3	1.7098e-3	3.7592e-3	3.1093e-3
		P3	—	1.5681e-3	8.1061e-5	8.3299e-5
		P4	3.2974e-3	4.2356e-3	5.9382e-3	4.4599e-3
		P5	4.6124e-4	8.6205e-2	9.9965e-2	1.9100e-7
		P6	1.5944e-2	1.2382e-2	1.3059e-2	1.3035e-2
		P7	—	—	—	—
		P8	8.9770e-2	3.9055e-7	8.9234e-8	2.1855e-7
		P9	4.0290e-2	1.1241e-1	9.7959e-8	4.6172e-9

Table A.16.: IQR values for IGD and HV for Ref_2 with the four SX versions of MRAND as grouping method and the WFG test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	1.0665e-1	4.5138e-2	4.1380e-2	4.9279e-2
		P2	2.1771e-2	2.9044e-2	1.8781e-2	2.2863e-2
		P3	2.6574e-2	2.3083e-2	1.7882e-2	1.7838e-2
		P4	2.7481e-4	2.5525e-4	2.1881e-4	2.0950e-4
		P5	2.6819e-4	2.4437e-4	1.9902e-4	1.8671e-4
		P6	1.3228e-3	1.4815e-3	1.8255e-3	1.1942e-3
		P7	2.7125e-4	3.0008e-4	3.6944e-4	4.4201e-4
		P8	6.2206e-3	6.3475e-3	8.1976e-3	8.3715e-3
		P9	7.2134e-3	1.2883e-2	1.3192e-2	1.5843e-2
	HV	P1	4.2555e-1	2.6168e-1	2.2487e-1	2.5285e-1
		P2	1.2210e-1	1.6430e-1	1.0953e-1	1.3326e-1
		P3	1.2820e-1	1.1606e-1	8.9997e-2	8.9705e-2
		P4	9.5274e-4	1.1595e-3	1.5160e-3	9.1638e-4
		P5	1.9558e-3	7.9418e-4	8.9323e-4	1.2132e-3
		P6	1.1142e-2	1.1530e-2	1.2679e-2	9.3049e-3
		P7	2.3340e-3	1.2308e-3	1.3244e-3	1.3840e-3
		P8	4.8952e-2	2.7232e-2	4.0864e-2	4.2697e-2
		P9	5.6164e-2	9.3942e-2	9.3900e-2	8.9332e-2
n=200	IGD	P1	8.1821e-2	6.3572e-2	4.4688e-2	2.9096e-2
		P2	1.8963e-2	2.6067e-2	2.4221e-2	1.9702e-2
		P3	2.8239e-2	2.8915e-2	2.4781e-2	2.0009e-2
		P4	2.2017e-4	1.8205e-4	3.0571e-4	1.7038e-4
		P5	6.0198e-3	2.5450e-4	2.1523e-4	2.0136e-4
		P6	5.4978e-4	4.9586e-4	5.0385e-4	3.6636e-4
		P7	2.1127e-3	3.7097e-4	3.6594e-4	3.2500e-4
		P8	5.4670e-3	4.8425e-3	6.2650e-3	6.0651e-3
		P9	1.3219e-2	2.8875e-3	3.7231e-3	1.4367e-2
	HV	P1	3.1511e-1	2.8810e-1	2.2973e-1	1.6106e-1
		P2	9.9503e-2	1.4002e-1	1.3445e-1	1.0438e-1
		P3	1.3409e-1	1.3832e-1	1.1944e-1	9.5713e-2
		P4	1.9942e-3	2.2376e-3	1.7635e-3	9.8953e-4
		P5	5.0729e-2	1.7923e-3	1.4902e-3	9.4638e-4
		P6	2.2200e-3	4.4969e-3	4.0553e-3	2.8870e-3
		P7	1.7378e-2	4.9947e-3	1.3213e-3	7.1950e-4
		P8	2.4459e-2	2.6956e-2	3.3485e-2	4.0733e-2
		P9	7.1992e-2	1.2073e-2	2.8586e-2	5.6487e-2

B. Multidimensional Delta Grouping

In this Section the tables to compare the different versions of the Multidimensional Delta Grouping (MDELTA) against each other are depicted. Each table compares four MDELTA versions (SX or NX with $X = \{2, 5, 20, 50\}$ respectively) against each other. Each table contains the IGD and HV values for the four MDELTA versions for 100 and 200 decision variables, this aspects of the tables remain the same. The varying parts of the tables are the reference algorithm, the version group and the problem suite. The combination of the two reference algorithms Ref_1 and Ref_2, the two test suites LSMOP and WFG and the two version groups SX and NX lead to a total of eight tables.

Table B.1.: Median IGD and HV values for Ref_1 with the four NX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	*2.8001e-1*$^{(-)}$	2.5264e-1$^{(-)}$	1.2307e-1$^{(-)}$	**5.7086e-2**$^{(r)}$
		P2	*8.1791e-2*$^{(-)}$	6.2095e-2$^{(-)}$	4.1969e-2$^{(-)}$	**3.5458e-2**$^{(r)}$
		P3	1.0940e+0$^{(*)}$	1.0811e+0$^{(*)}$	1.1285e+0$^{(*)}$	1.0579e+0$^{(*)}$
		P4	*1.9379e-1*$^{(-)}$	1.6050e-1$^{(-)}$	9.1372e-2$^{(-)}$	**6.8732e-2**$^{(r)}$
		P5	7.3049e-1$^{(o)}$	7.3205e-1$^{(o)}$	**6.8874e-1**$^{(r)}$	*7.4209e-1*$^{(o)}$
		P6	*9.5972e-1*$^{(-)}$	7.8291e-1$^{(o)}$	7.4931e-1$^{(-)}$	**7.4347e-1**$^{(r)}$
		P7	1.3662e+0$^{(*)}$	1.3487e+0$^{(*)}$	1.3542e+0$^{(*)}$	1.3494e+0$^{(*)}$
		P8	*3.7291e-1*$^{(-)}$	3.6910e-1$^{(-)}$	2.0076e-1$^{(-)}$	**1.4697e-1**$^{(r)}$
		P9	*6.7893e-1*$^{(-)}$	6.4194e-1$^{(-)}$	5.5438e-1$^{(-)}$	**5.0566e-1**$^{(r)}$
	HV	P1	*3.1499e-1*$^{(-)}$	3.5399e-1$^{(-)}$	5.1055e-1$^{(-)}$	**6.0237e-1**$^{(r)}$
		P2	*5.7611e-1*$^{(-)}$	6.0561e-1$^{(-)}$	6.3553e-1$^{(-)}$	**6.4618e-1**$^{(r)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	*4.3149e-1*$^{(-)}$	4.6985e-1$^{(-)}$	5.5282e-1$^{(-)}$	**5.8938e-1**$^{(r)}$
		P5	**1.1000e-1**$^{(r)}$	*1.0997e-1*$^{(o)}$	1.0998e-1$^{(o)}$	1.0997e-1$^{(o)}$
		P6	—$^{(-)}$	—$^{(-)}$	8.5639e-2$^{(o)}$	**9.1690e-2**$^{(r)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	1.1000e-1$^{(-)}$	*1.1000e-1*$^{(-)}$	2.0375e-1$^{(-)}$	**2.5470e-1**$^{(r)}$
		P9	*4.7500e-1*$^{(-)}$	5.0247e-1$^{(-)}$	5.7538e-1$^{(-)}$	**6.6581e-1**$^{(r)}$
n=200	IGD	P1	*3.9016e-1*$^{(-)}$	2.9958e-1$^{(-)}$	2.1160e-1$^{(-)}$	**1.4289e-1**$^{(r)}$
		P2	*7.4849e-2*$^{(-)}$	5.7214e-2$^{(-)}$	4.2073e-2$^{(-)}$	**3.6820e-2**$^{(r)}$
		P3	*1.1239e+0*$^{(o)}$	1.0861e+0$^{(o)}$	1.0388e+0$^{(o)}$	**9.8303e-1**$^{(r)}$
		P4	*1.3069e-1*$^{(-)}$	1.1621e-1$^{(-)}$	8.6290e-2$^{(-)}$	**6.5434e-2**$^{(r)}$
		P5	*7.2680e-1*$^{(o)}$	6.9194e-1$^{(o)}$	6.5072e-1$^{(o)}$	**5.7075e-1**$^{(r)}$
		P6	*1.6092e+0*$^{(-)}$	8.7827e-1$^{(-)}$	8.0014e-1$^{(o)}$	**7.6224e-1**$^{(r)}$
		P7	1.7094e+0$^{(*)}$	1.8083e+0$^{(*)}$	1.4525e+0$^{(*)}$	1.4455e+0$^{(*)}$
		P8	*3.5866e-1*$^{(-)}$	3.5282e-1$^{(-)}$	3.4965e-1$^{(-)}$	**1.6374e-1**$^{(r)}$
		P9	*7.6785e-1*$^{(-)}$	6.3499e-1$^{(-)}$	5.9526e-1$^{(-)}$	**5.4087e-1**$^{(r)}$
	HV	P1	*2.3722e-1*$^{(-)}$	2.9695e-1$^{(-)}$	4.1151e-1$^{(-)}$	**4.9332e-1**$^{(r)}$
		P2	*5.9056e-1*$^{(-)}$	6.1532e-1$^{(-)}$	6.3726e-1$^{(-)}$	**6.4493e-1**$^{(r)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	*5.1146e-1*$^{(-)}$	5.3076e-1$^{(-)}$	5.7100e-1$^{(-)}$	**5.9833e-1**$^{(r)}$
		P5	**1.0997e-1**$^{(r)}$	1.0974e-1$^{(o)}$	1.0980e-1$^{(o)}$	*1.0936e-1*$^{(-)}$
		P6	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	1.0994e-1$^{(-)}$	*1.0988e-1*$^{(-)}$	1.0998e-1$^{(-)}$	**2.2387e-1**$^{(r)}$
		P9	*3.6918e-1*$^{(-)}$	5.0442e-1$^{(-)}$	5.4179e-1$^{(-)}$	**6.2448e-1**$^{(r)}$

Table B.2.: Median IGD and HV values for Ref_1 with the four NX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	1.1194e+0$^{(*)}$	1.0919e+0$^{(*)}$	1.1052e+0$^{(*)}$	1.2147e+0$^{(*)}$
		P2	*4.2646e-2*$^{(-)}$	2.8816e-2$^{(-)}$	**1.9008e-2**$^{(r)}$	2.7542e-2$^{(-)}$
		P3	*4.4066e-2*$^{(-)}$	3.1300e-2$^{(-)}$	**1.8076e-2**$^{(r)}$	2.6069e-2$^{(-)}$
		P4	**1.3894e-2**$^{(r)}$	*1.4043e-2*$^{(o)}$	1.3896e-2$^{(o)}$	1.3982e-2$^{(o)}$
		P5	*6.9874e-2*$^{(o)}$	6.9856e-2$^{(o)}$	6.9847e-2$^{(o)}$	**6.9761e-2**$^{(r)}$
		P6	*1.9557e-2*$^{(o)}$	1.9428e-2$^{(o)}$	**1.9345e-2**$^{(r)}$	1.9385e-2$^{(o)}$
		P7	*8.0197e-2*$^{(o)}$	7.5540e-2$^{(o)}$	**7.4695e-2**$^{(r)}$	7.5930e-2$^{(o)}$
		P8	*6.1308e-2*$^{(-)}$	5.7893e-2$^{(-)}$	**5.0285e-2**$^{(r)}$	5.3970e-2$^{(-)}$
		P9	1.4107e-1$^{(-)}$	*1.5811e-1*$^{(-)}$	6.8259e-2$^{(o)}$	**6.5005e-2**$^{(r)}$
	HV	P1	2.4974e+0$^{(o)}$	**2.8097e+0**$^{(r)}$	2.7357e+0$^{(o)}$	1.7721e+0$^{(-)}$
		P2	5.9167e+0$^{(-)}$	6.0048e+0$^{(-)}$	**6.0731e+0**$^{(r)}$	6.0123e+0$^{(-)}$
		P3	5.4317e+0$^{(-)}$	5.5000e+0$^{(-)}$	**5.5790e+0**$^{(r)}$	5.5287e+0$^{(-)}$
		P4	3.3614e+0$^{(o)}$	3.3616e+0$^{(o)}$	3.3617e+0$^{(o)}$	**3.3617e+0**$^{(r)}$
		P5	2.9801e+0$^{(o)}$	2.9803e+0$^{(o)}$	**2.9813e+0**$^{(r)}$	2.9810e+0$^{(o)}$
		P6	3.2951e+0$^{(o)}$	3.2960e+0$^{(o)}$	**3.2965e+0**$^{(r)}$	3.2964e+0$^{(o)}$
		P7	2.9746e+0$^{(o)}$	**3.0225e+0**$^{(r)}$	3.0063e+0$^{(o)}$	3.0051e+0$^{(o)}$
		P8	3.0668e+0$^{(-)}$	3.0815e+0$^{(-)}$	**3.1521e+0**$^{(r)}$	3.1299e+0$^{(-)}$
		P9	2.6100e+0$^{(-)}$	2.5257e+0$^{(-)}$	3.0228e+0$^{(o)}$	**3.0543e+0**$^{(r)}$
n=200	IGD	P1	1.1796e+0$^{(*)}$	1.2091e+0$^{(*)}$	1.1459e+0$^{(*)}$	1.1901e+0$^{(*)}$
		P2	*1.0857e-1*$^{(-)}$	7.1972e-2$^{(-)}$	3.2732e-2$^{(-)}$	**2.3696e-2**$^{(r)}$
		P3	*1.2068e-1*$^{(-)}$	7.9659e-2$^{(-)}$	3.3261e-2$^{(-)}$	**2.2642e-2**$^{(r)}$
		P4	1.4836e-2$^{(o)}$	1.4882e-2$^{(o)}$	**1.4714e-2**$^{(r)}$	*1.4909e-2*$^{(o)}$
		P5	*6.9912e-2*$^{(-)}$	6.9870e-2$^{(o)}$	6.9875e-2$^{(o)}$	**6.9842e-2**$^{(r)}$
		P6	*1.5928e-2*$^{(o)}$	1.5835e-2$^{(o)}$	1.5794e-2$^{(o)}$	**1.5694e-2**$^{(r)}$
		P7	1.5363e-1$^{(o)}$	**1.4549e-1**$^{(r)}$	1.8191e-1$^{(o)}$	*1.8383e-1*$^{(o)}$
		P8	*6.6814e-2*$^{(-)}$	6.1813e-2$^{(-)}$	5.2221e-2$^{(o)}$	**5.0713e-2**$^{(r)}$
		P9	2.6067e-1$^{(-)}$	2.8009e-1$^{(-)}$	*2.9025e-1*$^{(-)}$	**1.0811e-1**$^{(r)}$
	HV	P1	2.2303e+0$^{(-)}$	2.1559e+0$^{(-)}$	**2.5327e+0**$^{(r)}$	2.1406e+0$^{(-)}$
		P2	5.5582e+0$^{(-)}$	5.7544e+0$^{(-)}$	5.9859e+0$^{(-)}$	**6.0472e+0**$^{(r)}$
		P3	5.0527e+0$^{(-)}$	5.2509e+0$^{(-)}$	5.4889e+0$^{(-)}$	**5.5509e+0**$^{(r)}$
		P4	3.3614e+0$^{(o)}$	**3.3615e+0**$^{(r)}$	3.3601e+0$^{(o)}$	3.3609e+0$^{(o)}$
		P5	2.9797e+0$^{(o)}$	2.9800e+0$^{(o)}$	2.9799e+0$^{(o)}$	**2.9800e+0**$^{(r)}$
		P6	3.3272e+0$^{(o)}$	3.3267e+0$^{(o)}$	**3.3275e+0**$^{(r)}$	3.3273e+0$^{(o)}$
		P7	2.5728e+0$^{(o)}$	**2.6143e+0**$^{(r)}$	2.4524e+0$^{(o)}$	2.4314e+0$^{(o)}$
		P8	3.0252e+0$^{(-)}$	3.0494e+0$^{(-)}$	3.1227e+0$^{(-)}$	**3.1382e+0**$^{(r)}$
		P9	2.0400e+0$^{(-)}$	1.9154e+0$^{(-)}$	1.8789e+0$^{(-)}$	**2.7903e+0**$^{(r)}$

Table B.3.: Median IGD and HV values for Ref_1 with the four SX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	**3.5625e-2**$^{(r)}$	1.1044e-1$^{(-)}$	2.1199e-1$^{(-)}$	*2.5128e-1*$^{(-)}$
		P2	**3.1553e-2**$^{(r)}$	4.0123e-2$^{(-)}$	6.2570e-2$^{(-)}$	*7.7912e-2*$^{(-)}$
		P3	1.0098e+0$^{(*)}$	1.1097e+0$^{(*)}$	1.0339e+0$^{(*)}$	1.0508e+0$^{(*)}$
		P4	**6.0935e-2**$^{(r)}$	8.3942e-2$^{(-)}$	1.5659e-1$^{(-)}$	*1.9050e-1*$^{(-)}$
		P5	**6.2532e-1**$^{(r)}$	*7.4210e-1*$^{(o)}$	7.3275e-1$^{(o)}$	7.3594e-1$^{(o)}$
		P6	**6.9817e-1**$^{(r)}$	7.4523e-1$^{(o)}$	7.8658e-1$^{(-)}$	*9.5786e-1*$^{(-)}$
		P7	1.3495e+0$^{(*)}$	1.3703e+0$^{(*)}$	1.3978e+0$^{(*)}$	1.6908e+0$^{(*)}$
		P8	**1.2810e-1**$^{(r)}$	2.0394e-1$^{(-)}$	*3.6918e-1*$^{(-)}$	3.6186e-1$^{(-)}$
		P9	**5.1701e-1**$^{(r)}$	5.3059e-1$^{(o)}$	6.4772e-1$^{(-)}$	*7.1188e-1*$^{(-)}$
	HV	P1	**6.4317e-1**$^{(r)}$	5.2910e-1$^{(-)}$	4.0005e-1$^{(-)}$	*3.4871e-1*$^{(-)}$
		P2	**6.5229e-1**$^{(r)}$	6.3937e-1$^{(-)}$	6.0505e-1$^{(-)}$	*5.8316e-1*$^{(-)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	**5.9980e-1**$^{(r)}$	5.6526e-1$^{(-)}$	4.7219e-1$^{(-)}$	*4.3107e-1*$^{(-)}$
		P5	**1.0998e-1**$^{(r)}$	1.0997e-1$^{(o)}$	1.0997e-1$^{(o)}$	*1.0996e-1*$^{(o)}$
		P6	8.8915e-2$^{(o)}$	**9.4427e-2**$^{(r)}$	—$^{(-)}$	—$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**2.5369e-1**$^{(r)}$	1.8897e-1$^{(-)}$	*1.1000e-1*$^{(-)}$	1.1012e-1$^{(-)}$
		P9	**6.5125e-1**$^{(r)}$	5.9625e-1$^{(-)}$	4.9592e-1$^{(-)}$	*4.3740e-1*$^{(-)}$
n=200	IGD	P1	**5.2145e-2**$^{(r)}$	1.4262e-1$^{(-)}$	2.4296e-1$^{(-)}$	*2.9160e-1*$^{(-)}$
		P2	**3.3090e-2**$^{(r)}$	3.5994e-2$^{(-)}$	4.8889e-2$^{(-)}$	*6.2518e-2*$^{(-)}$
		P3	1.0080e+0$^{(o)}$	**9.9999e-1**$^{(r)}$	1.0554e+0$^{(o)}$	*1.0813e+0*$^{(o)}$
		P4	**5.2774e-2**$^{(r)}$	6.5468e-2$^{(-)}$	1.0149e-1$^{(-)}$	*1.1818e-1*$^{(-)}$
		P5	**4.8445e-1**$^{(r)}$	5.3226e-1$^{(o)}$	6.8941e-1$^{(o)}$	*7.0149e-1*$^{(o)}$
		P6	**5.4734e-1**$^{(r)}$	7.6985e-1$^{(-)}$	7.9617e-1$^{(-)}$	*8.7198e-1*$^{(-)}$
		P7	1.4328e+0$^{(*)}$	1.4488e+0$^{(*)}$	1.6827e+0$^{(*)}$	1.8173e+0$^{(*)}$
		P8	**1.3432e-1**$^{(r)}$	2.4397e-1$^{(-)}$	3.5049e-1$^{(-)}$	*3.5139e-1*$^{(-)}$
		P9	**4.9035e-1**$^{(r)}$	5.1299e-1$^{(o)}$	5.9834e-1$^{(-)}$	*6.7884e-1*$^{(-)}$
	HV	P1	**6.1458e-1**$^{(r)}$	4.9601e-1$^{(-)}$	3.6197e-1$^{(-)}$	*2.8939e-1*$^{(-)}$
		P2	**6.5026e-1**$^{(r)}$	6.4665e-1$^{(-)}$	6.2659e-1$^{(-)}$	*6.0671e-1*$^{(-)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	**6.1816e-1**$^{(r)}$	5.9883e-1$^{(-)}$	5.5128e-1$^{(-)}$	*5.2841e-1*$^{(-)}$
		P5	*1.0920e-1*$^{(o)}$	1.0941e-1$^{(o)}$	**1.0992e-1**$^{(r)}$	1.0982e-1$^{(o)}$
		P6	**2.1413e-3**$^{(r)}$	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**2.4977e-1**$^{(r)}$	1.6710e-1$^{(-)}$	*1.0985e-1*$^{(-)}$	1.0988e-1$^{(-)}$
		P9	**6.7764e-1**$^{(r)}$	6.4539e-1$^{(o)}$	5.3773e-1$^{(-)}$	*4.5550e-1*$^{(-)}$

Table B.4.: Median IGD and HV values for Ref_1 with the four SX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	1.1881e+0$^{(*)}$	1.1193e+0$^{(*)}$	1.0899e+0$^{(*)}$	1.0967e+0$^{(*)}$
		P2	2.8966e-2$^{(-)}$	**1.8305e-2**$^{(r)}$	2.9062e-2$^{(-)}$	*4.7813e-2*$^{(-)}$
		P3	2.6337e-2$^{(-)}$	**1.7636e-2**$^{(r)}$	3.2632e-2$^{(-)}$	*5.1721e-2*$^{(-)}$
		P4	*1.3888e-2*$^{(o)}$	**1.3725e-2**$^{(r)}$	1.3788e-2$^{(o)}$	1.3838e-2$^{(o)}$
		P5	**6.9754e-2**$^{(r)}$	*6.9852e-2*$^{(o)}$	6.9838e-2$^{(o)}$	6.9803e-2$^{(o)}$
		P6	1.9432e-2$^{(o)}$	**1.9265e-2**$^{(r)}$	1.9435e-2$^{(o)}$	*1.9452e-2*$^{(o)}$
		P7	**7.1800e-2**$^{(r)}$	*7.8711e-2*$^{(o)}$	7.2421e-2$^{(o)}$	7.7800e-2$^{(o)}$
		P8	5.3835e-2$^{(o)}$	**5.1777e-2**$^{(r)}$	5.7866e-2$^{(-)}$	*6.0579e-2*$^{(-)}$
		P9	**8.5152e-2**$^{(r)}$	1.1224e-1$^{(o)}$	1.3901e-1$^{(-)}$	*1.3985e-1*$^{(-)}$
	HV	P1	1.9389e+0$^{(-)}$	2.6016e+0$^{(-)}$	**2.8420e+0**$^{(r)}$	2.7237e+0$^{(o)}$
		P2	6.0019e+0$^{(-)}$	**6.0768e+0**$^{(r)}$	5.9999e+0$^{(-)}$	5.8870e+0$^{(-)}$
		P3	5.5272e+0$^{(-)}$	**5.5821e+0**$^{(r)}$	5.4908e+0$^{(-)}$	5.3930e+0$^{(-)}$
		P4	3.3612e+0$^{(o)}$	**3.3616e+0**$^{(r)}$	3.3612e+0$^{(o)}$	3.3613e+0$^{(o)}$
		P5	**2.9811e+0**$^{(r)}$	2.9804e+0$^{(o)}$	2.9802e+0$^{(-)}$	2.9806e+0$^{(o)}$
		P6	3.2953e+0$^{(o)}$	3.2970e+0$^{(o)}$	3.2959e+0$^{(o)}$	**3.2972e+0**$^{(r)}$
		P7	**3.0350e+0**$^{(r)}$	2.9959e+0$^{(o)}$	3.0219e+0$^{(o)}$	2.9995e+0$^{(o)}$
		P8	3.1265e+0$^{(o)}$	**3.1412e+0**$^{(r)}$	3.0805e+0$^{(-)}$	3.0675e+0$^{(-)}$
		P9	**2.9118e+0**$^{(r)}$	2.7658e+0$^{(o)}$	2.6224e+0$^{(-)}$	2.6176e+0$^{(-)}$
n=200	IGD	P1	1.2502e+0$^{(*)}$	1.1653e+0$^{(*)}$	1.1701e+0$^{(*)}$	1.1942e+0$^{(*)}$
		P2	4.7517e-2$^{(-)}$	**2.5130e-2**$^{(r)}$	5.3633e-2$^{(-)}$	*8.5036e-2*$^{(-)}$
		P3	5.0570e-2$^{(-)}$	**2.3640e-2**$^{(r)}$	5.4132e-2$^{(-)}$	*9.0879e-2*$^{(-)}$
		P4	**1.4754e-2**$^{(r)}$	*1.5137e-2*$^{(o)}$	1.4944e-2$^{(o)}$	1.5007e-2$^{(o)}$
		P5	**6.9742e-2**$^{(r)}$	6.9890e-2$^{(-)}$	6.9866e-2$^{(-)}$	*6.9905e-2*$^{(-)}$
		P6	1.5759e-2$^{(o)}$	1.5733e-2$^{(o)}$	*1.5764e-2*$^{(o)}$	**1.5728e-2**$^{(r)}$
		P7	**1.3922e-1**$^{(r)}$	1.6647e-1$^{(o)}$	*1.8234e-1*$^{(o)}$	1.7357e-1$^{(o)}$
		P8	**5.1552e-2**$^{(r)}$	5.2042e-2$^{(o)}$	5.4040e-2$^{(-)}$	*6.2052e-2*$^{(-)}$
		P9	**1.2064e-1**$^{(r)}$	1.9857e-1$^{(o)}$	*2.8379e-1*$^{(-)}$	2.0753e-1$^{(o)}$
	HV	P1	1.5267e+0$^{(-)}$	2.3095e+0$^{(o)}$	**2.4841e+0**$^{(r)}$	2.2042e+0$^{(-)}$
		P2	5.8955e+0$^{(-)}$	**6.0390e+0**$^{(r)}$	5.8663e+0$^{(-)}$	5.6959e+0$^{(-)}$
		P3	5.3971e+0$^{(-)}$	**5.5377e+0**$^{(r)}$	5.3801e+0$^{(-)}$	5.1968e+0$^{(-)}$
		P4	3.3612e+0$^{(o)}$	3.3611e+0$^{(o)}$	**3.3615e+0**$^{(r)}$	3.3608e+0$^{(o)}$
		P5	**2.9812e+0**$^{(r)}$	2.9805e+0$^{(o)}$	2.9795e+0$^{(-)}$	2.9798e+0$^{(-)}$
		P6	**3.3282e+0**$^{(r)}$	3.3277e+0$^{(o)}$	3.3272e+0$^{(o)}$	3.3272e+0$^{(o)}$
		P7	**2.6686e+0**$^{(r)}$	2.5079e+0$^{(o)}$	2.4267e+0$^{(o)}$	2.4651e+0$^{(o)}$
		P8	**3.1231e+0**$^{(r)}$	3.1218e+0$^{(o)}$	3.1010e+0$^{(-)}$	3.0471e+0$^{(-)}$
		P9	**2.7313e+0**$^{(r)}$	2.2833e+0$^{(o)}$	1.9291e+0$^{(-)}$	2.2761e+0$^{(o)}$

Table B.5.: Median IGD and HV values for Ref_2 with the four NX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	**1.9653e-1**$^{(r)}$	*3.2941e-1*$^{(-)}$	3.2666e-1$^{(-)}$	2.2391e-1$^{(o)}$
		P2	*8.2326e-2*$^{(-)}$	5.1534e-2$^{(-)}$	3.2418e-2$^{(-)}$	**2.5347e-2**$^{(r)}$
		P3	**7.0696e-1**$^{(r)}$	7.0705e-1$^{(o)}$	7.0712e-1$^{(o)}$	*7.0718e-1*$^{(o)}$
		P4	*1.3297e-1*$^{(-)}$	1.1708e-1$^{(-)}$	5.5629e-2$^{(-)}$	**3.2884e-2**$^{(r)}$
		P5	*7.4206e-1*$^{(-)}$	3.4242e-1$^{(-)}$	3.4241e-1$^{(-)}$	**3.4119e-1**$^{(r)}$
		P6	*7.4339e-1*$^{(-)}$	7.4244e-1$^{(o)}$	7.4264e-1$^{(-)}$	**7.3679e-1**$^{(r)}$
		P7	1.2995e+0$^{(*)}$	1.3299e+0$^{(*)}$	1.2524e+0$^{(*)}$	1.9307e+0$^{(*)}$
		P8	*3.8740e-1*$^{(-)}$	3.8334e-1$^{(-)}$	3.7013e-1$^{(-)}$	**3.4822e-1**$^{(r)}$
		P9	*8.1004e-1*$^{(-)}$	8.1004e-1$^{(-)}$	7.3724e-1$^{(-)}$	**2.1146e-1**$^{(r)}$
	HV	P1	**4.2711e-1**$^{(r)}$	*2.4099e-1*$^{(-)}$	2.4410e-1$^{(-)}$	3.9223e-1$^{(o)}$
		P2	*5.7847e-1*$^{(-)}$	6.3027e-1$^{(-)}$	6.5745e-1$^{(-)}$	**6.6688e-1**$^{(r)}$
		P3	1.0992e-1$^{(-)}$	**1.0999e-1**$^{(r)}$	1.0998e-1$^{(-)}$	*1.0988e-1*$^{(-)}$
		P4	*5.0630e-1*$^{(-)}$	5.1415e-1$^{(-)}$	6.0415e-1$^{(-)}$	**6.4817e-1**$^{(r)}$
		P5	*1.1000e-1*$^{(-)}$	2.0996e-1$^{(-)}$	**2.1000e-1**$^{(r)}$	2.0900e-1$^{(-)}$
		P6	*1.0545e-1*$^{(-)}$	**1.0838e-1**$^{(r)}$	1.0746e-1$^{(-)}$	1.0812e-1$^{(o)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	*1.1000e-1*$^{(-)}$	1.1000e-1$^{(o)}$	1.1000e-1$^{(o)}$	**1.2476e-1**$^{(r)}$
		P9	*3.7814e-1*$^{(-)}$	3.7814e-1$^{(-)}$	4.0310e-1$^{(-)}$	**7.0508e-1**$^{(r)}$
n=200	IGD	P1	*3.8497e-1*$^{(-)}$	**2.7652e-1**$^{(r)}$	3.4073e-1$^{(-)}$	3.3809e-1$^{(-)}$
		P2	*8.6734e-2*$^{(-)}$	4.4336e-2$^{(-)}$	3.7465e-2$^{(-)}$	**3.2204e-2**$^{(r)}$
		P3	*7.2305e-1*$^{(-)}$	**7.0718e-1**$^{(r)}$	7.0764e-1$^{(-)}$	7.1253e-1$^{(-)}$
		P4	*8.0908e-2*$^{(-)}$	7.3163e-2$^{(-)}$	4.7857e-2$^{(-)}$	**2.7268e-2**$^{(r)}$
		P5	7.4204e-1$^{(-)}$	*7.4205e-1*$^{(-)}$	**3.4175e-1**$^{(r)}$	5.3798e-1$^{(-)}$
		P6	*7.5745e-1*$^{(o)}$	7.5494e-1$^{(o)}$	7.5407e-1$^{(o)}$	**7.5316e-1**$^{(r)}$
		P7	1.6222e+0$^{(*)}$	1.6808e+0$^{(*)}$	1.7488e+0$^{(*)}$	1.8963e+0$^{(*)}$
		P8	3.6482e-1$^{(-)}$	*3.6490e-1*$^{(-)}$	3.6397e-1$^{(-)}$	**3.6291e-1**$^{(r)}$
		P9	8.1004e-1$^{(o)}$	**8.1004e-1**$^{(r)}$	8.1004e-1$^{(-)}$	*8.1004e-1*$^{(o)}$
	HV	P1	**3.3147e-1**$^{(r)}$	3.0402e-1$^{(-)}$	*2.2530e-1*$^{(-)}$	2.2966e-1$^{(-)}$
		P2	*5.7453e-1*$^{(-)}$	6.4124e-1$^{(-)}$	6.5133e-1$^{(-)}$	**6.5899e-1**$^{(r)}$
		P3	*1.0987e-2*$^{(-)}$	**1.0977e-1**$^{(r)}$	1.0902e-1$^{(-)}$	1.0165e-1$^{(-)}$
		P4	*5.8133e-1*$^{(-)}$	5.9001e-1$^{(-)}$	6.2514e-1$^{(-)}$	**6.5953e-1**$^{(r)}$
		P5	1.1000e-1$^{(-)}$	1.1000e-1$^{(-)}$	**2.0961e-1**$^{(r)}$	*1.0892e-1*$^{(-)}$
		P6	*5.0344e-2*$^{(-)}$	7.5224e-2$^{(o)}$	7.7284e-2$^{(o)}$	**7.9688e-2**$^{(r)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	1.1000e-1$^{(-)}$	**1.1000e-1**$^{(r)}$	1.1000e-1$^{(-)}$	*1.1000e-1*$^{(o)}$
		P9	3.7814e-1$^{(-)}$	**3.7814e-1**$^{(r)}$	3.7814e-1$^{(-)}$	*3.7814e-1*$^{(o)}$

Table B.6.: Median IGD and HV values for Ref_2 with the four NX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	2.1182e-1$^{(-)}$	**1.5240e-1**$^{(r)}$	2.4056e-1$^{(-)}$	*4.8361e-1*$^{(-)}$
		P2	5.9319e-2$^{(o)}$	**5.7433e-2**$^{(r)}$	6.3756e-2$^{(o)}$	*1.1074e-1*$^{(-)}$
		P3	5.8659e-2$^{(o)}$	6.2754e-2$^{(o)}$	**5.7153e-2**$^{(r)}$	*1.1131e-1*$^{(-)}$
		P4	1.3838e-2$^{(o)}$	1.3888e-2$^{(o)}$	*1.3922e-2*$^{(o)}$	**1.3824e-2**$^{(r)}$
		P5	6.3767e-2$^{(-)}$	6.3729e-2$^{(o)}$	**6.3674e-2**$^{(r)}$	*6.3779e-2*$^{(o)}$
		P6	**1.6211e-2**$^{(r)}$	1.7182e-2$^{(-)}$	1.8990e-2$^{(-)}$	*1.9515e-2*$^{(-)}$
		P7	1.4054e-2$^{(o)}$	**1.4048e-2**$^{(r)}$	1.4144e-2$^{(o)}$	*1.4815e-2*$^{(-)}$
		P8	*8.1564e-2*$^{(-)}$	6.3311e-2$^{(-)}$	**4.8141e-2**$^{(r)}$	5.4244e-2$^{(-)}$
		P9	**2.1880e-2**$^{(r)}$	2.4782e-2$^{(o)}$	3.4331e-2$^{(-)}$	*3.4837e-2*$^{(-)}$
	HV	P1	5.6569e+0$^{(-)}$	**5.9291e+0**$^{(r)}$	5.6184e+0$^{(-)}$	4.2910e+0$^{(-)}$
		P2	5.8187e+0$^{(o)}$	**5.8339e+0**$^{(r)}$	5.7971e+0$^{(o)}$	5.5297e+0$^{(-)}$
		P3	5.3575e+0$^{(o)}$	5.3370e+0$^{(o)}$	**5.3648e+0**$^{(r)}$	5.0980e+0$^{(-)}$
		P4	3.3615e+0$^{(o)}$	**3.3615e+0**$^{(r)}$	3.3601e+0$^{(-)}$	3.3594e+0$^{(-)}$
		P5	**3.0344e+0**$^{(r)}$	3.0338e+0$^{(-)}$	3.0337e+0$^{(-)}$	3.0327e+0$^{(-)}$
		P6	**3.3246e+0**$^{(r)}$	3.3143e+0$^{(-)}$	3.2982e+0$^{(-)}$	3.2966e+0$^{(-)}$
		P7	**3.3633e+0**$^{(r)}$	3.3631e+0$^{(o)}$	3.3615e+0$^{(-)}$	3.3471e+0$^{(-)}$
		P8	2.9468e+0$^{(-)}$	3.0483e+0$^{(-)}$	**3.1403e+0**$^{(r)}$	3.1136e+0$^{(-)}$
		P9	**3.2651e+0**$^{(r)}$	3.2539e+0$^{(o)}$	3.1722e+0$^{(-)}$	3.1717e+0$^{(-)}$
n=200	IGD	P1	2.6249e-1$^{(-)}$	**2.0281e-1**$^{(r)}$	3.1121e-1$^{(-)}$	*5.1642e-1*$^{(-)}$
		P2	**1.0877e-1**$^{(r)}$	1.1602e-1$^{(-)}$	1.1532e-1$^{(-)}$	*1.4092e-1*$^{(-)}$
		P3	1.1579e-1$^{(o)}$	1.2141e-1$^{(-)}$	**1.0837e-1**$^{(r)}$	*1.2830e-1*$^{(-)}$
		P4	**1.4010e-2**$^{(r)}$	1.4089e-2$^{(o)}$	1.4279e-2$^{(-)}$	*1.4296e-2*$^{(-)}$
		P5	*6.3770e-2*$^{(-)}$	6.3698e-2$^{(o)}$	**6.3653e-2**$^{(r)}$	6.3717e-2$^{(-)}$
		P6	**1.4628e-2**$^{(r)}$	1.4740e-2$^{(o)}$	1.5442e-2$^{(-)}$	*1.5690e-2*$^{(-)}$
		P7	1.4037e-2$^{(o)}$	**1.3982e-2**$^{(r)}$	1.4318e-2$^{(-)}$	*1.4925e-2*$^{(-)}$
		P8	*9.6655e-2*$^{(-)}$	7.4555e-2$^{(-)}$	5.2662e-2$^{(o)}$	**5.1243e-2**$^{(r)}$
		P9	**1.9970e-2**$^{(r)}$	2.1796e-2$^{(-)}$	2.5095e-2$^{(-)}$	*2.9612e-2*$^{(-)}$
	HV	P1	5.3973e+0$^{(-)}$	**5.6717e+0**$^{(r)}$	5.2073e+0$^{(-)}$	4.1296e+0$^{(-)}$
		P2	**5.5428e+0**$^{(r)}$	5.5052e+0$^{(-)}$	5.5045e+0$^{(-)}$	5.3678e+0$^{(-)}$
		P3	5.0770e+0$^{(o)}$	5.0483e+0$^{(-)}$	**5.1122e+0**$^{(r)}$	5.0158e+0$^{(-)}$
		P4	**3.3575e+0**$^{(r)}$	3.3573e+0$^{(o)}$	3.3527e+0$^{(-)}$	3.3514e+0$^{(-)}$
		P5	**3.0342e+0**$^{(r)}$	3.0341e+0$^{(o)}$	3.0340e+0$^{(o)}$	3.0329e+0$^{(-)}$
		P6	**3.3438e+0**$^{(r)}$	3.3404e+0$^{(-)}$	3.3318e+0$^{(-)}$	3.3292e+0$^{(-)}$
		P7	**3.3628e+0**$^{(r)}$	3.3624e+0$^{(o)}$	3.3549e+0$^{(-)}$	3.3444e+0$^{(-)}$
		P8	2.8671e+0$^{(-)}$	2.9861e+0$^{(-)}$	3.1024e+0$^{(o)}$	**3.1131e+0**$^{(r)}$
		P9	**3.2981e+0**$^{(r)}$	3.2634e+0$^{(-)}$	3.2346e+0$^{(-)}$	3.2196e+0$^{(-)}$

Table B.7.: Median IGD and HV values for Ref_2 with the four SX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	**1.4202e-1**$^{(r)}$	3.3130e-1$^{(-)}$	*3.3222e-1*$^{(-)}$	2.0004e-1$^{(-)}$
		P2	**2.2768e-2**$^{(r)}$	2.9504e-2$^{(-)}$	4.9999e-2$^{(-)}$	*6.0579e-2*$^{(-)}$
		P3	*7.0762e-1*$^{(-)}$	7.0707e-1$^{(o)}$	7.0706e-1$^{(o)}$	**7.0696e-1**$^{(r)}$
		P4	**2.9015e-2**$^{(r)}$	4.9007e-2$^{(-)}$	1.2001e-1$^{(-)}$	*1.3653e-1*$^{(-)}$
		P5	**3.4177e-1**$^{(r)}$	3.4242e-1$^{(o)}$	*7.4206e-1*$^{(-)}$	4.4713e-1$^{(-)}$
		P6	**6.7058e-1**$^{(r)}$	7.4250e-1$^{(-)}$	7.4258e-1$^{(-)}$	*7.4344e-1*$^{(-)}$
		P7	1.7244e+0$^{(*)}$	1.3248e+0$^{(*)}$	1.3322e+0$^{(*)}$	1.3013e+0$^{(*)}$
		P8	**3.7049e-1**$^{(r)}$	3.7799e-1$^{(-)}$	3.8624e-1$^{(-)}$	*3.8895e-1*$^{(-)}$
		P9	**1.7731e-1**$^{(r)}$	8.1004e-1$^{(-)}$	8.1004e-1$^{(-)}$	*8.1004e-1*$^{(-)}$
	HV	P1	**5.0794e-1**$^{(r)}$	2.4047e-1$^{(-)}$	*2.3837e-1*$^{(-)}$	4.2088e-1$^{(-)}$
		P2	**6.7071e-1**$^{(r)}$	6.6200e-1$^{(-)}$	6.3253e-1$^{(-)}$	*6.1677e-1*$^{(-)}$
		P3	*1.0901e-1*$^{(-)}$	1.0997e-1$^{(-)}$	**1.0999e-1**$^{(r)}$	1.0994e-1$^{(-)}$
		P4	**6.5500e-1**$^{(r)}$	6.1675e-1$^{(-)}$	5.1142e-1$^{(-)}$	*4.9953e-1*$^{(-)}$
		P5	2.0935e-1$^{(-)}$	**2.1000e-1**$^{(r)}$	1.1000e-1$^{(-)}$	*1.1000e-1*$^{(-)}$
		P6	1.0676e-1$^{(-)}$	1.0796e-1$^{(o)}$	**1.0829e-1**$^{(r)}$	*1.0561e-1*$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	1.1000e-1$^{(o)}$	**1.1000e-1**$^{(r)}$	1.1000e-1$^{(o)}$	*1.1000e-1*$^{(-)}$
		P9	**7.2486e-1**$^{(r)}$	3.7814e-1$^{(-)}$	3.7814e-1$^{(-)}$	*3.7814e-1*$^{(-)}$
n=200	IGD	P1	**1.4154e-1**$^{(r)}$	*3.4083e-1*$^{(-)}$	3.3868e-1$^{(-)}$	2.8531e-1$^{(-)}$
		P2	3.2788e-2$^{(o)}$	**3.2298e-2**$^{(r)}$	4.0877e-2$^{(-)}$	*4.6121e-2*$^{(-)}$
		P3	*8.5948e-1*$^{(-)}$	7.1149e-1$^{(-)}$	**7.0724e-1**$^{(r)}$	7.0726e-1$^{(o)}$
		P4	**2.0585e-2**$^{(r)}$	2.8511e-2$^{(-)}$	6.2990e-2$^{(-)}$	*7.6146e-2*$^{(-)}$
		P5	*7.4234e-1*$^{(-)}$	**3.4013e-1**$^{(r)}$	3.4243e-1$^{(-)}$	7.4205e-1$^{(-)}$
		P6	**7.2720e-1**$^{(r)}$	*7.5397e-1*$^{(o)}$	7.5359e-1$^{(o)}$	7.5323e-1$^{(o)}$
		P7	2.2251e+0$^{(*)}$	2.0421e+0$^{(*)}$	1.7846e+0$^{(*)}$	1.7551e+0$^{(*)}$
		P8	**3.6138e-1**$^{(r)}$	3.6410e-1$^{(-)}$	3.6475e-1$^{(-)}$	*3.6509e-1*$^{(-)}$
		P9	**8.9466e-2**$^{(r)}$	8.1004e-1$^{(-)}$	8.1004e-1$^{(-)}$	*8.1004e-1*$^{(-)}$
	HV	P1	**5.0702e-1**$^{(r)}$	2.2637e-1$^{(-)}$	*2.2480e-1*$^{(-)}$	2.9215e-1$^{(-)}$
		P2	6.5493e-1$^{(-)}$	**6.5869e-1**$^{(r)}$	6.4638e-1$^{(-)}$	*6.3859e-1*$^{(-)}$
		P3	—$^{(-)}$	1.0324e-1$^{(-)}$	**1.0964e-1**$^{(r)}$	1.0953e-1$^{(-)}$
		P4	**6.7100e-1**$^{(r)}$	6.5750e-1$^{(-)}$	6.0266e-1$^{(-)}$	*5.8576e-1*$^{(-)}$
		P5	*1.0920e-1*$^{(-)}$	1.8515e-1$^{(o)}$	**2.0983e-1**$^{(r)}$	1.1000e-1$^{(-)}$
		P6	*5.3938e-2*$^{(-)}$	**7.9288e-2**$^{(r)}$	7.8672e-2$^{(o)}$	6.5235e-2$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**1.1006e-1**$^{(r)}$	*1.1000e-1*$^{(o)}$	1.1000e-1$^{(o)}$	1.1000e-1$^{(o)}$
		P9	**8.6941e-1**$^{(r)}$	3.7814e-1$^{(-)}$	3.7814e-1$^{(-)}$	*3.7814e-1*$^{(-)}$

Table B.8.: Median IGD and HV values for Ref_2 with the four SX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	*5.1361e-1*$^{(-)}$	2.3838e-1$^{(-)}$	**1.5044e-1**$^{(r)}$	2.2009e-1$^{(-)}$
		P2	*1.0905e-1*$^{(-)}$	6.7895e-2$^{(-)}$	6.4794e-2$^{(o)}$	**6.1473e-2**$^{(r)}$
		P3	*1.1043e-1*$^{(-)}$	**4.8416e-2**$^{(r)}$	5.9993e-2$^{(-)}$	6.6809e-2$^{(-)}$
		P4	1.3877e-2$^{(o)}$	*1.3933e-2*$^{(o)}$	1.3868e-2$^{(o)}$	**1.3857e-2**$^{(r)}$
		P5	6.3731e-2$^{(-)}$	**6.3674e-2**$^{(r)}$	*6.3735e-2*$^{(-)}$	6.3710e-2$^{(-)}$
		P6	*1.9354e-2*$^{(-)}$	1.8936e-2$^{(-)}$	1.7673e-2$^{(-)}$	**1.6241e-2**$^{(r)}$
		P7	*1.4198e-2*$^{(-)}$	1.4103e-2$^{(-)}$	1.4047e-2$^{(-)}$	**1.3844e-2**$^{(r)}$
		P8	5.6867e-2$^{(-)}$	**4.7993e-2**$^{(r)}$	6.2410e-2$^{(-)}$	*8.3788e-2*$^{(-)}$
		P9	*3.4366e-2*$^{(-)}$	3.4179e-2$^{(-)}$	**2.1105e-2**$^{(r)}$	2.6175e-2$^{(-)}$
	HV	P1	4.1373e+0$^{(-)}$	5.6013e+0$^{(-)}$	**5.9388e+0**$^{(r)}$	5.6068e+0$^{(-)}$
		P2	5.5402e+0$^{(-)}$	5.7678e+0$^{(-)}$	5.7866e+0$^{(o)}$	**5.8126e+0**$^{(r)}$
		P3	5.1023e+0$^{(-)}$	**5.4093e+0**$^{(r)}$	5.3503e+0$^{(-)}$	5.3161e+0$^{(-)}$
		P4	3.3594e+0$^{(-)}$	3.3604e+0$^{(-)}$	3.3617e+0$^{(o)}$	**3.3617e+0**$^{(r)}$
		P5	3.0321e+0$^{(-)}$	3.0339e+0$^{(o)}$	3.0341e+0$^{(o)}$	**3.0341e+0**$^{(r)}$
		P6	3.2964e+0$^{(-)}$	3.2999e+0$^{(-)}$	3.3103e+0$^{(-)}$	**3.3250e+0**$^{(r)}$
		P7	3.3599e+0$^{(-)}$	3.3626e+0$^{(-)}$	**3.3633e+0**$^{(r)}$	3.3625e+0$^{(-)}$
		P8	3.1042e+0$^{(-)}$	**3.1344e+0**$^{(r)}$	3.0521e+0$^{(-)}$	2.9391e+0$^{(-)}$
		P9	3.1717e+0$^{(-)}$	3.1730e+0$^{(-)}$	**3.2738e+0**$^{(r)}$	3.2479e+0$^{(-)}$
n=200	IGD	P1	*8.1476e-1*$^{(-)}$	4.5738e-1$^{(-)}$	2.1177e-1$^{(o)}$	**2.0270e-1**$^{(r)}$
		P2	*1.6831e-1*$^{(-)}$	1.2554e-1$^{(-)}$	1.1497e-1$^{(o)}$	**1.1197e-1**$^{(r)}$
		P3	*1.7431e-1*$^{(-)}$	1.3023e-1$^{(o)}$	**1.1950e-1**$^{(r)}$	1.2281e-1$^{(o)}$
		P4	1.4192e-2$^{(-)}$	*1.4244e-2*$^{(-)}$	1.4084e-2$^{(o)}$	**1.3984e-2**$^{(r)}$
		P5	*6.3903e-2*$^{(-)}$	**6.3642e-2**$^{(r)}$	6.3653e-2$^{(o)}$	6.3752e-2$^{(o)}$
		P6	*1.5823e-2*$^{(-)}$	1.5670e-2$^{(-)}$	1.4949e-2$^{(o)}$	**1.4796e-2**$^{(r)}$
		P7	*1.7883e-2*$^{(-)}$	1.4315e-2$^{(-)}$	1.4116e-2$^{(o)}$	**1.4073e-2**$^{(r)}$
		P8	5.4638e-2$^{(-)}$	**4.9655e-2**$^{(r)}$	6.3766e-2$^{(-)}$	*7.5890e-2*$^{(-)}$
		P9	*2.8471e-2*$^{(-)}$	2.5029e-2$^{(-)}$	2.2013e-2$^{(o)}$	**2.1759e-2**$^{(r)}$
	HV	P1	2.8440e+0$^{(-)}$	4.4159e+0$^{(-)}$	5.6155e+0$^{(o)}$	**5.6966e+0**$^{(r)}$
		P2	5.2220e+0$^{(-)}$	5.4534e+0$^{(-)}$	5.5104e+0$^{(o)}$	**5.5243e+0**$^{(r)}$
		P3	4.7961e+0$^{(-)}$	5.0077e+0$^{(o)}$	**5.0591e+0**$^{(r)}$	5.0420e+0$^{(o)}$
		P4	3.3524e+0$^{(-)}$	3.3512e+0$^{(-)}$	3.3552e+0$^{(-)}$	**3.3573e+0**$^{(r)}$
		P5	3.0280e+0$^{(-)}$	3.0331e+0$^{(-)}$	**3.0343e+0**$^{(r)}$	3.0341e+0$^{(o)}$
		P6	3.3289e+0$^{(-)}$	3.3298e+0$^{(-)}$	3.3357e+0$^{(-)}$	**3.3408e+0**$^{(r)}$
		P7	3.3174e+0$^{(-)}$	3.3553e+0$^{(-)}$	3.3624e+0$^{(-)}$	**3.3631e+0**$^{(r)}$
		P8	3.1008e+0$^{(-)}$	**3.1241e+0**$^{(r)}$	3.0378e+0$^{(-)}$	2.9691e+0$^{(-)}$
		P9	3.2199e+0$^{(-)}$	3.2342e+0$^{(-)}$	3.2706e+0$^{(o)}$	**3.2719e+0**$^{(r)}$

Table B.9.: IQR values for IGD and HV for Ref_1 with the four NX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	1.9695e-1	8.6683e-2	3.5275e-2	4.2203e-2
		P2	9.9121e-3	4.5824e-3	2.3622e-3	1.8786e-3
		P3	1.6399e-1	1.2018e-1	1.3345e-1	8.9179e-2
		P4	1.0768e-2	8.9189e-3	1.1711e-2	7.7234e-3
		P5	4.0079e-1	4.0000e-1	4.2559e-1	4.0209e-1
		P6	5.5904e-1	1.2500e+0	1.9004e-2	8.7848e-2
		P7	7.8160e-1	6.7908e-1	5.0890e-2	1.4115e-1
		P8	3.2614e-2	4.4169e-2	7.5784e-2	5.6260e-2
		P9	2.3518e-1	2.0872e-1	1.5385e-1	4.3459e-1
	HV	P1	2.0422e-1	1.0311e-1	4.8120e-2	5.4068e-2
		P2	1.3335e-2	6.2279e-3	3.7590e-3	3.7305e-3
		P3	—	—	—	—
		P4	1.4196e-2	9.4099e-3	1.5441e-2	1.5752e-2
		P5	9.8463e-2	9.2091e-2	1.0092e-1	7.5409e-2
		P6	—	1.6776e-2	5.0473e-2	8.9477e-2
		P7	—	—	—	—
		P8	2.4037e-2	4.4681e-2	5.7880e-2	4.1333e-2
		P9	2.0615e-1	1.7913e-1	1.3642e-1	1.9964e-1
n=200	IGD	P1	2.1579e-1	1.6930e-1	6.7239e-2	4.9289e-2
		P2	4.1729e-3	4.7734e-3	2.0423e-3	2.6282e-3
		P3	4.5434e-1	2.1960e-1	1.9192e-1	1.1703e-1
		P4	7.1900e-3	6.7302e-3	7.2078e-3	4.9650e-3
		P5	2.4935e-1	2.5419e-1	2.2931e-1	3.2591e-1
		P6	9.2523e-1	1.0159e+0	6.3789e-1	6.4090e-1
		P7	6.2725e-1	7.6376e-1	3.0275e-1	3.7525e-2
		P8	7.6419e-2	4.3760e-2	5.4620e-2	7.3291e-2
		P9	1.4792e-1	1.7761e-1	1.1484e-1	3.7968e-1
	HV	P1	1.2646e-1	1.4296e-1	4.3026e-2	5.1339e-2
		P2	6.3204e-3	6.6371e-3	3.1553e-3	4.1756e-3
		P3	—	—	—	—
		P4	7.1769e-3	7.7777e-3	9.4052e-3	6.3497e-3
		P5	4.0605e-4	7.8293e-4	7.4355e-4	1.2818e-3
		P6	—	—	—	—
		P7	—	—	—	—
		P8	1.7380e-2	2.1113e-2	4.2408e-2	5.3748e-2
		P9	1.5061e-1	1.5164e-1	9.9183e-2	1.3979e-1

Table B.10.: IQR values for IGD and HV for Ref_1 with the four NX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	8.5997e-2	5.6417e-2	5.9167e-2	8.4151e-2
		P2	1.2295e-2	7.2281e-3	3.3423e-3	3.9171e-3
		P3	1.1377e-2	1.0801e-2	2.3124e-3	5.5201e-3
		P4	7.7230e-4	1.1983e-3	9.4147e-4	1.0107e-3
		P5	3.3634e-4	2.1749e-4	5.1749e-4	5.3973e-4
		P6	7.9087e-4	4.5036e-4	5.4708e-4	4.8091e-4
		P7	2.0957e-2	2.0013e-2	4.3562e-2	4.6751e-2
		P8	7.8530e-3	6.0895e-3	5.3197e-3	4.5078e-3
		P9	3.5602e-2	6.0105e-2	7.8872e-2	6.2864e-2
	HV	P1	6.3222e-1	4.7028e-1	3.8510e-1	5.0476e-1
		P2	6.8332e-2	3.9251e-2	2.8497e-2	2.6536e-2
		P3	6.2092e-2	5.9978e-2	1.5553e-2	3.1400e-2
		P4	1.7963e-3	2.1020e-3	1.0658e-3	2.0137e-3
		P5	1.7792e-3	1.3251e-3	3.0497e-3	3.3090e-3
		P6	3.9978e-3	3.6832e-3	3.1913e-3	3.3944e-3
		P7	1.2843e-1	1.5471e-1	2.3124e-1	2.3625e-1
		P8	4.3377e-2	4.1239e-2	4.0173e-2	1.8173e-2
		P9	1.8741e-1	3.1384e-1	4.3117e-1	3.5202e-1
n=200	IGD	P1	9.3016e-2	1.4920e-1	7.6374e-2	9.5634e-2
		P2	1.2753e-2	1.2467e-2	4.6370e-3	6.8119e-3
		P3	1.3883e-2	9.3895e-3	4.0197e-3	2.6571e-3
		P4	2.2340e-3	1.8828e-3	2.3597e-3	1.8623e-3
		P5	2.1152e-4	2.5594e-4	2.9874e-4	1.8286e-4
		P6	6.4481e-4	5.1653e-4	3.0742e-4	5.7606e-4
		P7	7.6254e-2	6.9264e-2	8.1227e-2	9.4654e-2
		P8	7.3498e-3	4.5312e-3	7.0311e-3	5.2837e-3
		P9	8.0547e-2	6.8038e-2	7.6723e-2	1.2724e-1
	HV	P1	4.7734e-1	5.0838e-1	5.2856e-1	4.5244e-1
		P2	6.4395e-2	6.5590e-2	1.8617e-2	2.3828e-2
		P3	6.7618e-2	5.0079e-2	2.5578e-2	1.6096e-2
		P4	2.3545e-3	2.9916e-3	2.9834e-3	2.5565e-3
		P5	2.5586e-3	1.4607e-3	2.5415e-3	1.7142e-3
		P6	2.3662e-3	2.4226e-3	1.6633e-3	2.3998e-3
		P7	3.9888e-1	3.6244e-1	4.3885e-1	5.3930e-1
		P8	3.5640e-2	2.8642e-2	3.1732e-2	3.6920e-2
		P9	3.8849e-1	3.2287e-1	3.8475e-1	7.1557e-1

Table B.11.: IQR values for IGD and HV for Ref_1 with the four SX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	1.5432e-2	5.8401e-2	6.5531e-2	9.9397e-2
		P2	3.2974e-3	2.7215e-3	4.8356e-3	7.5871e-3
		P3	1.7298e-1	1.0090e-1	1.8052e-1	1.9918e-1
		P4	8.0478e-3	9.7542e-3	1.2894e-2	1.4377e-2
		P5	3.8186e-1	4.1227e-1	2.9889e-1	2.0925e-1
		P6	2.3844e-1	1.9177e-2	6.7599e-1	1.4700e+0
		P7	6.3112e-2	7.1938e-1	7.5904e-1	6.5628e-1
		P8	4.4984e-2	6.4647e-2	8.0769e-2	8.0678e-2
		P9	4.2288e-1	3.9911e-1	2.0485e-1	1.7876e-1
	HV	P1	2.3727e-2	5.7820e-2	5.8483e-2	1.1207e-1
		P2	5.7789e-3	4.1669e-3	6.5112e-3	1.1059e-2
		P3	—	—	—	—
		P4	1.2270e-2	1.4274e-2	1.0380e-2	1.7604e-2
		P5	7.5305e-2	9.9076e-2	1.1664e-4	5.9583e-4
		P6	5.4578e-2	5.6798e-2	9.9601e-3	—
		P7	—	—	—	—
		P8	4.1542e-2	7.6840e-2	1.5351e-2	4.8797e-2
		P9	1.7514e-1	1.3158e-1	1.8855e-1	1.2063e-1
n=200	IGD	P1	1.1718e-2	3.6205e-2	8.7738e-2	1.7554e-1
		P2	1.9423e-3	2.3587e-3	3.3620e-3	4.9556e-3
		P3	2.5384e-1	1.8814e-1	2.5541e-1	2.6075e-1
		P4	5.9824e-3	5.6566e-3	6.5431e-3	5.4925e-3
		P5	3.9280e-1	2.8714e-1	3.6870e-1	2.1796e-1
		P6	2.0323e-1	4.5963e-1	4.4512e-1	8.5057e-1
		P7	8.0327e-2	2.8651e-1	7.9065e-1	7.9302e-1
		P8	4.4884e-2	1.5620e-1	4.0184e-2	4.4379e-2
		P9	4.2684e-1	3.5047e-1	3.0511e-1	1.6299e-1
	HV	P1	2.7549e-2	3.6695e-2	1.1774e-1	1.5035e-1
		P2	3.0989e-3	3.5004e-3	4.2871e-3	7.4822e-3
		P3	—	—	—	—
		P4	9.9004e-3	8.2533e-3	1.0016e-2	7.6016e-3
		P5	7.8709e-2	1.0806e-3	3.0441e-2	6.6179e-4
		P6	1.7781e-2	—	—	—
		P7	—	—	—	—
		P8	6.0578e-2	8.3238e-2	8.3071e-3	7.7650e-3
		P9	1.8939e-1	1.2668e-1	9.8154e-2	1.4099e-1

Table B.12.: IQR values for IGD and HV for Ref_1 with the four SX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	7.5147e-2	5.8149e-2	6.5788e-2	9.0611e-2
		P2	6.0070e-3	2.6117e-3	1.0217e-2	1.4360e-2
		P3	6.8316e-3	2.4121e-3	6.6662e-3	1.5531e-2
		P4	7.7411e-4	3.9139e-4	1.0304e-3	8.1209e-4
		P5	8.0231e-4	1.9108e-4	2.1569e-4	2.3915e-3
		P6	3.8462e-4	5.4013e-4	6.0851e-4	9.6551e-4
		P7	5.2842e-2	2.0670e-2	3.9246e-2	3.0626e-2
		P8	4.6666e-3	8.4704e-3	7.1796e-3	9.9715e-3
		P9	8.9771e-2	8.8287e-2	4.8766e-2	2.9377e-2
	HV	P1	5.3074e-1	3.7228e-1	4.5643e-1	7.1622e-1
		P2	3.8549e-2	1.7221e-2	6.6745e-2	8.6215e-2
		P3	3.8171e-2	1.9010e-2	3.6080e-2	7.8589e-2
		P4	1.2446e-3	1.6951e-3	1.5276e-3	1.4509e-3
		P5	4.4490e-3	1.9652e-3	1.6742e-3	1.5158e-2
		P6	3.6753e-3	2.9731e-3	2.7475e-3	4.5386e-3
		P7	2.9201e-1	1.1025e-1	2.0259e-1	1.8565e-1
		P8	3.1949e-2	3.9178e-2	4.7897e-2	4.7949e-2
		P9	4.6277e-1	4.6572e-1	2.5730e-1	1.5320e-1
n=200	IGD	P1	6.2751e-2	9.2436e-2	1.3400e-1	1.1339e-1
		P2	8.5608e-3	9.4178e-3	6.6688e-3	1.0006e-2
		P3	5.6135e-3	2.7237e-3	8.4960e-3	1.1999e-2
		P4	1.7109e-3	1.5915e-3	2.0811e-3	1.7364e-3
		P5	4.4957e-4	2.6324e-4	2.3297e-4	2.4864e-4
		P6	6.9801e-4	6.9172e-4	4.2482e-4	4.6195e-4
		P7	7.8728e-2	8.1576e-2	9.4434e-2	7.7467e-2
		P8	3.9563e-3	6.3496e-3	6.3398e-3	5.6094e-3
		P9	2.1565e-1	2.1358e-1	7.2881e-2	1.1080e-1
	HV	P1	4.7891e-1	5.9221e-1	6.6300e-1	4.2954e-1
		P2	4.0734e-2	2.3769e-2	2.9531e-2	5.4095e-2
		P3	3.1800e-2	1.5433e-2	4.3387e-2	6.0556e-2
		P4	2.4352e-3	2.9151e-3	1.0716e-3	4.0091e-3
		P5	3.8374e-3	2.1051e-3	2.6218e-3	2.3561e-3
		P6	2.7819e-3	4.1141e-3	1.9976e-3	2.8416e-3
		P7	4.4821e-1	4.6368e-1	5.0799e-1	3.8798e-1
		P8	2.3561e-2	3.6349e-2	4.2272e-2	4.0438e-2
		P9	1.1297e+0	1.0911e+0	3.3575e-1	5.2431e-1

Table B.13.: IQR values for IGD and HV for Ref_2 with the four NX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	3.5138e-2	1.0036e-1	6.5259e-2	6.7473e-2
		P2	4.6201e-2	3.2186e-3	2.9801e-3	2.8746e-3
		P3	2.5168e-1	2.0803e-4	8.3582e-5	4.6764e-4
		P4	1.0692e-2	1.3001e-2	9.2106e-3	5.5087e-3
		P5	3.9965e-1	3.9966e-1	1.7570e-5	3.4140e-3
		P6	9.1237e-4	1.7933e-2	2.3472e-2	1.6977e-1
		P7	8.5763e-2	1.5791e-1	7.8614e-1	7.0360e-1
		P8	9.0425e-3	1.4141e-2	3.0711e-2	1.4055e-1
		P9	3.1086e-15	1.7764e-15	3.0253e-1	3.5486e-1
	HV	P1	5.9344e-2	1.4349e-1	9.2661e-2	9.5598e-2
		P2	6.9126e-2	4.6074e-3	4.0865e-3	4.3416e-3
		P3	1.1000e-1	2.7530e-5	3.4367e-5	1.6681e-4
		P4	1.2968e-2	1.3376e-2	1.7413e-2	1.0699e-2
		P5	9.9910e-2	9.9995e-2	8.3597e-7	8.4014e-3
		P6	3.7783e-3	4.7851e-4	9.5165e-4	1.6220e-3
		P7	—	—	—	—
		P8	8.0321e-8	3.2862e-8	3.6265e-2	8.7358e-2
		P9	3.1086e-15	1.7208e-15	2.5149e-1	1.5355e-1
n=200	IGD	P1	3.7849e-2	7.6496e-2	7.9952e-3	7.3160e-3
		P2	3.7512e-3	2.3407e-3	2.3379e-3	2.1754e-3
		P3	8.5905e-1	9.4767e-4	7.2860e-4	1.2680e-2
		P4	5.8375e-3	3.8890e-3	3.0104e-3	2.4589e-3
		P5	3.6394e-5	3.9962e-1	1.3349e-3	2.0638e-1
		P6	1.9889e-2	5.8664e-3	4.2377e-3	6.0692e-3
		P7	3.1479e-1	3.4678e-1	4.7276e-1	7.4701e-1
		P8	5.5681e-3	1.7208e-3	2.4537e-3	5.8050e-3
		P9	2.9191e-9	1.9531e-10	2.5714e-8	4.6391e-1
	HV	P1	1.5527e-2	1.0340e-1	6.1804e-3	9.9223e-3
		P2	4.8645e-3	3.1777e-3	3.4784e-3	3.1327e-3
		P3	1.0859e-1	1.2807e-4	8.4216e-4	1.9181e-2
		P4	8.7517e-3	6.1099e-3	5.1754e-3	3.9909e-3
		P5	3.1961e-7	9.9757e-2	1.4477e-3	9.4189e-4
		P6	3.2732e-2	2.0018e-2	8.8073e-3	1.3658e-2
		P7	—	—	—	—
		P8	8.6227e-3	1.4486e-7	1.4041e-3	6.3081e-3
		P9	9.1576e-9	3.4475e-10	4.4382e-8	2.5638e-1

Table B.14.: IQR values for IGD and HV for Ref_2 with the four NX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			N2	N5	N20	N50
n=100	IGD	P1	7.1828e-2	4.9105e-2	4.7588e-2	9.3887e-2
		P2	2.6035e-2	1.8999e-2	2.0985e-2	1.8339e-2
		P3	1.9696e-2	2.2613e-2	2.1292e-2	3.3101e-2
		P4	2.9743e-4	2.3811e-4	3.8105e-4	2.5368e-4
		P5	2.0627e-4	1.7097e-4	2.0687e-4	3.0030e-4
		P6	5.3440e-4	9.0106e-4	1.1925e-3	1.2375e-3
		P7	2.4759e-4	2.2849e-4	3.3913e-4	9.0492e-4
		P8	9.3579e-3	6.6379e-3	5.4783e-3	8.4832e-3
		P9	1.5250e-2	1.2099e-2	9.4752e-3	2.4482e-3
	HV	P1	3.5623e-1	2.5353e-1	2.9076e-1	3.3040e-1
		P2	1.4294e-1	1.1087e-1	1.1813e-1	9.9159e-2
		P3	9.9266e-2	1.1272e-1	1.0690e-1	1.5876e-1
		P4	1.5909e-3	1.3834e-3	1.5105e-3	1.2996e-3
		P5	9.9175e-4	7.0122e-4	8.7517e-4	3.2566e-3
		P6	6.7804e-3	1.0005e-2	8.0553e-3	1.0177e-2
		P7	1.0700e-3	9.6319e-4	2.0180e-3	1.8390e-2
		P8	4.8230e-2	2.8575e-2	3.0216e-2	4.0663e-2
		P9	1.0944e-1	9.5806e-2	7.2142e-2	2.2063e-2
n=200	IGD	P1	4.1028e-2	5.2401e-2	7.2668e-2	1.0782e-1
		P2	1.0184e-2	2.2326e-2	2.3672e-2	2.2008e-2
		P3	1.8116e-2	1.5954e-2	2.4473e-2	1.9359e-2
		P4	2.4079e-4	2.3204e-4	4.8429e-4	2.4655e-4
		P5	1.4294e-4	1.6464e-4	2.0544e-4	2.3596e-4
		P6	2.7145e-4	3.1662e-4	3.1878e-4	2.6437e-4
		P7	2.9018e-4	2.1534e-4	4.6219e-4	4.0298e-3
		P8	8.3550e-3	6.9417e-3	7.4554e-3	4.8730e-3
		P9	3.3049e-3	3.7869e-3	5.6312e-3	2.4525e-2
	HV	P1	2.1559e-1	2.4898e-1	3.4085e-1	4.8946e-1
		P2	6.0818e-2	1.2128e-1	1.3092e-1	1.1815e-1
		P3	8.7799e-2	7.6795e-2	1.1837e-1	9.2436e-2
		P4	1.7120e-3	1.0134e-3	1.8298e-3	1.6012e-3
		P5	1.1118e-3	1.5245e-3	9.4623e-4	2.5340e-3
		P6	1.9765e-3	3.4774e-3	4.4602e-3	4.0812e-3
		P7	1.0799e-3	1.2649e-3	9.4369e-3	3.7762e-2
		P8	4.2192e-2	3.7412e-2	4.3484e-2	2.7344e-2
		P9	4.6976e-2	3.1539e-2	2.1341e-2	1.3723e-1

Table B.15.: IQR values for IGD and HV for Ref_2 with the four SX versions of MDELTA as grouping method and the LSMOP test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	5.6562e-2	5.6243e-2	1.0249e-1	7.9445e-2
		P2	1.8252e-3	2.6090e-3	4.1325e-3	1.2533e-2
		P3	1.1881e-1	2.2412e-4	2.9467e-4	1.6446e-1
		P4	3.7586e-3	1.1764e-2	1.0967e-2	1.1099e-2
		P5	1.8610e-3	1.4518e-6	3.9966e-1	3.9966e-1
		P6	2.7276e-1	1.3800e-2	5.3589e-3	1.5454e-3
		P7	7.1893e-1	5.5347e-1	1.1352e-1	1.6233e-1
		P8	7.4695e-2	1.3419e-2	1.2297e-2	1.6723e-2
		P9	1.8593e-1	3.5611e-1	1.7764e-15	3.2196e-15
	HV	P1	7.8852e-2	7.8801e-2	1.4176e-1	1.1135e-1
		P2	3.6089e-3	4.3005e-3	5.5739e-3	1.8687e-2
		P3	1.0959e-1	5.3861e-5	2.1228e-5	4.9043e-2
		P4	6.9457e-3	2.2361e-2	1.4567e-2	1.5901e-2
		P5	4.1625e-3	5.6554e-7	9.9957e-2	9.9999e-2
		P6	3.4064e-3	8.0475e-4	7.6143e-4	1.8936e-3
		P7	—	—	—	—
		P8	1.6638e-2	3.7767e-8	3.6908e-8	9.7634e-8
		P9	7.6467e-2	2.6068e-1	2.3870e-15	4.1633e-15
n=200	IGD	P1	6.4369e-2	3.2140e-2	1.3263e-2	9.2921e-2
		P2	2.3313e-3	2.3957e-3	2.9302e-3	4.7891e-3
		P3	1.1488e-1	6.0316e-3	2.4739e-4	4.8638e-3
		P4	2.7747e-3	2.0370e-3	3.1984e-3	4.0456e-3
		P5	4.1209e-2	8.0580e-3	3.8856e-1	3.6140e-5
		P6	2.2195e-1	5.3405e-3	6.7349e-3	2.1727e-2
		P7	9.6207e-1	5.9675e-1	6.8207e-1	3.7432e-1
		P8	1.1173e-1	2.0407e-3	1.1541e-3	1.4942e-3
		P9	7.9304e-2	4.7357e-1	7.4140e-9	1.2422e-8
	HV	P1	8.8870e-2	4.3581e-2	1.0212e-2	1.3640e-1
		P2	4.4093e-3	3.4392e-3	4.1652e-3	6.7215e-3
		P3	—	9.2048e-3	2.5495e-4	1.7399e-2
		P4	5.1134e-3	4.0629e-3	4.0440e-3	5.6367e-3
		P5	4.0106e-4	5.4985e-2	9.9869e-2	1.4212e-7
		P6	2.6949e-2	1.2530e-2	1.4006e-2	1.7661e-2
		P7	—	—	—	—
		P8	5.8067e-2	3.7040e-4	2.1009e-7	2.0506e-7
		P9	1.0173e-1	2.1424e-1	1.3053e-8	2.2218e-8

Table B.16.: IQR values for IGD and HV for Ref_2 with the four SX versions of MDELTA as grouping method and the WFG test suite with both 100 and 200 decision variables

			S2	S5	S20	S50
n=100	IGD	P1	8.5114e-2	5.1943e-2	4.4100e-2	6.5210e-2
		P2	1.7077e-2	2.2882e-2	1.6776e-2	1.8670e-2
		P3	1.8367e-2	2.0402e-2	1.9796e-2	1.7216e-2
		P4	2.9051e-4	3.7059e-4	3.2874e-4	3.0751e-4
		P5	2.4574e-4	1.6295e-4	2.0594e-4	1.2203e-4
		P6	1.3861e-3	1.3017e-3	1.3745e-3	4.4093e-4
		P7	3.6623e-4	2.2082e-4	2.7642e-4	3.3526e-4
		P8	8.2821e-3	5.9650e-3	6.5604e-3	1.1488e-2
		P9	8.3935e-3	1.2421e-2	1.9613e-3	1.5635e-2
	HV	P1	3.9344e-1	2.2848e-1	2.3165e-1	3.0346e-1
		P2	9.0587e-2	1.2797e-1	9.0912e-2	1.1027e-1
		P3	8.9947e-2	1.0485e-1	9.9409e-2	8.5341e-2
		P4	1.3005e-3	2.0756e-3	1.1348e-3	1.4194e-3
		P5	4.1108e-3	7.8154e-4	1.2711e-3	9.2888e-4
		P6	7.6507e-3	1.0598e-2	1.0667e-2	3.3008e-3
		P7	3.5992e-3	9.9455e-4	1.1542e-3	1.5862e-3
		P8	5.1483e-2	3.0473e-2	4.6189e-2	5.9470e-2
		P9	6.4974e-2	9.6141e-2	1.7666e-2	1.0355e-1
n=200	IGD	P1	9.2293e-2	7.0315e-2	4.0139e-2	4.9592e-2
		P2	3.1555e-2	1.5335e-2	2.3991e-2	1.9159e-2
		P3	3.0430e-2	2.8298e-2	2.4025e-2	2.1563e-2
		P4	2.2510e-4	2.7758e-4	3.0050e-4	4.3609e-4
		P5	4.6430e-3	2.6728e-4	1.5445e-4	1.6778e-4
		P6	3.6751e-4	3.4488e-4	4.9873e-4	5.2092e-4
		P7	2.6470e-3	3.8045e-4	2.8964e-4	4.1017e-4
		P8	5.1368e-3	5.5044e-3	5.8035e-3	9.0842e-3
		P9	1.4691e-2	3.3860e-3	4.6021e-3	9.3292e-3
	HV	P1	3.6740e-1	3.5187e-1	2.1024e-1	2.3965e-1
		P2	1.5559e-1	8.6988e-2	1.2891e-1	9.9651e-2
		P3	1.4600e-1	1.3529e-1	1.1578e-1	1.0286e-1
		P4	1.2102e-3	2.3327e-3	1.5798e-3	1.2571e-3
		P5	4.5603e-2	2.4577e-3	1.8002e-3	1.0011e-3
		P6	2.9084e-3	2.3368e-3	4.7564e-3	2.5997e-3
		P7	2.2705e-2	6.0094e-3	8.2105e-4	1.1018e-3
		P8	2.8484e-2	2.3617e-2	3.5364e-2	4.7040e-2
		P9	7.5350e-2	9.4739e-3	4.2888e-2	7.4747e-2

C. Comparison of Multidimensional Random and Delta Grouping

In this section the tables to compare versions of MRAND and MDELTA are depicted. Each table contains information about the two test suites LSMOP and WFG, the two quality criteria HV and IGD and eight versions of MRAND and MDELTA.

Table C.1.: Median IGD and HV values for Ref_1 with MRAND and MDELTA with S2 and N50 versions for the LSMOP and WFG test suite and 100 decision variables

			MRAND+S2	MRAND+N50	MDELTA+S2	MDELTA+N50	MRAND+S2	MDELTA+S2	MRAND+N50	MDELTA+N50
LSMOP	IGD	P1	**3.5644e-2**$^{(r)}$	6.2296e-2$^{(-)}$	**3.5625e-2**$^{(r)}$	5.7086e-2$^{(-)}$	3.5644e-2$^{(o)}$	**3.5625e-2**$^{(r)}$	6.2296e-2$^{(o)}$	**5.7086e-2**$^{(r)}$
		P2	**3.5003e-2**$^{(r)}$	3.7551e-2$^{(-)}$	**3.1553e-2**$^{(r)}$	3.5458e-2$^{(-)}$	3.5003e-2$^{(-)}$	**3.1553e-2**$^{(r)}$	3.7551e-2$^{(-)}$	**3.5458e-2**$^{(r)}$
		P3	**9.6557e-1**$^{(r)}$	1.0425e+0$^{(-)}$	**1.0098e+0**$^{(r)}$	1.0579e+0$^{(-)}$	**9.6557e-1**$^{(r)}$	1.0098e+0$^{(o)}$	**1.0425e+0**$^{(r)}$	1.0579e+0$^{(o)}$
		P4	**4.9708e-2**$^{(r)}$	5.1230e-2$^{(o)}$	**6.0935e-2**$^{(r)}$	6.8732e-2$^{(-)}$	**4.9708e-2**$^{(r)}$	6.0935e-2$^{(-)}$	**5.1230e-2**$^{(r)}$	6.8732e-2$^{(-)}$
		P5	**4.7698e-1**$^{(r)}$	7.3897e-1$^{(o)}$	**6.2532e-1**$^{(r)}$	7.4209e-1$^{(o)}$	**4.7698e-1**$^{(r)}$	6.2532e-1$^{(o)}$	**7.3897e-1**$^{(r)}$	7.4209e-1$^{(o)}$
		P6	**7.3005e-1**$^{(r)}$	7.4366e-1$^{(o)}$	**6.9817e-1**$^{(r)}$	7.4347e-1$^{(o)}$	7.3005e-1$^{(o)}$	**6.9817e-1**$^{(r)}$	7.4366e-1$^{(o)}$	**7.4347e-1**$^{(r)}$
		P7	1.3473e+0$^{(o)}$	**1.3463e+0**$^{(r)}$	1.3495e+0$^{(o)}$	**1.3494e+0**$^{(r)}$	**1.3473e+0**$^{(r)}$	1.3495e+0$^{(o)}$	**1.3463e+0**$^{(r)}$	1.3494e+0$^{(o)}$
		P8	**1.3515e-1**$^{(r)}$	1.4120e-1$^{(o)}$	**1.2810e-1**$^{(r)}$	1.4697e-1$^{(o)}$	1.3515e-1$^{(o)}$	**1.2810e-1**$^{(r)}$	**1.4120e-1**$^{(r)}$	1.4697e-1$^{(o)}$
		P9	**4.8592e-1**$^{(r)}$	5.5040e-1$^{(o)}$	5.1701e-1$^{(o)}$	**5.0566e-1**$^{(r)}$	**4.8592e-1**$^{(r)}$	5.1701e-1$^{(o)}$	5.5040e-1$^{(o)}$	**5.0566e-1**$^{(r)}$
	HV	P1	**6.4324e-1**$^{(r)}$	6.0019e-1$^{(-)}$	**6.4317e-1**$^{(r)}$	6.0237e-1$^{(-)}$	**6.4324e-1**$^{(r)}$	6.4317e-1$^{(o)}$	6.0019e-1$^{(o)}$	**6.0237e-1**$^{(r)}$
		P2	**6.4725e-1**$^{(r)}$	6.4287e-1$^{(-)}$	**6.5229e-1**$^{(r)}$	6.4618e-1$^{(-)}$	6.4725e-1$^{(-)}$	**6.5229e-1**$^{(r)}$	6.4287e-1$^{(-)}$	**6.4618e-1**$^{(r)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	**6.1720e-1**$^{(r)}$	6.1560e-1$^{(o)}$	**5.9980e-1**$^{(r)}$	5.8938e-1$^{(-)}$	**6.1720e-1**$^{(r)}$	5.9980e-1$^{(-)}$	**6.1560e-1**$^{(r)}$	5.8938e-1$^{(-)}$
		P5	1.0997e-1$^{(o)}$	**1.0998e-1**$^{(r)}$	**1.0998e-1**$^{(r)}$	1.0997e-1$^{(o)}$	1.0997e-1$^{(o)}$	**1.0998e-1**$^{(r)}$	**1.0998e-1**$^{(r)}$	1.0997e-1$^{(o)}$
		P6	8.9504e-2$^{(o)}$	**9.3188e-2**$^{(r)}$	8.8915e-2$^{(o)}$	**9.1690e-2**$^{(r)}$	**8.9504e-2**$^{(r)}$	8.8915e-2$^{(o)}$	**9.3188e-2**$^{(r)}$	9.1690e-2$^{(o)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**2.5430e-1**$^{(r)}$	2.4004e-1$^{(o)}$	2.5369e-1$^{(o)}$	**2.5470e-1**$^{(r)}$	**2.5430e-1**$^{(r)}$	2.5369e-1$^{(o)}$	2.4004e-1$^{(o)}$	**2.5470e-1**$^{(r)}$
		P9	**6.8276e-1**$^{(r)}$	6.1810e-1$^{(o)}$	6.5125e-1$^{(o)}$	**6.6581e-1**$^{(r)}$	**6.8276e-1**$^{(r)}$	6.5125e-1$^{(o)}$	6.1810e-1$^{(o)}$	**6.6581e-1**$^{(r)}$
WFG	IGD	P1	**1.1869e+0**$^{(r)}$	1.2005e+0$^{(o)}$	**1.1881e+0**$^{(r)}$	1.2147e+0$^{(o)}$	**1.1869e+0**$^{(r)}$	1.1881e+0$^{(o)}$	**1.2005e+0**$^{(r)}$	1.2147e+0$^{(o)}$
		P2	**3.5706e-2**$^{(r)}$	3.6181e-2$^{(o)}$	2.8966e-2$^{(o)}$	**2.7542e-2**$^{(r)}$	3.5706e-2$^{(-)}$	**2.8966e-2**$^{(r)}$	3.6181e-2$^{(-)}$	**2.7542e-2**$^{(r)}$
		P3	3.4219e-2$^{(o)}$	**3.2809e-2**$^{(r)}$	2.6337e-2$^{(o)}$	**2.6069e-2**$^{(r)}$	3.4219e-2$^{(-)}$	**2.6337e-2**$^{(r)}$	3.2809e-2$^{(-)}$	**2.6069e-2**$^{(r)}$
		P4	1.4053e-2$^{(o)}$	**1.3967e-2**$^{(r)}$	**1.3888e-2**$^{(r)}$	1.3982e-2$^{(o)}$	1.4053e-2$^{(o)}$	**1.3888e-2**$^{(r)}$	**1.3967e-2**$^{(r)}$	1.3982e-2$^{(o)}$
		P5	**6.9494e-2**$^{(r)}$	6.9698e-2$^{(o)}$	**6.9754e-2**$^{(r)}$	6.9761e-2$^{(o)}$	**6.9494e-2**$^{(r)}$	6.9754e-2$^{(o)}$	**6.9698e-2**$^{(r)}$	6.9761e-2$^{(o)}$
		P6	**1.9226e-2**$^{(r)}$	1.9477e-2$^{(-)}$	1.9432e-2$^{(o)}$	**1.9385e-2**$^{(r)}$	**1.9226e-2**$^{(r)}$	1.9432e-2$^{(-)}$	1.9477e-2$^{(o)}$	**1.9385e-2**$^{(r)}$
		P7	**7.1881e-2**$^{(r)}$	8.5730e-2$^{(-)}$	**7.1800e-2**$^{(r)}$	7.5930e-2$^{(o)}$	7.1881e-2$^{(o)}$	**7.1800e-2**$^{(r)}$	8.5730e-2$^{(o)}$	**7.5930e-2**$^{(r)}$
		P8	5.5949e-2$^{(o)}$	**5.5039e-2**$^{(r)}$	**5.3835e-2**$^{(r)}$	5.3970e-2$^{(o)}$	5.5949e-2$^{(o)}$	**5.3835e-2**$^{(r)}$	5.5039e-2$^{(-)}$	**5.3970e-2**$^{(r)}$
		P9	**7.7393e-2**$^{(r)}$	9.7740e-2$^{(o)}$	8.5152e-2$^{(o)}$	**6.5005e-2**$^{(r)}$	**7.7393e-2**$^{(r)}$	8.5152e-2$^{(o)}$	9.7740e-2$^{(o)}$	**6.5005e-2**$^{(r)}$
	HV	P1	**1.8236e+0**$^{(r)}$	1.7517e+0$^{(o)}$	**1.9389e+0**$^{(r)}$	1.7721e+0$^{(o)}$	1.8236e+0$^{(o)}$	**1.9389e+0**$^{(r)}$	1.7517e+0$^{(o)}$	**1.7721e+0**$^{(r)}$
		P2	5.9602e+0$^{(o)}$	**5.9608e+0**$^{(r)}$	6.0019e+0$^{(o)}$	**6.0123e+0**$^{(r)}$	5.9602e+0$^{(-)}$	**6.0019e+0**$^{(r)}$	5.9608e+0$^{(-)}$	**6.0123e+0**$^{(r)}$
		P3	5.4836e+0$^{(o)}$	**5.4923e+0**$^{(r)}$	5.5272e+0$^{(o)}$	**5.5287e+0**$^{(r)}$	5.4836e+0$^{(-)}$	**5.5272e+0**$^{(r)}$	5.4923e+0$^{(-)}$	**5.5287e+0**$^{(r)}$
		P4	**3.3620e+0**$^{(r)}$	3.3615e+0$^{(o)}$	3.3612e+0$^{(o)}$	**3.3617e+0**$^{(r)}$	**3.3620e+0**$^{(r)}$	3.3612e+0$^{(o)}$	3.3615e+0$^{(o)}$	**3.3617e+0**$^{(r)}$
		P5	**2.9825e+0**$^{(r)}$	2.9809e+0$^{(o)}$	**2.9811e+0**$^{(r)}$	2.9810e+0$^{(o)}$	**2.9825e+0**$^{(r)}$	2.9811e+0$^{(o)}$	2.9809e+0$^{(o)}$	**2.9810e+0**$^{(r)}$
		P6	**3.2959e+0**$^{(r)}$	3.2953e+0$^{(o)}$	3.2953e+0$^{(o)}$	**3.2964e+0**$^{(r)}$	**3.2959e+0**$^{(r)}$	3.2953e+0$^{(o)}$	3.2953e+0$^{(o)}$	**3.2964e+0**$^{(r)}$
		P7	**3.0374e+0**$^{(r)}$	2.9484e+0$^{(-)}$	**3.0350e+0**$^{(r)}$	3.0051e+0$^{(o)}$	**3.0374e+0**$^{(r)}$	3.0350e+0$^{(o)}$	2.9484e+0$^{(-)}$	**3.0051e+0**$^{(r)}$
		P8	3.1133e+0$^{(o)}$	**3.1162e+0**$^{(r)}$	3.1265e+0$^{(o)}$	**3.1299e+0**$^{(r)}$	3.1133e+0$^{(-)}$	**3.1265e+0**$^{(r)}$	3.1162e+0$^{(-)}$	**3.1299e+0**$^{(r)}$
		P9	**2.9488e+0**$^{(r)}$	2.9121e+0$^{(o)}$	2.9118e+0$^{(o)}$	**3.0543e+0**$^{(r)}$	**2.9488e+0**$^{(r)}$	2.9118e+0$^{(o)}$	2.9121e+0$^{(o)}$	**3.0543e+0**$^{(r)}$

Table C.2.: Median IGD and HV values for Ref_2 with MRAND and MDELTA with S2 and N50 versions for the LSMOP and WFG test suite and 100 decision variables

			MRAND+S2	MRAND+N50	MDELTA+S2	MDELTA+N50	MRAND+S2	MDELTA+S2	MRAND+N50	MDELTA+N50
LSMOP	IGD	P1	**1.4588e-1**$^{(r)}$	*2.3010e-1*$^{(-)}$	**1.4202e-1**$^{(r)}$	*2.2391e-1*$^{(-)}$	*1.4588e-1*$^{(o)}$	**1.4202e-1**$^{(r)}$	*2.3010e-1*$^{(o)}$	**2.2391e-1**$^{(r)}$
		P2	**2.3937e-2**$^{(r)}$	*2.6401e-2*$^{(-)}$	**2.2768e-2**$^{(r)}$	*2.5347e-2*$^{(-)}$	*2.3937e-2*$^{(-)}$	**2.2768e-2**$^{(r)}$	*2.6401e-2*$^{(-)}$	**2.5347e-2**$^{(r)}$
		P3	*7.0735e-1*$^{(-)}$	**7.0714e-1**$^{(r)}$	*7.0762e-1*$^{(-)}$	**7.0718e-1**$^{(r)}$	**7.0735e-1**$^{(r)}$	*7.0762e-1*$^{(o)}$	**7.0714e-1**$^{(r)}$	*7.0718e-1*$^{(o)}$
		P4	**2.4374e-2**$^{(r)}$	*2.7863e-2*$^{(-)}$	**2.9015e-2**$^{(r)}$	*3.2884e-2*$^{(-)}$	**2.4374e-2**$^{(r)}$	*2.9015e-2*$^{(-)}$	**2.7863e-2**$^{(r)}$	*3.2884e-2*$^{(-)}$
		P5	**3.4110e-1**$^{(r)}$	*3.4134e-1*$^{(o)}$	*3.4177e-1*$^{(o)}$	**3.4119e-1**$^{(r)}$	**3.4110e-1**$^{(r)}$	*3.4177e-1*$^{(o)}$	*3.4134e-1*$^{(o)}$	**3.4119e-1**$^{(r)}$
		P6	**6.3628e-1**$^{(r)}$	*7.4225e-1*$^{(o)}$	**6.7058e-1**$^{(r)}$	*7.3679e-1*$^{(o)}$	**6.3628e-1**$^{(r)}$	*6.7058e-1*$^{(o)}$	*7.4225e-1*$^{(o)}$	**7.3679e-1**$^{(r)}$
		P7	*1.3645e+0*$^{(o)}$	**1.3443e+0**$^{(r)}$	**1.7244e+0**$^{(r)}$	*1.9307e+0*$^{(o)}$	**1.3645e+0**$^{(r)}$	*1.7244e+0*$^{(o)}$	**1.3443e+0**$^{(r)}$	*1.9307e+0*$^{(-)}$
		P8	**3.7289e-1**$^{(r)}$	*3.7439e-1*$^{(o)}$	*3.7049e-1*$^{(o)}$	**3.4822e-1**$^{(r)}$	*3.7289e-1*$^{(o)}$	**3.7049e-1**$^{(r)}$	*3.7439e-1*$^{(o)}$	**3.4822e-1**$^{(r)}$
		P9	**1.5321e-1**$^{(r)}$	*1.9003e-1*$^{(-)}$	**1.7731e-1**$^{(r)}$	*2.1146e-1*$^{(o)}$	**1.5321e-1**$^{(r)}$	*1.7731e-1*$^{(-)}$	**1.9003e-1**$^{(r)}$	*2.1146e-1*$^{(o)}$
	HV	P1	**5.0182e-1**$^{(r)}$	*3.8431e-1*$^{(-)}$	**5.0794e-1**$^{(r)}$	*3.9223e-1*$^{(-)}$	*5.0182e-1*$^{(o)}$	**5.0794e-1**$^{(r)}$	*3.8431e-1*$^{(o)}$	**3.9223e-1**$^{(r)}$
		P2	**6.6844e-1**$^{(r)}$	*6.6535e-1*$^{(-)}$	**6.7071e-1**$^{(r)}$	*6.6688e-1*$^{(-)}$	*6.6844e-1*$^{(-)}$	**6.7071e-1**$^{(r)}$	*6.6535e-1*$^{(o)}$	**6.6688e-1**$^{(r)}$
		P3	*1.0962e-1*$^{(-)}$	**1.0992e-1**$^{(r)}$	*1.0901e-1*$^{(-)}$	**1.0988e-1**$^{(r)}$	**1.0962e-1**$^{(r)}$	*1.0901e-1*$^{(o)}$	**1.0992e-1**$^{(r)}$	*1.0988e-1*$^{(o)}$
		P4	**6.6468e-1**$^{(r)}$	*6.5861e-1*$^{(-)}$	**6.5500e-1**$^{(r)}$	*6.4817e-1*$^{(-)}$	**6.6468e-1**$^{(r)}$	*6.5500e-1*$^{(-)}$	**6.5861e-1**$^{(r)}$	*6.4817e-1*$^{(-)}$
		P5	*2.0730e-1*$^{(o)}$	**2.0928e-1**$^{(r)}$	**2.0935e-1**$^{(r)}$	*2.0900e-1*$^{(o)}$	*2.0730e-1*$^{(o)}$	**2.0935e-1**$^{(r)}$	**2.0928e-1**$^{(r)}$	*2.0900e-1*$^{(o)}$
		P6	*1.0528e-1*$^{(-)}$	**1.0789e-1**$^{(r)}$	*1.0676e-1*$^{(-)}$	**1.0812e-1**$^{(r)}$	*1.0528e-1*$^{(o)}$	**1.0676e-1**$^{(r)}$	*1.0789e-1*$^{(o)}$	**1.0812e-1**$^{(r)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**1.1000e-1**$^{(r)}$	*1.1000e-1*$^{(o)}$	*1.1000e-1*$^{(o)}$	**1.2476e-1**$^{(r)}$	**1.1000e-1**$^{(r)}$	*1.1000e-1*$^{(o)}$	*1.1000e-1*$^{(o)}$	**1.2476e-1**$^{(r)}$
		P9	**7.6316e-1**$^{(r)}$	*7.2415e-1*$^{(-)}$	**7.2486e-1**$^{(r)}$	*7.0508e-1*$^{(o)}$	**7.6316e-1**$^{(r)}$	*7.2486e-1*$^{(-)}$	**7.2415e-1**$^{(r)}$	*7.0508e-1*$^{(o)}$
WFG	IGD	P1	**4.8609e-1**$^{(r)}$	*4.9699e-1*$^{(o)}$	*5.1361e-1*$^{(o)}$	**4.8361e-1**$^{(r)}$	**4.8609e-1**$^{(r)}$	*5.1361e-1*$^{(o)}$	*4.9699e-1*$^{(o)}$	**4.8361e-1**$^{(r)}$
		P2	*1.0999e-1*$^{(o)}$	**1.0970e-1**$^{(r)}$	**1.0905e-1**$^{(r)}$	*1.1074e-1*$^{(o)}$	*1.0999e-1*$^{(o)}$	**1.0905e-1**$^{(r)}$	**1.0970e-1**$^{(r)}$	*1.1074e-1*$^{(o)}$
		P3	**1.1817e-1**$^{(r)}$	*1.1851e-1*$^{(o)}$	**1.1043e-1**$^{(r)}$	*1.1131e-1*$^{(o)}$	*1.1817e-1*$^{(-)}$	**1.1043e-1**$^{(r)}$	*1.1851e-1*$^{(o)}$	**1.1131e-1**$^{(r)}$
		P4	*1.3899e-2*$^{(o)}$	**1.3867e-2**$^{(r)}$	*1.3877e-2*$^{(o)}$	**1.3824e-2**$^{(r)}$	*1.3899e-2*$^{(o)}$	**1.3877e-2**$^{(r)}$	*1.3867e-2*$^{(o)}$	**1.3824e-2**$^{(r)}$
		P5	**6.3712e-2**$^{(r)}$	*6.3731e-2*$^{(o)}$	**6.3731e-2**$^{(r)}$	*6.3779e-2*$^{(o)}$	**6.3712e-2**$^{(r)}$	*6.3731e-2*$^{(o)}$	**6.3731e-2**$^{(r)}$	*6.3779e-2*$^{(o)}$
		P6	**1.9132e-2**$^{(r)}$	*1.9287e-2*$^{(o)}$	**1.9354e-2**$^{(r)}$	*1.9515e-2*$^{(o)}$	**1.9132e-2**$^{(r)}$	*1.9354e-2*$^{(o)}$	**1.9287e-2**$^{(r)}$	*1.9515e-2*$^{(o)}$
		P7	**1.4207e-2**$^{(r)}$	*1.4385e-2*$^{(o)}$	**1.4198e-2**$^{(r)}$	*1.4815e-2*$^{(-)}$	*1.4207e-2*$^{(o)}$	**1.4198e-2**$^{(r)}$	**1.4385e-2**$^{(r)}$	*1.4815e-2*$^{(o)}$
		P8	*5.5198e-2*$^{(o)}$	**5.4206e-2**$^{(r)}$	*5.6867e-2*$^{(o)}$	**5.4244e-2**$^{(r)}$	**5.5198e-2**$^{(r)}$	*5.6867e-2*$^{(o)}$	**5.4206e-2**$^{(r)}$	*5.4244e-2*$^{(o)}$
		P9	**3.4599e-2**$^{(r)}$	*3.4939e-2*$^{(o)}$	**3.4366e-2**$^{(r)}$	*3.4837e-2*$^{(o)}$	*3.4599e-2*$^{(o)}$	**3.4366e-2**$^{(r)}$	*3.4939e-2*$^{(o)}$	**3.4837e-2**$^{(r)}$
	HV	P1	4.2942e+0$^{(o)}$	**4.3094e+0**$^{(r)}$	4.1373e+0$^{(o)}$	**4.2910e+0**$^{(r)}$	**4.2942e+0**$^{(r)}$	4.1373e+0$^{(o)}$	**4.3094e+0**$^{(r)}$	4.2910e+0$^{(o)}$
		P2	5.5364e+0$^{(o)}$	**5.5377e+0**$^{(r)}$	**5.5402e+0**$^{(r)}$	5.5297e+0$^{(o)}$	5.5364e+0$^{(o)}$	**5.5402e+0**$^{(r)}$	**5.5377e+0**$^{(r)}$	5.5297e+0$^{(o)}$
		P3	**5.0641e+0**$^{(r)}$	5.0627e+0$^{(o)}$	**5.1023e+0**$^{(r)}$	5.0980e+0$^{(o)}$	5.0641e+0$^{(-)}$	**5.1023e+0**$^{(r)}$	5.0627e+0$^{(o)}$	**5.0980e+0**$^{(r)}$
		P4	**3.3599e+0**$^{(r)}$	3.3594e+0$^{(-)}$	**3.3594e+0**$^{(r)}$	3.3594e+0$^{(o)}$	**3.3599e+0**$^{(r)}$	3.3594e+0$^{(o)}$	3.3594e+0$^{(o)}$	**3.3594e+0**$^{(r)}$
		P5	**3.0329e+0**$^{(r)}$	3.0327e+0$^{(o)}$	3.0321e+0$^{(o)}$	**3.0327e+0**$^{(r)}$	**3.0329e+0**$^{(r)}$	3.0321e+0$^{(o)}$	**3.0327e+0**$^{(r)}$	3.0327e+0$^{(o)}$
		P6	**3.2998e+0**$^{(r)}$	3.2958e+0$^{(o)}$	3.2964e+0$^{(o)}$	**3.2966e+0**$^{(r)}$	**3.2998e+0**$^{(r)}$	3.2964e+0$^{(o)}$	3.2958e+0$^{(o)}$	**3.2966e+0**$^{(r)}$
		P7	**3.3592e+0**$^{(r)}$	3.3566e+0$^{(-)}$	**3.3599e+0**$^{(r)}$	3.3471e+0$^{(-)}$	3.3592e+0$^{(o)}$	**3.3599e+0**$^{(r)}$	**3.3566e+0**$^{(r)}$	3.3471e+0$^{(o)}$
		P8	3.1068e+0$^{(o)}$	**3.1109e+0**$^{(r)}$	3.1042e+0$^{(o)}$	**3.1136e+0**$^{(r)}$	**3.1068e+0**$^{(r)}$	3.1042e+0$^{(o)}$	3.1109e+0$^{(o)}$	**3.1136e+0**$^{(r)}$
		P9	**3.1717e+0**$^{(r)}$	3.1708e+0$^{(o)}$	**3.1717e+0**$^{(r)}$	3.1717e+0$^{(o)}$	3.1717e+0$^{(o)}$	**3.1717e+0**$^{(r)}$	3.1708e+0$^{(o)}$	**3.1717e+0**$^{(r)}$

Table C.3.: Median IGD and HV values for Ref_1 with MRAND and MDELTA with S2 and N50 versions for the LSMOP and WFG test suite and 200 decision variables

			MRAND+S2	MRAND+N50	MDELTA+S2	MDELTA+N50	MRAND+S2	MDELTA+S2	MRAND+N50	MDELTA+N50
LSMOP	IGD	P1	**5.3233e-2**$^{(r)}$	1.5515e-1$^{(-)}$	**5.2145e-2**$^{(r)}$	1.4289e-1$^{(-)}$	5.3233e-2$^{(o)}$	**5.2145e-2**$^{(r)}$	1.5515e-1$^{(o)}$	**1.4289e-1**$^{(r)}$
		P2	**3.4332e-2**$^{(r)}$	3.7901e-2$^{(-)}$	**3.3090e-2**$^{(r)}$	3.6820e-2$^{(-)}$	3.4332e-2$^{(-)}$	**3.3090e-2**$^{(r)}$	3.7901e-2$^{(-)}$	**3.6820e-2**$^{(r)}$
		P3	**9.6753e-1**$^{(r)}$	9.8849e-1$^{(o)}$	1.0080e+0$^{(o)}$	**9.8303e-1**$^{(r)}$	**9.6753e-1**$^{(r)}$	1.0080e+0$^{(o)}$	9.8849e-1$^{(o)}$	**9.8303e-1**$^{(r)}$
		P4	**4.6468e-2**$^{(r)}$	5.4709e-2$^{(-)}$	**5.2774e-2**$^{(r)}$	6.5434e-2$^{(-)}$	**4.6468e-2**$^{(r)}$	5.2774e-2$^{(-)}$	**5.4709e-2**$^{(r)}$	6.5434e-2$^{(-)}$
		P5	**4.8913e-1**$^{(r)}$	6.2002e-1$^{(o)}$	**4.8445e-1**$^{(r)}$	5.7075e-1$^{(o)}$	4.8913e-1$^{(o)}$	**4.8445e-1**$^{(r)}$	6.2002e-1$^{(o)}$	**5.7075e-1**$^{(r)}$
		P6	**6.5753e-1**$^{(r)}$	7.2950e-1$^{(-)}$	**5.4734e-1**$^{(r)}$	7.6224e-1$^{(-)}$	6.5753e-1$^{(o)}$	**5.4734e-1**$^{(r)}$	**7.2950e-1**$^{(r)}$	7.6224e-1$^{(o)}$
		P7	**1.4339e+0**$^{(r)}$	1.4343e+0$^{(o)}$	**1.4328e+0**$^{(r)}$	1.4455e+0$^{(-)}$	1.4339e+0$^{(o)}$	**1.4328e+0**$^{(r)}$	**1.4343e+0**$^{(r)}$	1.4455e+0$^{(o)}$
		P8	**1.1585e-1**$^{(r)}$	1.5607e-1$^{(-)}$	**1.3432e-1**$^{(r)}$	1.6374e-1$^{(-)}$	**1.1585e-1**$^{(r)}$	1.3432e-1$^{(o)}$	**1.5607e-1**$^{(r)}$	1.6374e-1$^{(o)}$
		P9	**4.9482e-1**$^{(r)}$	5.2326e-1$^{(-)}$	**4.9035e-1**$^{(r)}$	5.4087e-1$^{(-)}$	4.9482e-1$^{(o)}$	**4.9035e-1**$^{(r)}$	**5.2326e-1**$^{(r)}$	5.4087e-1$^{(o)}$
	HV	P1	**6.1394e-1**$^{(r)}$	4.8203e-1$^{(-)}$	**6.1458e-1**$^{(r)}$	4.9332e-1$^{(-)}$	6.1394e-1$^{(o)}$	**6.1458e-1**$^{(r)}$	4.8203e-1$^{(o)}$	**4.9332e-1**$^{(r)}$
		P2	**6.4892e-1**$^{(r)}$	6.4354e-1$^{(-)}$	**6.5026e-1**$^{(r)}$	6.4493e-1$^{(-)}$	6.4892e-1$^{(-)}$	**6.5026e-1**$^{(r)}$	6.4354e-1$^{(o)}$	**6.4493e-1**$^{(r)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	**6.2738e-1**$^{(r)}$	6.1430e-1$^{(-)}$	**6.1816e-1**$^{(r)}$	5.9833e-1$^{(-)}$	**6.2738e-1**$^{(r)}$	6.1816e-1$^{(-)}$	**6.1430e-1**$^{(r)}$	5.9833e-1$^{(-)}$
		P5	1.0929e-1$^{(o)}$	**1.0960e-1**$^{(r)}$	1.0920e-1$^{(o)}$	**1.0936e-1**$^{(r)}$	**1.0929e-1**$^{(r)}$	1.0920e-1$^{(o)}$	**1.0960e-1**$^{(r)}$	1.0936e-1$^{(o)}$
		P6	**1.1120e-2**$^{(r)}$	8.6111e-3$^{(o)}$	**2.1413e-3**$^{(r)}$	—$^{(-)}$	**1.1120e-2**$^{(r)}$	2.1413e-3$^{(o)}$	**8.6111e-3**$^{(r)}$	—$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**2.5841e-1**$^{(r)}$	2.2505e-1$^{(-)}$	**2.4977e-1**$^{(r)}$	2.2387e-1$^{(-)}$	**2.5841e-1**$^{(r)}$	2.4977e-1$^{(o)}$	**2.2505e-1**$^{(r)}$	2.2387e-1$^{(o)}$
		P9	**6.7104e-1**$^{(r)}$	6.2226e-1$^{(-)}$	**6.7764e-1**$^{(r)}$	6.2448e-1$^{(-)}$	6.7104e-1$^{(o)}$	**6.7764e-1**$^{(r)}$	6.2226e-1$^{(o)}$	**6.2448e-1**$^{(r)}$
WFG	IGD	P1	1.2919e+0$^{(-)}$	**1.1804e+0**$^{(r)}$	1.2502e+0$^{(-)}$	**1.1901e+0**$^{(r)}$	1.2919e+0$^{(o)}$	**1.2502e+0**$^{(r)}$	**1.1804e+0**$^{(r)}$	1.1901e+0$^{(o)}$
		P2	5.4538e-2$^{(-)}$	**2.6449e-2**$^{(r)}$	4.7517e-2$^{(-)}$	**2.3696e-2**$^{(r)}$	5.4538e-2$^{(-)}$	**4.7517e-2**$^{(r)}$	2.6449e-2$^{(-)}$	**2.3696e-2**$^{(r)}$
		P3	5.5815e-2$^{(-)}$	**2.4979e-2**$^{(r)}$	5.0570e-2$^{(-)}$	**2.2642e-2**$^{(r)}$	5.5815e-2$^{(-)}$	**5.0570e-2**$^{(r)}$	2.4979e-2$^{(-)}$	**2.2642e-2**$^{(r)}$
		P4	1.5040e-2$^{(o)}$	**1.4996e-2**$^{(r)}$	**1.4754e-2**$^{(r)}$	1.4909e-2$^{(o)}$	1.5040e-2$^{(o)}$	**1.4754e-2**$^{(r)}$	1.4996e-2$^{(o)}$	**1.4909e-2**$^{(r)}$
		P5	**6.9703e-2**$^{(r)}$	6.9889e-2$^{(-)}$	**6.9742e-2**$^{(r)}$	6.9842e-2$^{(o)}$	**6.9703e-2**$^{(r)}$	6.9742e-2$^{(o)}$	6.9889e-2$^{(o)}$	**6.9842e-2**$^{(r)}$
		P6	1.5865e-2$^{(o)}$	**1.5697e-2**$^{(r)}$	1.5759e-2$^{(o)}$	**1.5694e-2**$^{(r)}$	1.5865e-2$^{(o)}$	**1.5759e-2**$^{(r)}$	1.5697e-2$^{(o)}$	**1.5694e-2**$^{(r)}$
		P7	**1.4802e-1**$^{(r)}$	1.8817e-1$^{(o)}$	**1.3922e-1**$^{(r)}$	1.8383e-1$^{(o)}$	1.4802e-1$^{(o)}$	**1.3922e-1**$^{(r)}$	1.8817e-1$^{(o)}$	**1.8383e-1**$^{(r)}$
		P8	**5.1535e-2**$^{(r)}$	5.2341e-2$^{(o)}$	5.1552e-2$^{(o)}$	**5.0713e-2**$^{(r)}$	**5.1535e-2**$^{(r)}$	5.1552e-2$^{(o)}$	5.2341e-2$^{(o)}$	**5.0713e-2**$^{(r)}$
		P9	1.5105e-1$^{(o)}$	**1.1877e-1**$^{(r)}$	1.2064e-1$^{(o)}$	**1.0811e-1**$^{(r)}$	1.5105e-1$^{(o)}$	**1.2064e-1**$^{(r)}$	1.1877e-1$^{(o)}$	**1.0811e-1**$^{(r)}$
	HV	P1	1.3132e+0$^{(-)}$	**2.2030e+0**$^{(r)}$	1.5267e+0$^{(-)}$	**2.1406e+0**$^{(r)}$	1.3132e+0$^{(-)}$	**1.5267e+0**$^{(r)}$	**2.2030e+0**$^{(r)}$	2.1406e+0$^{(o)}$
		P2	5.8528e+0$^{(-)}$	**6.0286e+0**$^{(r)}$	5.8955e+0$^{(-)}$	**6.0472e+0**$^{(r)}$	5.8528e+0$^{(-)}$	**5.8955e+0**$^{(r)}$	6.0286e+0$^{(-)}$	**6.0472e+0**$^{(r)}$
		P3	5.3696e+0$^{(-)}$	**5.5349e+0**$^{(r)}$	5.3971e+0$^{(-)}$	**5.5509e+0**$^{(r)}$	5.3696e+0$^{(-)}$	**5.3971e+0**$^{(r)}$	5.5349e+0$^{(-)}$	**5.5509e+0**$^{(r)}$
		P4	**3.3616e+0**$^{(r)}$	3.3612e+0$^{(o)}$	**3.3612e+0**$^{(r)}$	3.3609e+0$^{(o)}$	**3.3616e+0**$^{(r)}$	3.3612e+0$^{(o)}$	**3.3612e+0**$^{(r)}$	3.3609e+0$^{(o)}$
		P5	**2.9801e+0**$^{(r)}$	2.9798e+0$^{(o)}$	**2.9812e+0**$^{(r)}$	2.9800e+0$^{(-)}$	2.9801e+0$^{(o)}$	**2.9812e+0**$^{(r)}$	2.9798e+0$^{(o)}$	**2.9800e+0**$^{(r)}$
		P6	3.3268e+0$^{(o)}$	**3.3275e+0**$^{(r)}$	**3.3282e+0**$^{(r)}$	3.3273e+0$^{(o)}$	3.3268e+0$^{(o)}$	**3.3282e+0**$^{(r)}$	**3.3275e+0**$^{(r)}$	3.3273e+0$^{(o)}$
		P7	**2.6022e+0**$^{(r)}$	2.3980e+0$^{(o)}$	**2.6686e+0**$^{(r)}$	2.4314e+0$^{(o)}$	2.6022e+0$^{(o)}$	**2.6686e+0**$^{(r)}$	2.3980e+0$^{(o)}$	**2.4314e+0**$^{(r)}$
		P8	3.1166e+0$^{(o)}$	**3.1226e+0**$^{(r)}$	3.1231e+0$^{(-)}$	**3.1382e+0**$^{(r)}$	3.1166e+0$^{(o)}$	**3.1231e+0**$^{(r)}$	3.1226e+0$^{(-)}$	**3.1382e+0**$^{(r)}$
		P9	2.5635e+0$^{(o)}$	**2.7248e+0**$^{(r)}$	2.7313e+0$^{(o)}$	**2.7903e+0**$^{(r)}$	2.5635e+0$^{(o)}$	**2.7313e+0**$^{(r)}$	2.7248e+0$^{(o)}$	**2.7903e+0**$^{(r)}$

Table C.4.: Median IGD and HV values for Ref_2 with MRAND and MDELTA with S2 and N50 versions for the LSMOP and WFG test suite and 200 decision variables

			MRAND+S2	MRAND+N50	MDELTA+S2	MDELTA+N50	MRAND+S2	MDELTA+S2	MRAND+N50	MDELTA+N50
LSMOP	IGD	P1	**1.3970e-1**$^{(r)}$	*3.3773e-1*$^{(-)}$	**1.4154e-1**$^{(r)}$	*3.3809e-1*$^{(-)}$	**1.3970e-1**$^{(r)}$	*1.4154e-1*$^{(o)}$	**3.3773e-1**$^{(r)}$	*3.3809e-1*$^{(o)}$
		P2	*3.2586e-2*$^{(o)}$	**3.1986e-2**$^{(r)}$	*3.2788e-2*$^{(-)}$	**3.2204e-2**$^{(r)}$	**3.2586e-2**$^{(r)}$	*3.2788e-2*$^{(o)}$	**3.1986e-2**$^{(r)}$	*3.2204e-2*$^{(o)}$
		P3	*9.5222e-1*$^{(-)}$	**7.0856e-1**$^{(r)}$	*8.5948e-1*$^{(-)}$	**7.1253e-1**$^{(r)}$	*9.5222e-1*$^{(-)}$	**8.5948e-1**$^{(r)}$	**7.0856e-1**$^{(r)}$	*7.1253e-1*$^{(-)}$
		P4	**1.7460e-2**$^{(r)}$	*2.1436e-2*$^{(-)}$	**2.0585e-2**$^{(r)}$	*2.7268e-2*$^{(-)}$	**1.7460e-2**$^{(r)}$	*2.0585e-2*$^{(-)}$	**2.1436e-2**$^{(r)}$	*2.7268e-2*$^{(-)}$
		P5	*7.4235e-1*$^{(-)}$	**4.6949e-1**$^{(r)}$	*7.4234e-1*$^{(-)}$	**5.3798e-1**$^{(r)}$	*7.4235e-1*$^{(o)}$	**7.4234e-1**$^{(r)}$	**4.6949e-1**$^{(r)}$	*5.3798e-1*$^{(o)}$
		P6	*7.5622e-1*$^{(o)}$	**7.5073e-1**$^{(r)}$	**7.2720e-1**$^{(r)}$	*7.5316e-1*$^{(o)}$	*7.5622e-1*$^{(o)}$	**7.2720e-1**$^{(r)}$	**7.5073e-1**$^{(r)}$	*7.5316e-1*$^{(o)}$
		P7	*2.1555e+0*$^{(o)}$	**2.1140e+0**$^{(r)}$	*2.2251e+0*$^{(o)}$	**1.8963e+0**$^{(r)}$	**2.1555e+0**$^{(r)}$	*2.2251e+0*$^{(o)}$	*2.1140e+0*$^{(o)}$	**1.8963e+0**$^{(r)}$
		P8	**3.4422e-1**$^{(r)}$	*3.6345e-1*$^{(-)}$	**3.6138e-1**$^{(r)}$	*3.6291e-1*$^{(o)}$	**3.4422e-1**$^{(r)}$	*3.6138e-1*$^{(o)}$	*3.6345e-1*$^{(o)}$	**3.6291e-1**$^{(r)}$
		P9	**6.5119e-2**$^{(r)}$	*2.1315e-1*$^{(-)}$	**8.9466e-2**$^{(r)}$	*8.1004e-1*$^{(-)}$	**6.5119e-2**$^{(r)}$	*8.9466e-2*$^{(-)}$	**2.1315e-1**$^{(r)}$	*8.1004e-1*$^{(-)}$
	HV	P1	**5.1006e-1**$^{(r)}$	*2.3005e-1*$^{(-)}$	**5.0702e-1**$^{(r)}$	*2.2966e-1*$^{(-)}$	**5.1006e-1**$^{(r)}$	*5.0702e-1*$^{(o)}$	**2.3005e-1**$^{(r)}$	*2.2966e-1*$^{(o)}$
		P2	*6.5570e-1*$^{(-)}$	**6.5918e-1**$^{(r)}$	*6.5493e-1*$^{(-)}$	**6.5899e-1**$^{(r)}$	**6.5570e-1**$^{(r)}$	*6.5493e-1*$^{(o)}$	**6.5918e-1**$^{(r)}$	*6.5899e-1*$^{(o)}$
		P3	—$^{(-)}$	**1.0774e-1**$^{(r)}$	—$^{(-)}$	**1.0165e-1**$^{(r)}$	—$^{(*)}$	—$^{(*)}$	**1.0774e-1**$^{(r)}$	*1.0165e-1*$^{(-)}$
		P4	**6.7795e-1**$^{(r)}$	*6.7075e-1*$^{(-)}$	**6.7100e-1**$^{(r)}$	*6.5953e-1*$^{(-)}$	**6.7795e-1**$^{(r)}$	*6.7100e-1*$^{(-)}$	**6.7075e-1**$^{(r)}$	*6.5953e-1*$^{(-)}$
		P5	**1.0918e-1**$^{(r)}$	*1.0892e-1*$^{(-)}$	**1.0920e-1**$^{(r)}$	*1.0892e-1*$^{(-)}$	*1.0918e-1*$^{(o)}$	**1.0920e-1**$^{(r)}$	**1.0892e-1**$^{(r)}$	*1.0892e-1*$^{(o)}$
		P6	*6.5976e-2*$^{(-)}$	**8.5659e-2**$^{(r)}$	*5.3938e-2*$^{(-)}$	**7.9688e-2**$^{(r)}$	**6.5976e-2**$^{(r)}$	*5.3938e-2*$^{(-)}$	**8.5659e-2**$^{(r)}$	*7.9688e-2*$^{(o)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**1.3437e-1**$^{(r)}$	*1.1000e-1*$^{(-)}$	**1.1006e-1**$^{(r)}$	*1.1000e-1*$^{(o)}$	**1.3437e-1**$^{(r)}$	*1.1006e-1*$^{(o)}$	*1.1000e-1*$^{(o)}$	**1.1000e-1**$^{(r)}$
		P9	**8.8806e-1**$^{(r)}$	*7.0092e-1*$^{(-)}$	**8.6941e-1**$^{(r)}$	*3.7814e-1*$^{(-)}$	**8.8806e-1**$^{(r)}$	*8.6941e-1*$^{(o)}$	**7.0092e-1**$^{(r)}$	*3.7814e-1*$^{(-)}$
WFG	IGD	P1	*8.2091e-1*$^{(-)}$	**4.9224e-1**$^{(r)}$	*8.1476e-1*$^{(-)}$	**5.1642e-1**$^{(r)}$	*8.2091e-1*$^{(o)}$	**8.1476e-1**$^{(r)}$	**4.9224e-1**$^{(r)}$	*5.1642e-1*$^{(o)}$
		P2	*1.7438e-1*$^{(-)}$	**1.4078e-1**$^{(r)}$	*1.6831e-1*$^{(-)}$	**1.4092e-1**$^{(r)}$	*1.7438e-1*$^{(o)}$	**1.6831e-1**$^{(r)}$	**1.4078e-1**$^{(r)}$	*1.4092e-1*$^{(o)}$
		P3	*1.8317e-1*$^{(-)}$	**1.4268e-1**$^{(r)}$	*1.7431e-1*$^{(-)}$	**1.2830e-1**$^{(r)}$	*1.8317e-1*$^{(o)}$	**1.7431e-1**$^{(r)}$	*1.4268e-1*$^{(-)}$	**1.2830e-1**$^{(r)}$
		P4	**1.4190e-2**$^{(r)}$	*1.4277e-2*$^{(o)}$	**1.4192e-2**$^{(r)}$	*1.4296e-2*$^{(o)}$	**1.4190e-2**$^{(r)}$	*1.4192e-2*$^{(o)}$	**1.4277e-2**$^{(r)}$	*1.4296e-2*$^{(o)}$
		P5	*6.4049e-2*$^{(-)}$	**6.3791e-2**$^{(r)}$	*6.3903e-2*$^{(-)}$	**6.3717e-2**$^{(r)}$	*6.4049e-2*$^{(o)}$	**6.3903e-2**$^{(r)}$	*6.3791e-2*$^{(o)}$	**6.3717e-2**$^{(r)}$
		P6	*1.5716e-2*$^{(o)}$	**1.5554e-2**$^{(r)}$	*1.5823e-2*$^{(o)}$	**1.5690e-2**$^{(r)}$	**1.5716e-2**$^{(r)}$	*1.5823e-2*$^{(o)}$	**1.5554e-2**$^{(r)}$	*1.5690e-2*$^{(o)}$
		P7	*1.7377e-2*$^{(-)}$	**1.5187e-2**$^{(r)}$	*1.7883e-2*$^{(-)}$	**1.4925e-2**$^{(r)}$	**1.7377e-2**$^{(r)}$	*1.7883e-2*$^{(o)}$	*1.5187e-2*$^{(o)}$	**1.4925e-2**$^{(r)}$
		P8	*5.4534e-2*$^{(-)}$	**5.3328e-2**$^{(r)}$	*5.4638e-2*$^{(-)}$	**5.1243e-2**$^{(r)}$	**5.4534e-2**$^{(r)}$	*5.4638e-2*$^{(o)}$	*5.3328e-2*$^{(o)}$	**5.1243e-2**$^{(r)}$
		P9	**2.8217e-2**$^{(r)}$	*3.1725e-2*$^{(o)}$	**2.8471e-2**$^{(r)}$	*2.9612e-2*$^{(o)}$	**2.8217e-2**$^{(r)}$	*2.8471e-2*$^{(o)}$	*3.1725e-2*$^{(o)}$	**2.9612e-2**$^{(r)}$
	HV	P1	2.8247e+0$^{(-)}$	**4.2666e+0**$^{(r)}$	2.8440e+0$^{(-)}$	**4.1296e+0**$^{(r)}$	2.8247e+0$^{(o)}$	**2.8440e+0**$^{(r)}$	**4.2666e+0**$^{(r)}$	4.1296e+0$^{(o)}$
		P2	5.1906e+0$^{(-)}$	**5.3704e+0**$^{(r)}$	5.2220e+0$^{(-)}$	**5.3678e+0**$^{(r)}$	5.1906e+0$^{(o)}$	**5.2220e+0**$^{(r)}$	**5.3704e+0**$^{(r)}$	5.3678e+0$^{(o)}$
		P3	4.7534e+0$^{(-)}$	**4.9481e+0**$^{(r)}$	4.7961e+0$^{(-)}$	**5.0158e+0**$^{(r)}$	4.7534e+0$^{(o)}$	**4.7961e+0**$^{(r)}$	4.9481e+0$^{(-)}$	**5.0158e+0**$^{(r)}$
		P4	**3.3522e+0**$^{(r)}$	3.3508e+0$^{(-)}$	**3.3524e+0**$^{(r)}$	3.3514e+0$^{(-)}$	3.3522e+0$^{(o)}$	**3.3524e+0**$^{(r)}$	3.3508e+0$^{(o)}$	**3.3514e+0**$^{(r)}$
		P5	3.0292e+0$^{(-)}$	**3.0326e+0**$^{(r)}$	3.0280e+0$^{(-)}$	**3.0329e+0**$^{(r)}$	**3.0292e+0**$^{(r)}$	3.0280e+0$^{(o)}$	3.0326e+0$^{(o)}$	**3.0329e+0**$^{(r)}$
		P6	3.3285e+0$^{(-)}$	**3.3306e+0**$^{(r)}$	3.3289e+0$^{(o)}$	**3.3292e+0**$^{(r)}$	3.3285e+0$^{(o)}$	**3.3289e+0**$^{(r)}$	**3.3306e+0**$^{(r)}$	3.3292e+0$^{(-)}$
		P7	3.3214e+0$^{(o)}$	**3.3420e+0**$^{(r)}$	3.3174e+0$^{(-)}$	**3.3444e+0**$^{(r)}$	**3.3214e+0**$^{(r)}$	3.3174e+0$^{(o)}$	3.3420e+0$^{(o)}$	**3.3444e+0**$^{(r)}$
		P8	3.1003e+0$^{(o)}$	**3.1126e+0**$^{(r)}$	3.1008e+0$^{(-)}$	**3.1131e+0**$^{(r)}$	3.1003e+0$^{(o)}$	**3.1008e+0**$^{(r)}$	3.1126e+0$^{(o)}$	**3.1131e+0**$^{(r)}$
		P9	**3.2218e+0**$^{(r)}$	3.2014e+0$^{(o)}$	**3.2199e+0**$^{(r)}$	3.2196e+0$^{(o)}$	**3.2218e+0**$^{(r)}$	3.2199e+0$^{(o)}$	3.2014e+0$^{(o)}$	**3.2196e+0**$^{(r)}$

D. MDG2 Transfer Strategies

In this section the tables to compare Multidimensional Differential Grouping 2 (MDG2) with the different Transfer Strategies against each other are depicted. Each table compares MDG2 with the four Transfer Strategies against each other. Each table contains the IGD and HV values for the four Transfer Strategies.

Table D.1.: Median IGD and HV values for Ref_1 with MDG2 and the four Transfer Strategies for the LSMOP test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	1.3749e-2$^{(o)}$	**1.3550e-2**$^{(r)}$	1.4594e-2$^{(o)}$	*1.5101e-2*$^{(o)}$
		P2	4.2302e-2$^{(-)}$	*4.2361e-2*$^{(-)}$	**2.7792e-2**$^{(r)}$	2.8625e-2$^{(o)}$
		P3	*9.5886e-1*$^{(-)}$	8.9326e-1$^{(-)}$	**5.9298e-1**$^{(r)}$	6.0404e-1$^{(o)}$
		P4	*1.4990e-1*$^{(-)}$	1.4602e-1$^{(-)}$	5.7010e-2$^{(o)}$	**5.4147e-2**$^{(r)}$
		P5	4.7878e-1$^{(o)}$	*6.5130e-1*$^{(-)}$	**4.3436e-1**$^{(r)}$	5.1016e-1$^{(o)}$
		P6	*7.3982e-1*$^{(-)}$	**5.9514e-1**$^{(r)}$	6.8864e-1$^{(o)}$	7.2588e-1$^{(o)}$
		P7	**1.2873e+0**$^{(r)}$	1.3063e+0$^{(o)}$	*1.5162e+0*$^{(o)}$	1.3345e+0$^{(o)}$
		P8	9.3913e-2$^{(o)}$	**9.2113e-2**$^{(r)}$	*9.4772e-2*$^{(o)}$	9.3089e-2$^{(o)}$
		P9	5.9746e-1$^{(-)}$	*6.3058e-1*$^{(-)}$	4.8033e-1$^{(o)}$	**4.7855e-1**$^{(r)}$
	HV	P1	**6.8238e-1**$^{(r)}$	6.8209e-1$^{(o)}$	6.8195e-1$^{(o)}$	*6.8021e-1*$^{(o)}$
		P2	6.3565e-1$^{(-)}$	*6.3544e-1*$^{(-)}$	**6.5762e-1**$^{(r)}$	6.5673e-1$^{(o)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	*4.8260e-1*$^{(-)}$	4.8871e-1$^{(-)}$	6.0909e-1$^{(o)}$	**6.1136e-1**$^{(r)}$
		P5	1.0999e-1$^{(o)}$	*1.0995e-1*$^{(-)}$	**1.1003e-1**$^{(r)}$	1.0998e-1$^{(-)}$
		P6	—$^{(-)}$	2.4308e-2$^{(-)}$	1.0155e-1$^{(o)}$	**1.0459e-1**$^{(r)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	*2.8500e-1*$^{(o)}$	2.8626e-1$^{(o)}$	**2.8763e-1**$^{(r)}$	2.8597e-1$^{(o)}$
		P9	5.4732e-1$^{(-)}$	*5.2678e-1*$^{(-)}$	6.8928e-1$^{(o)}$	**6.9437e-1**$^{(r)}$
n=200	IGD	P1	**2.1061e-2**$^{(r)}$	*2.2916e-2*$^{(-)}$	2.1635e-2$^{(o)}$	2.1415e-2$^{(o)}$
		P2	*4.4804e-2*$^{(-)}$	4.3915e-2$^{(-)}$	3.1111e-2$^{(o)}$	**3.0500e-2**$^{(r)}$
		P3	*1.0426e+0*$^{(-)}$	**9.1020e-1**$^{(r)}$	1.0107e+0$^{(-)}$	9.4873e-1$^{(o)}$
		P4	1.1519e-1$^{(-)}$	*1.1641e-1*$^{(-)}$	4.8278e-2$^{(o)}$	**4.7211e-2**$^{(r)}$
		P5	4.7051e-1$^{(o)}$	*4.8380e-1*$^{(o)}$	4.4314e-1$^{(o)}$	**4.4286e-1**$^{(r)}$
		P6	*1.0156e+0*$^{(-)}$	6.1255e-1$^{(-)}$	**4.6935e-1**$^{(r)}$	5.0772e-1$^{(o)}$
		P7	1.8124e+0$^{(-)}$	**1.4297e+0**$^{(r)}$	*1.9499e+0*$^{(-)}$	1.4340e+0$^{(o)}$
		P8	6.6225e-2$^{(o)}$	*6.7041e-2*$^{(o)}$	6.6891e-2$^{(o)}$	**6.6119e-2**$^{(r)}$
		P9	5.6868e-1$^{(-)}$	*5.7630e-1*$^{(-)}$	4.8185e-1$^{(o)}$	**4.8098e-1**$^{(r)}$
	HV	P1	**6.6684e-1**$^{(r)}$	*6.6465e-1*$^{(-)}$	6.6580e-1$^{(o)}$	6.6653e-1$^{(o)}$
		P2	*6.3375e-1*$^{(-)}$	6.3435e-1$^{(-)}$	6.5377e-1$^{(o)}$	**6.5478e-1**$^{(r)}$
		P3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P4	5.3563e-1$^{(-)}$	*5.3299e-1*$^{(-)}$	6.2532e-1$^{(o)}$	**6.2689e-1**$^{(r)}$
		P5	1.0897e-1$^{(o)}$	**1.0909e-1**$^{(r)}$	*1.0868e-1*$^{(o)}$	1.0901e-1$^{(o)}$
		P6	—$^{(-)}$	—$^{(-)}$	**3.2601e-3**$^{(r)}$	—$^{(o)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**3.1538e-1**$^{(r)}$	3.1448e-1$^{(o)}$	*3.1351e-1*$^{(o)}$	3.1470e-1$^{(o)}$
		P9	5.8259e-1$^{(-)}$	*5.6038e-1*$^{(-)}$	6.8503e-1$^{(o)}$	**6.8915e-1**$^{(r)}$

Table D.2.: Median IGD and HV values for Ref_1 with MDG2 and the four Transfer Strategies for the WFG test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	1.2721e+0$^{(*)}$	1.2675e+0$^{(*)}$	1.2755e+0$^{(*)}$	1.2939e+0$^{(*)}$
		P2	*1.3555e-2*$^{(o)}$	1.3527e-2$^{(o)}$	1.3273e-2$^{(o)}$	**1.3163e-2**$^{(r)}$
		P3	1.3041e-2$^{(o)}$	1.3092e-2$^{(o)}$	**1.3008e-2**$^{(r)}$	*1.3170e-2*$^{(o)}$
		P4	1.4055e-2$^{(o)}$	1.3956e-2$^{(o)}$	*1.4252e-2*$^{(-)}$	**1.3932e-2**$^{(r)}$
		P5	6.8977e-2$^{(o)}$	*6.9257e-2*$^{(o)}$	6.9116e-2$^{(-)}$	**6.8790e-2**$^{(r)}$
		P6	*1.9710e-2*$^{(-)}$	**1.9228e-2**$^{(r)}$	1.9333e-2$^{(o)}$	1.9534e-2$^{(o)}$
		P7	*8.6450e-2*$^{(o)}$	8.1438e-2$^{(o)}$	7.7506e-2$^{(o)}$	**7.4459e-2**$^{(r)}$
		P8	*6.2279e-2*$^{(-)}$	**5.5432e-2**$^{(r)}$	5.7599e-2$^{(-)}$	5.5533e-2$^{(o)}$
		P9	1.3954e-1$^{(o)}$	**1.3254e-1**$^{(r)}$	*1.4439e-1*$^{(o)}$	1.3619e-1$^{(o)}$
	HV	P1	1.2346e+0$^{(o)}$	1.2224e+0$^{(o)}$	**1.2427e+0**$^{(r)}$	1.1351e+0$^{(o)}$
		P2	**6.1306e+0**$^{(r)}$	6.1300e+0$^{(o)}$	6.1305e+0$^{(o)}$	6.1306e+0$^{(o)}$
		P3	5.6297e+0$^{(o)}$	5.6296e+0$^{(o)}$	**5.6299e+0**$^{(r)}$	5.6291e+0$^{(o)}$
		P4	3.3615e+0$^{(o)}$	3.3613e+0$^{(o)}$	**3.3616e+0**$^{(r)}$	3.3612e+0$^{(o)}$
		P5	2.9851e+0$^{(o)}$	2.9838e+0$^{(-)}$	2.9841e+0$^{(o)}$	**2.9861e+0**$^{(r)}$
		P6	3.2941e+0$^{(-)}$	**3.2980e+0**$^{(r)}$	3.2971e+0$^{(o)}$	3.2971e+0$^{(o)}$
		P7	2.9509e+0$^{(o)}$	3.0040e+0$^{(o)}$	3.0132e+0$^{(o)}$	**3.0268e+0**$^{(r)}$
		P8	3.0612e+0$^{(-)}$	**3.0985e+0**$^{(r)}$	3.0824e+0$^{(-)}$	3.0954e+0$^{(o)}$
		P9	2.6351e+0$^{(o)}$	**2.6634e+0**$^{(r)}$	2.6016e+0$^{(o)}$	2.6515e+0$^{(o)}$
n=200	IGD	P1	1.4132e+0$^{(*)}$	1.4108e+0$^{(*)}$	1.3999e+0$^{(*)}$	1.4033e+0$^{(*)}$
		P2	1.4584e-2$^{(o)}$	**1.4137e-2**$^{(r)}$	1.5517e-2$^{(-)}$	*1.5806e-2*$^{(o)}$
		P3	1.3360e-2$^{(o)}$	1.3404e-2$^{(o)}$	**1.3300e-2**$^{(r)}$	*1.3643e-2*$^{(o)}$
		P4	1.5181e-2$^{(o)}$	*1.6167e-2*$^{(-)}$	**1.4592e-2**$^{(r)}$	1.4681e-2$^{(o)}$
		P5	6.9294e-2$^{(o)}$	6.9300e-2$^{(o)}$	*6.9400e-2*$^{(o)}$	**6.9288e-2**$^{(r)}$
		P6	1.6041e-2$^{(o)}$	*1.6194e-2*$^{(o)}$	**1.5790e-2**$^{(r)}$	1.5872e-2$^{(o)}$
		P7	1.7435e-1$^{(o)}$	1.7658e-1$^{(o)}$	*1.8999e-1*$^{(o)}$	**1.7425e-1**$^{(r)}$
		P8	*5.7467e-2*$^{(-)}$	4.2103e-2$^{(o)}$	5.7425e-2$^{(-)}$	**4.1838e-2**$^{(r)}$
		P9	2.6891e-1$^{(o)}$	**2.5994e-1**$^{(r)}$	*2.9208e-1*$^{(o)}$	2.6427e-1$^{(o)}$
	HV	P1	*6.0500e-1*$^{(o)}$	6.2606e-1$^{(o)}$	**6.5210e-1**$^{(r)}$	6.4523e-1$^{(o)}$
		P2	6.1291e+0$^{(o)}$	**6.1292e+0**$^{(r)}$	6.1275e+0$^{(o)}$	6.1277e+0$^{(o)}$
		P3	5.6272e+0$^{(o)}$	5.6275e+0$^{(o)}$	**5.6283e+0**$^{(r)}$	5.6264e+0$^{(-)}$
		P4	3.3604e+0$^{(o)}$	**3.3613e+0**$^{(r)}$	3.3605e+0$^{(o)}$	3.3604e+0$^{(o)}$
		P5	**2.9833e+0**$^{(r)}$	2.9826e+0$^{(o)}$	2.9827e+0$^{(o)}$	2.9826e+0$^{(o)}$
		P6	3.3268e+0$^{(o)}$	3.3252e+0$^{(o)}$	3.3252e+0$^{(o)}$	**3.3270e+0**$^{(r)}$
		P7	2.4635e+0$^{(o)}$	2.4575e+0$^{(o)}$	2.4186e+0$^{(o)}$	**2.4857e+0**$^{(r)}$
		P8	3.0750e+0$^{(-)}$	**3.1760e+0**$^{(r)}$	3.0814e+0$^{(-)}$	3.1725e+0$^{(o)}$
		P9	1.9976e+0$^{(o)}$	**2.0367e+0**$^{(r)}$	1.8903e+0$^{(-)}$	2.0011e+0$^{(o)}$

Table D.3.: Median IGD and HV values for Ref_2 with MDG2 and the four Transfer Strategies for the LSMOP test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	*4.9400e-2* $^{(o)}$	**4.2126e-2** $^{(r)}$	4.2778e-2 $^{(o)}$	4.3647e-2 $^{(o)}$
		P2	*2.5206e-2* $^{(-)}$	2.5082e-2 $^{(-)}$	2.2858e-2 $^{(o)}$	**2.2836e-2** $^{(r)}$
		P3	*6.5317e-1* $^{(-)}$	5.0935e-1 $^{(o)}$	4.9420e-1 $^{(-)}$	**4.5374e-1** $^{(r)}$
		P4	*1.3994e-1* $^{(-)}$	1.3877e-1 $^{(-)}$	**2.5655e-2** $^{(r)}$	2.5683e-2 $^{(o)}$
		P5	**3.4246e-1** $^{(r)}$	3.8915e-1 $^{(-)}$	3.6967e-1 $^{(o)}$	*4.1671e-1* $^{(o)}$
		P6	*6.6970e-1* $^{(-)}$	5.3033e-1 $^{(o)}$	4.3171e-1 $^{(o)}$	**4.0620e-1** $^{(r)}$
		P7	1.2893e+0 $^{(*)}$	1.5628e+0 $^{(*)}$	1.2999e+0 $^{(*)}$	2.0325e+0 $^{(*)}$
		P8	1.9835e-1 $^{(o)}$	**1.7292e-1** $^{(r)}$	*2.0641e-1* $^{(o)}$	1.9979e-1 $^{(o)}$
		P9	3.3452e-1 $^{(-)}$	*3.5736e-1* $^{(-)}$	1.0933e-1 $^{(o)}$	**8.1321e-2** $^{(r)}$
	HV	P1	*6.3865e-1* $^{(o)}$	**6.4800e-1** $^{(r)}$	6.4748e-1 $^{(o)}$	6.4533e-1 $^{(o)}$
		P2	6.6903e-1 $^{(o)}$	**6.6907e-1** $^{(r)}$	*6.6755e-1* $^{(o)}$	6.6792e-1 $^{(o)}$
		P3	1.0998e-1 $^{(o)}$	**1.3265e-1** $^{(r)}$	*6.5100e-2* $^{(-)}$	1.2505e-1 $^{(o)}$
		P4	*4.8785e-1* $^{(-)}$	4.8872e-1 $^{(-)}$	**6.6220e-1** $^{(r)}$	6.6206e-1 $^{(o)}$
		P5	**1.4439e-1** $^{(r)}$	*1.0999e-1* $^{(o)}$	1.0999e-1 $^{(o)}$	1.1045e-1 $^{(o)}$
		P6	**1.0776e-1** $^{(r)}$	9.9507e-2 $^{(-)}$	*4.4427e-2* $^{(-)}$	6.1107e-2 $^{(-)}$
		P7	— $^{(*)}$	— $^{(*)}$	— $^{(*)}$	— $^{(*)}$
		P8	*1.9189e-1* $^{(o)}$	**2.1949e-1** $^{(r)}$	1.9985e-1 $^{(o)}$	2.0983e-1 $^{(o)}$
		P9	*5.0462e-1* $^{(-)}$	5.1033e-1 $^{(-)}$	8.7179e-1 $^{(o)}$	**8.9882e-1** $^{(r)}$
n=200	IGD	P1	4.1708e-2 $^{(o)}$	4.0746e-2 $^{(o)}$	*4.9521e-2* $^{(o)}$	**3.9865e-2** $^{(r)}$
		P2	**2.0698e-2** $^{(r)}$	2.1550e-2 $^{(-)}$	5.0271e-2 $^{(-)}$	*5.0365e-2* $^{(-)}$
		P3	7.2281e-1 $^{(-)}$	**7.0383e-1** $^{(r)}$	7.8059e-1 $^{(-)}$	*7.8600e-1* $^{(-)}$
		P4	1.1392e-1 $^{(-)}$	*1.2347e-1* $^{(-)}$	**1.9582e-2** $^{(r)}$	1.9792e-2 $^{(o)}$
		P5	7.4214e-1 $^{(o)}$	7.4214e-1 $^{(o)}$	*7.4215e-1* $^{(o)}$	**7.4212e-1** $^{(r)}$
		P6	**4.0873e-1** $^{(r)}$	4.1677e-1 $^{(o)}$	4.4172e-1 $^{(o)}$	*4.7271e-1* $^{(o)}$
		P7	1.3835e+0 $^{(*)}$	2.1501e+0 $^{(*)}$	1.3587e+0 $^{(*)}$	1.8201e+0 $^{(*)}$
		P8	2.0250e-1 $^{(o)}$	*2.0920e-1* $^{(o)}$	**1.9654e-1** $^{(r)}$	1.9737e-1 $^{(o)}$
		P9	3.9139e-1 $^{(-)}$	*3.9608e-1* $^{(-)}$	4.7749e-2 $^{(o)}$	**4.0729e-2** $^{(r)}$
	HV	P1	6.4795e-1 $^{(o)}$	6.4736e-1 $^{(o)}$	*6.3661e-1* $^{(o)}$	**6.4972e-1** $^{(r)}$
		P2	**6.7571e-1** $^{(r)}$	6.7447e-1 $^{(-)}$	*6.2626e-1* $^{(-)}$	6.2651e-1 $^{(-)}$
		P3	**8.5679e-2** $^{(r)}$	7.7373e-2 $^{(o)}$	— $^{(-)}$	— $^{(-)}$
		P4	5.3159e-1 $^{(-)}$	*5.1939e-1* $^{(-)}$	**6.7333e-1** $^{(r)}$	6.7272e-1 $^{(o)}$
		P5	*1.0980e-1* $^{(o)}$	1.0983e-1 $^{(o)}$	1.0983e-1 $^{(o)}$	**1.0989e-1** $^{(r)}$
		P6	**8.0354e-2** $^{(r)}$	6.1649e-2 $^{(-)}$	2.1618e-2 $^{(-)}$	*8.8879e-3* $^{(-)}$
		P7	— $^{(*)}$	— $^{(*)}$	— $^{(*)}$	— $^{(*)}$
		P8	1.8843e-1 $^{(o)}$	1.9095e-1 $^{(o)}$	**1.9988e-1** $^{(r)}$	*1.8020e-1* $^{(o)}$
		P9	3.8412e-1 $^{(-)}$	*3.7923e-1* $^{(-)}$	9.3436e-1 $^{(o)}$	**9.4300e-1** $^{(r)}$

Table D.4.: Median IGD and HV values for Ref_2 with MDG2 and the four Transfer Strategies for the WFG test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	**8.4828e-1** $^{(r)}$	8.5021e-1 $^{(o)}$	8.4947e-1 $^{(o)}$	*8.5100e-1* $^{(o)}$
		P2	1.5417e-2 $^{(o)}$	1.5432e-2 $^{(o)}$	*1.5574e-2* $^{(o)}$	**1.4874e-2** $^{(r)}$
		P3	*1.4968e-2* $^{(-)}$	**1.3652e-2** $^{(r)}$	1.3916e-2 $^{(o)}$	1.3762e-2 $^{(o)}$
		P4	**1.3745e-2** $^{(r)}$	*1.3926e-2* $^{(-)}$	1.3850e-2 $^{(o)}$	1.3801e-2 $^{(o)}$
		P5	*6.4712e-2* $^{(-)}$	**6.3867e-2** $^{(r)}$	6.4624e-2 $^{(-)}$	6.3988e-2 $^{(o)}$
		P6	*1.6132e-2* $^{(o)}$	1.6130e-2 $^{(o)}$	**1.6017e-2** $^{(r)}$	1.6104e-2 $^{(o)}$
		P7	1.4261e-2 $^{(o)}$	**1.4253e-2** $^{(r)}$	1.4801e-2 $^{(-)}$	*1.4884e-2* $^{(-)}$
		P8	*1.1370e-1* $^{(-)}$	1.0370e-1 $^{(o)}$	1.1213e-1 $^{(-)}$	**1.0097e-1** $^{(r)}$
		P9	*2.5763e-2* $^{(o)}$	2.4425e-2 $^{(o)}$	2.4310e-2 $^{(o)}$	**2.3788e-2** $^{(r)}$
	HV	P1	2.7027e+0 $^{(o)}$	2.6923e+0 $^{(o)}$	**2.7039e+0** $^{(r)}$	2.6978e+0 $^{(o)}$
		P2	6.1187e+0 $^{(o)}$	**6.1190e+0** $^{(r)}$	6.1008e+0 $^{(o)}$	6.1133e+0 $^{(o)}$
		P3	5.6063e+0 $^{(-)}$	**5.6221e+0** $^{(r)}$	5.6143e+0 $^{(-)}$	5.6220e+0 $^{(o)}$
		P4	3.3597e+0 $^{(o)}$	3.3597e+0 $^{(o)}$	**3.3598e+0** $^{(r)}$	3.3597e+0 $^{(o)}$
		P5	3.0154e+0 $^{(-)}$	**3.0313e+0** $^{(r)}$	3.0187e+0 $^{(-)}$	3.0275e+0 $^{(o)}$
		P6	3.3268e+0 $^{(o)}$	3.3263e+0 $^{(o)}$	3.3258e+0 $^{(o)}$	**3.3285e+0** $^{(r)}$
		P7	**3.3552e+0** $^{(r)}$	3.3550e+0 $^{(o)}$	3.3452e+0 $^{(-)}$	3.3471e+0 $^{(-)}$
		P8	2.7791e+0 $^{(-)}$	2.8354e+0 $^{(o)}$	2.7866e+0 $^{(-)}$	**2.8399e+0** $^{(r)}$
		P9	3.2451e+0 $^{(o)}$	**3.2596e+0** $^{(r)}$	3.2586e+0 $^{(o)}$	3.2562e+0 $^{(o)}$
n=200	IGD	P1	1.1425e+0 $^{(*)}$	1.1575e+0 $^{(*)}$	1.1671e+0 $^{(*)}$	1.1567e+0 $^{(*)}$
		P2	1.4582e-2 $^{(o)}$	1.4743e-2 $^{(o)}$	*1.5088e-2* $^{(o)}$	**1.4394e-2** $^{(r)}$
		P3	1.3518e-2 $^{(o)}$	1.3446e-2 $^{(o)}$	**1.3374e-2** $^{(r)}$	*1.3532e-2* $^{(o)}$
		P4	1.3984e-2 $^{(o)}$	1.4061e-2 $^{(o)}$	**1.3927e-2** $^{(r)}$	*1.4109e-2* $^{(o)}$
		P5	**6.6503e-2** $^{(r)}$	6.6671e-2 $^{(o)}$	6.7780e-2 $^{(o)}$	*6.9691e-2* $^{(o)}$
		P6	1.4851e-2 $^{(o)}$	1.4900e-2 $^{(o)}$	*1.4998e-2* $^{(-)}$	**1.4828e-2** $^{(r)}$
		P7	*2.9674e-2* $^{(o)}$	2.8874e-2 $^{(o)}$	**2.7075e-2** $^{(r)}$	2.8966e-2 $^{(o)}$
		P8	1.7617e-1 $^{(-)}$	1.5651e-1 $^{(o)}$	*1.7815e-1* $^{(-)}$	**1.5515e-1** $^{(r)}$
		P9	*2.8400e-2* $^{(-)}$	2.6177e-2 $^{(o)}$	2.5588e-2 $^{(o)}$	**2.3841e-2** $^{(r)}$
	HV	P1	**1.6298e+0** $^{(r)}$	1.5749e+0 $^{(o)}$	1.5350e+0 $^{(o)}$	1.5648e+0 $^{(o)}$
		P2	6.1183e+0 $^{(o)}$	6.1182e+0 $^{(o)}$	**6.1207e+0** $^{(r)}$	6.1128e+0 $^{(o)}$
		P3	5.6245e+0 $^{(o)}$	5.6210e+0 $^{(o)}$	**5.6271e+0** $^{(r)}$	5.6218e+0 $^{(o)}$
		P4	**3.3547e+0** $^{(r)}$	3.3546e+0 $^{(o)}$	3.3543e+0 $^{(o)}$	3.3545e+0 $^{(o)}$
		P5	**3.0029e+0** $^{(r)}$	2.9988e+0 $^{(o)}$	2.9904e+0 $^{(o)}$	2.9819e+0 $^{(o)}$
		P6	**3.3419e+0** $^{(r)}$	3.3412e+0 $^{(o)}$	3.3415e+0 $^{(o)}$	3.3413e+0 $^{(o)}$
		P7	3.2363e+0 $^{(o)}$	3.2412e+0 $^{(o)}$	**3.2517e+0** $^{(r)}$	3.2431e+0 $^{(o)}$
		P8	2.4512e+0 $^{(-)}$	2.5524e+0 $^{(o)}$	2.4400e+0 $^{(-)}$	**2.5638e+0** $^{(r)}$
		P9	3.2562e+0 $^{(o)}$	**3.2640e+0** $^{(r)}$	3.2586e+0 $^{(o)}$	3.2527e+0 $^{(o)}$

Table D.5.: Median IGD and HV values for Ref_1 and Ref_2 with MDG2 and the four Transfer Strategies for the LSMOP test suite and 500 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
Ref_1	IGD	P1	**1.1531e-2**$^{(r)}$	*1.3105e-2*$^{(o)}$	1.2651e-2$^{(o)}$	1.2684e-2$^{(o)}$
		P2	2.0292e-2$^{(-)}$	*2.0549e-2*$^{(-)}$	1.4511e-2$^{(o)}$	**1.4505e-2**$^{(r)}$
		P3	*7.5657e-1*$^{(-)}$	7.3110e-1$^{(-)}$	**3.8550e-1**$^{(r)}$	7.2370e-1$^{(-)}$
		P4	*5.6787e-2*$^{(-)}$	5.6332e-2$^{(-)}$	**2.0417e-2**$^{(r)}$	2.0623e-2$^{(o)}$
		P5	*6.4406e-1*$^{(o)}$	5.4559e-1$^{(o)}$	**4.5672e-1**$^{(r)}$	4.7360e-1$^{(o)}$
		P6	*4.9515e-1*$^{(-)}$	4.1686e-1$^{(o)}$	**4.0106e-1**$^{(r)}$	4.0887e-1$^{(o)}$
		P7	*1.6219e+0*$^{(-)}$	1.4748e+0$^{(o)}$	1.5072e+0$^{(-)}$	**1.4704e+0**$^{(r)}$
		P8	**3.4902e-2**$^{(r)}$	*3.8289e-2*$^{(-)}$	3.5635e-2$^{(o)}$	3.6743e-2$^{(-)}$
		P9	5.0278e-1$^{(-)}$	*5.0335e-1*$^{(-)}$	4.5686e-1$^{(o)}$	**4.5647e-1**$^{(r)}$
	HV	P1	**6.8504e-1**$^{(r)}$	*6.8333e-1*$^{(o)}$	6.8377e-1$^{(o)}$	6.8413e-1$^{(o)}$
		P2	6.7431e-1$^{(-)}$	*6.7393e-1*$^{(-)}$	6.8406e-1$^{(o)}$	**6.8420e-1**$^{(r)}$
		P3	*1.9245e-2*$^{(-)}$	6.3999e-2$^{(o)}$	**7.8136e-2**$^{(r)}$	6.0408e-2$^{(o)}$
		P4	*6.1697e-1*$^{(-)}$	6.1776e-1$^{(-)}$	**6.7140e-1**$^{(r)}$	6.7108e-1$^{(o)}$
		P5	*1.0979e-1*$^{(o)}$	1.0980e-1$^{(o)}$	**1.0994e-1**$^{(r)}$	1.0988e-1$^{(o)}$
		P6	—$^{(-)}$	1.7152e-2$^{(-)}$	**1.0814e-1**$^{(r)}$	1.0188e-1$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	**3.6704e-1**$^{(r)}$	*3.6156e-1*$^{(-)}$	3.6638e-1$^{(o)}$	3.6335e-1$^{(-)}$
		P9	6.5834e-1$^{(-)}$	*6.5596e-1*$^{(-)}$	7.1868e-1$^{(o)}$	**7.1994e-1**$^{(r)}$
Ref_2	IGD	P1	3.3420e-2$^{(o)}$	3.1687e-2$^{(o)}$	*3.7860e-2*$^{(o)}$	**2.8595e-2**$^{(r)}$
		P2	*2.7682e-2*$^{(-)}$	2.7397e-2$^{(-)}$	**2.6139e-2**$^{(r)}$	2.6369e-2$^{(o)}$
		P3	*7.0807e-1*$^{(-)}$	5.0581e-1$^{(-)}$	**4.6396e-1**$^{(r)}$	4.7960e-1$^{(o)}$
		P4	6.2552e-2$^{(-)}$	*6.3717e-2*$^{(-)}$	**6.1712e-3**$^{(r)}$	6.4966e-3$^{(o)}$
		P5	7.4212e-1$^{(-)}$	**7.4211e-1**$^{(r)}$	*7.4212e-1*$^{(o)}$	7.4212e-1$^{(-)}$
		P6	3.9854e-1$^{(o)}$	*4.3118e-1*$^{(-)}$	**3.8878e-1**$^{(r)}$	4.0529e-1$^{(o)}$
		P7	1.4452e+0$^{(o)}$	1.6987e+0$^{(-)}$	**1.3643e+0**$^{(r)}$	*1.7426e+0*$^{(-)}$
		P8	*2.0016e-1*$^{(o)}$	1.8425e-1$^{(o)}$	**1.7696e-1**$^{(r)}$	1.8504e-1$^{(o)}$
		P9	*1.7462e-1*$^{(-)}$	1.7179e-1$^{(-)}$	**2.5060e-2**$^{(r)}$	3.3474e-2$^{(-)}$
	HV	P1	6.6175e-1$^{(o)}$	6.6393e-1$^{(o)}$	*6.5529e-1*$^{(o)}$	**6.6711e-1**$^{(r)}$
		P2	*6.6225e-1*$^{(-)}$	6.6269e-1$^{(-)}$	**6.6490e-1**$^{(r)}$	6.6438e-1$^{(o)}$
		P3	*1.0851e-1*$^{(-)}$	**1.6409e-1**$^{(r)}$	1.1706e-1$^{(-)}$	1.0933e-1$^{(-)}$
		P4	6.0987e-1$^{(-)}$	*6.0831e-1*$^{(-)}$	**6.9937e-1**$^{(r)}$	6.9878e-1$^{(o)}$
		P5	1.0993e-1$^{(o)}$	**1.0993e-1**$^{(r)}$	1.0992e-1$^{(o)}$	*1.0992e-1*$^{(o)}$
		P6	**8.1130e-2**$^{(r)}$	8.0274e-2$^{(o)}$	*6.6705e-2*$^{(-)}$	6.8552e-2$^{(-)}$
		P7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		P8	2.2788e-1$^{(o)}$	2.3132e-1$^{(o)}$	**2.4040e-1**$^{(r)}$	*2.2723e-1*$^{(o)}$
		P9	6.8592e-1$^{(-)}$	*6.8558e-1*$^{(-)}$	**9.8438e-1**$^{(r)}$	9.7266e-1$^{(-)}$

Table D.6.: Median IGD and HV values for Ref_1 and Ref_2 with MDG2 and the four Transfer Strategies for the WFG test suite and 500 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
Ref_1	IGD	P1	1.2568e+0$^{(o)}$	*1.2832e+0*$^{(-)}$	1.2790e+0$^{(-)}$	**1.2265e+0**$^{(r)}$
		P2	1.3092e-2$^{(o)}$	*1.3753e-2*$^{(-)}$	**1.3024e-2**$^{(r)}$	1.3234e-2$^{(o)}$
		P3	1.2923e-2$^{(o)}$	*1.3065e-2*$^{(o)}$	1.3049e-2$^{(o)}$	**1.2857e-2**$^{(r)}$
		P4	1.3747e-2$^{(o)}$	1.3755e-2$^{(o)}$	**1.3740e-2**$^{(r)}$	*1.3893e-2*$^{(o)}$
		P5	6.8932e-2$^{(o)}$	6.8918e-2$^{(o)}$	*6.9105e-2*$^{(o)}$	**6.8696e-2**$^{(r)}$
		P6	*1.4054e-2*$^{(o)}$	**1.3943e-2**$^{(r)}$	1.4014e-2$^{(o)}$	1.4008e-2$^{(o)}$
		P7	**1.4630e-1**$^{(r)}$	1.5038e-1$^{(o)}$	*1.6931e-1*$^{(o)}$	1.6852e-1$^{(o)}$
		P8	*3.1999e-2*$^{(-)}$	**2.8175e-2**$^{(r)}$	3.1873e-2$^{(-)}$	2.9418e-2$^{(o)}$
		P9	**2.4502e-1**$^{(r)}$	2.5224e-1$^{(o)}$	2.5035e-1$^{(o)}$	*2.5278e-1*$^{(o)}$
	HV	P1	1.3098e+0$^{(o)}$	1.1857e+0$^{(-)}$	1.1894e+0$^{(-)}$	**1.4029e+0**$^{(r)}$
		P2	6.1315e+0$^{(o)}$	6.1305e+0$^{(-)}$	**6.1315e+0**$^{(r)}$	6.1311e+0$^{(o)}$
		P3	5.6309e+0$^{(o)}$	5.6303e+0$^{(o)}$	5.6302e+0$^{(o)}$	**5.6309e+0**$^{(r)}$
		P4	3.3607e+0$^{(o)}$	3.3613e+0$^{(o)}$	3.3613e+0$^{(o)}$	**3.3617e+0**$^{(r)}$
		P5	2.9847e+0$^{(o)}$	2.9854e+0$^{(o)}$	2.9849e+0$^{(o)}$	**2.9858e+0**$^{(r)}$
		P6	3.3485e+0$^{(o)}$	**3.3489e+0**$^{(r)}$	3.3482e+0$^{(o)}$	3.3482e+0$^{(o)}$
		P7	**2.6160e+0**$^{(r)}$	2.5812e+0$^{(o)}$	2.4917e+0$^{(o)}$	2.4887e+0$^{(o)}$
		P8	3.2220e+0$^{(-)}$	3.2480e+0$^{(o)}$	3.2293e+0$^{(-)}$	**3.2481e+0**$^{(r)}$
		P9	**2.1008e+0**$^{(r)}$	2.0464e+0$^{(o)}$	2.0722e+0$^{(o)}$	2.0631e+0$^{(o)}$
Ref_2	IGD	P1	**8.5918e-1**$^{(r)}$	8.6530e-1$^{(o)}$	*8.8230e-1*$^{(o)}$	8.7295e-1$^{(o)}$
		P2	**1.4088e-2**$^{(r)}$	1.4574e-2$^{(o)}$	1.4433e-2$^{(o)}$	*1.4640e-2*$^{(o)}$
		P3	*1.3566e-2*$^{(o)}$	**1.3219e-2**$^{(r)}$	1.3266e-2$^{(o)}$	1.3349e-2$^{(o)}$
		P4	*1.3894e-2*$^{(o)}$	**1.3784e-2**$^{(r)}$	1.3862e-2$^{(o)}$	1.3793e-2$^{(o)}$
		P5	*6.3978e-2*$^{(o)}$	6.3921e-2$^{(o)}$	6.3769e-2$^{(o)}$	**6.3706e-2**$^{(r)}$
		P6	**1.4144e-2**$^{(r)}$	1.4184e-2$^{(o)}$	1.4181e-2$^{(o)}$	*1.4215e-2*$^{(o)}$
		P7	*3.0626e-2*$^{(o)}$	2.9983e-2$^{(o)}$	**2.9010e-2**$^{(r)}$	2.9821e-2$^{(o)}$
		P8	6.1838e-2$^{(-)}$	5.8367e-2$^{(o)}$	*6.2505e-2*$^{(-)}$	**5.6590e-2**$^{(r)}$
		P9	**1.6196e-2**$^{(r)}$	*1.7316e-2*$^{(o)}$	1.6602e-2$^{(o)}$	1.7215e-2$^{(o)}$
	HV	P1	**2.6630e+0**$^{(r)}$	2.6340e+0$^{(o)}$	2.5714e+0$^{(o)}$	2.5994e+0$^{(o)}$
		P2	6.1254e+0$^{(o)}$	**6.1292e+0**$^{(r)}$	6.1283e+0$^{(o)}$	6.1266e+0$^{(o)}$
		P3	5.6270e+0$^{(o)}$	5.6278e+0$^{(o)}$	**5.6285e+0**$^{(r)}$	5.6265e+0$^{(o)}$
		P4	**3.3598e+0**$^{(r)}$	3.3589e+0$^{(o)}$	3.3594e+0$^{(o)}$	3.3587e+0$^{(-)}$
		P5	**3.0311e+0**$^{(r)}$	3.0307e+0$^{(o)}$	3.0309e+0$^{(o)}$	3.0296e+0$^{(o)}$
		P6	3.3480e+0$^{(o)}$	3.3474e+0$^{(-)}$	3.3480e+0$^{(o)}$	**3.3485e+0**$^{(r)}$
		P7	3.2265e+0$^{(o)}$	3.2325e+0$^{(o)}$	**3.2412e+0**$^{(r)}$	3.2373e+0$^{(o)}$
		P8	3.0483e+0$^{(-)}$	3.0685e+0$^{(o)}$	3.0467e+0$^{(-)}$	**3.0791e+0**$^{(r)}$
		P9	3.3367e+0$^{(o)}$	3.3290e+0$^{(o)}$	**3.3397e+0**$^{(r)}$	3.3331e+0$^{(o)}$

Table D.7.: IQR values of IGD and HV for Ref_1 with MDG2 and the four Transfer Strategies for the LSMOP test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	1.8613e-3	2.4253e-3	3.9458e-3	2.9737e-3
		P2	4.3432e-3	3.0429e-3	1.9557e-3	2.6788e-3
		P3	1.4010e-1	4.4587e-1	2.6642e-1	3.5120e-1
		P4	1.1085e-2	1.7123e-2	8.2148e-3	9.4498e-3
		P5	6.5918e-1	5.7146e-1	6.8840e-1	5.5120e-1
		P6	6.7556e-1	3.7882e-1	2.2332e-1	1.0120e-1
		P7	7.8980e-1	1.3994e-1	8.1471e-1	6.6176e-2
		P8	7.2256e-3	7.1406e-3	6.9977e-3	7.7912e-3
		P9	1.5762e-1	1.0197e-1	4.3721e-2	3.4156e-2
	HV	P1	2.5573e-3	5.7678e-3	8.7444e-3	7.0675e-3
		P2	5.5160e-3	4.7434e-3	2.9796e-3	5.0624e-3
		P3	—	3.2781e-2	4.5748e-2	9.4785e-2
		P4	1.0154e-2	1.7578e-2	1.0849e-2	1.3136e-2
		P5	2.4242e-1	1.6439e-1	2.6651e-1	1.5835e-1
		P6	—	1.0466e-1	2.2497e-2	6.2108e-3
		P7	—	—	—	—
		P8	8.2768e-3	1.0031e-2	6.8291e-3	5.9609e-3
		P9	1.5739e-1	9.9542e-2	4.6644e-2	3.8073e-2
n=200	IGD	P1	2.5600e-3	3.8882e-3	3.5342e-3	3.9961e-3
		P2	4.5289e-3	3.9684e-3	2.0798e-3	2.1503e-3
		P3	7.7251e-1	1.8457e-1	2.0055e-1	2.0722e-1
		P4	6.5701e-3	4.5970e-3	3.8023e-3	5.5915e-3
		P5	2.6497e-1	2.7565e-1	1.6311e-1	2.9381e-1
		P6	6.0724e-1	4.0605e-1	1.9193e-1	4.0968e-1
		P7	6.9418e-1	4.1829e-2	6.3931e-1	7.2794e-2
		P8	8.1867e-3	5.5514e-3	6.9728e-3	9.2413e-3
		P9	4.8871e-2	9.7377e-2	3.4103e-2	2.5956e-2
	HV	P1	4.8027e-3	6.9843e-3	4.9484e-3	9.3427e-3
		P2	7.1724e-3	5.2882e-3	3.2458e-3	3.9288e-3
		P3	—	—	—	—
		P4	9.7512e-3	7.0657e-3	5.5019e-3	8.3446e-3
		P5	1.5926e-3	9.9169e-4	1.4347e-3	1.0818e-1
		P6	—	—	6.1330e-2	3.1782e-2
		P7	—	—	—	—
		P8	6.7344e-3	6.8945e-3	8.6944e-3	8.9399e-3
		P9	6.2620e-2	9.8578e-2	3.7667e-2	2.6677e-2

Table D.8.: IQR values of IGD and HV for Ref_1 with MDG2 and the four Transfer Strategies for the WFG test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	8.8250e-2	1.0165e-1	8.4489e-2	7.4186e-2
		P2	1.3458e-3	1.9117e-3	1.0359e-3	8.3219e-4
		P3	5.5295e-4	3.2943e-4	3.0162e-4	3.9280e-4
		P4	7.5777e-4	7.9474e-4	1.1924e-3	1.0510e-3
		P5	7.0009e-4	1.1999e-3	7.2489e-4	1.3992e-3
		P6	8.7077e-4	9.5905e-4	9.2686e-4	8.8502e-4
		P7	2.0016e-2	2.1667e-2	2.2801e-2	2.8894e-2
		P8	8.2576e-3	5.8549e-3	1.0481e-2	7.7915e-3
		P9	4.3259e-2	3.7597e-2	4.2718e-2	5.7988e-2
	HV	P1	3.9655e-1	3.8303e-1	3.6004e-1	3.4033e-1
		P2	1.7402e-3	2.1507e-3	1.4420e-3	1.5907e-3
		P3	2.6750e-3	1.4926e-3	1.0726e-3	1.8807e-3
		P4	1.8394e-3	1.8643e-3	2.0315e-3	1.8102e-3
		P5	5.0724e-3	6.7068e-3	4.9154e-3	7.0010e-3
		P6	6.2229e-3	7.5333e-3	6.0786e-3	6.1926e-3
		P7	1.0373e-1	1.2713e-1	1.2515e-1	1.8992e-1
		P8	5.3875e-2	3.4205e-2	5.1794e-2	5.2636e-2
		P9	2.1286e-1	2.0903e-1	2.2259e-1	3.0265e-1
n=200	IGD	P1	9.5721e-2	7.8695e-2	7.1195e-2	1.5983e-1
		P2	2.6700e-3	2.3664e-3	3.9659e-3	9.9178e-3
		P3	5.6617e-4	6.1676e-4	5.9175e-4	8.1797e-4
		P4	1.8992e-3	3.8132e-3	2.0154e-3	3.3569e-3
		P5	6.5755e-4	6.1590e-4	7.0646e-4	3.6951e-4
		P6	5.5133e-4	8.9858e-4	6.7557e-4	6.5443e-4
		P7	8.1382e-2	9.3314e-2	5.4763e-2	8.2631e-2
		P8	7.3597e-3	4.1999e-3	8.9005e-3	5.8692e-3
		P9	4.9990e-2	1.2068e-1	4.8643e-2	8.9410e-2
	HV	P1	3.9968e-1	3.3592e-1	3.1030e-1	6.6297e-1
		P2	1.9227e-3	3.8951e-3	3.0780e-3	5.9553e-3
		P3	4.8875e-3	3.3569e-3	2.3135e-3	3.7129e-3
		P4	2.3352e-3	3.0762e-3	2.5680e-3	3.8913e-3
		P5	4.3675e-3	4.6717e-3	3.8726e-3	3.3184e-3
		P6	4.0832e-3	4.3136e-3	5.6832e-3	5.2600e-3
		P7	4.1282e-1	4.3987e-1	3.1657e-1	4.5374e-1
		P8	4.5584e-2	2.3056e-2	5.1805e-2	3.2545e-2
		P9	2.3736e-1	5.9867e-1	2.1817e-1	4.1494e-1

Table D.9.: IQR values of IGD and HV for Ref_2 with MDG2 and the four Transfer Strategies for the LSMOP test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	2.4716e-2	2.7195e-2	2.3402e-2	2.4576e-2
		P2	2.0455e-3	1.9692e-3	4.2971e-3	4.5869e-3
		P3	1.3279e-1	2.5995e-1	1.4490e-1	1.2407e-1
		P4	1.1520e-2	8.4590e-3	3.5652e-3	3.8410e-3
		P5	3.2015e-1	1.8392e-1	1.1811e-1	1.5952e-1
		P6	1.4393e-1	2.1398e-1	2.4644e-1	1.5619e-1
		P7	1.0565e-1	7.8908e-1	7.4415e-1	1.0114e+0
		P8	1.1770e-1	1.4081e-1	6.7335e-2	9.3607e-2
		P9	1.0148e-1	1.0746e-1	1.2517e-1	1.0252e-1
	HV	P1	3.4125e-2	3.7521e-2	3.3149e-2	3.4260e-2
		P2	3.1269e-3	2.7620e-3	8.2898e-3	8.4030e-3
		P3	8.2115e-4	9.3720e-2	1.1137e-1	9.9279e-2
		P4	1.1872e-2	9.4848e-3	6.6323e-3	5.6650e-3
		P5	1.6713e-1	8.2215e-2	9.9789e-2	9.9675e-2
		P6	7.3189e-4	1.2337e-2	4.7879e-2	3.3281e-2
		P7	—	—	—	—
		P8	8.5551e-2	1.0486e-1	8.5741e-2	6.6029e-2
		P9	1.8123e-1	1.4013e-1	1.5055e-1	7.2189e-2
n=200	IGD	P1	1.9306e-2	1.8576e-2	2.6087e-2	2.8266e-2
		P2	3.2731e-3	4.1592e-3	2.9313e-3	2.9255e-3
		P3	1.3441e-2	1.6172e-1	1.1171e-1	1.1835e-1
		P4	3.7333e-3	3.8119e-3	2.5578e-3	2.9662e-3
		P5	2.6317e-1	2.1947e-1	1.8829e-2	5.2152e-1
		P6	5.0323e-2	3.4828e-2	4.6421e-2	8.8476e-2
		P7	3.6262e-1	8.3599e-1	3.5372e-1	4.3513e-1
		P8	1.0359e-1	9.3096e-2	5.3288e-2	1.2685e-1
		P9	3.6775e-2	2.7768e-2	2.3512e-2	3.2908e-2
	HV	P1	2.7165e-2	2.4975e-2	3.7542e-2	3.7493e-2
		P2	4.7797e-3	6.2796e-3	4.0022e-3	4.3445e-3
		P3	1.9578e-2	9.8720e-2	2.2833e-5	1.1084e-3
		P4	6.1002e-3	4.9176e-3	4.5139e-3	5.1194e-3
		P5	9.9752e-4	3.6754e-4	1.6282e-4	1.3768e-1
		P6	2.2227e-2	2.5260e-2	3.0822e-2	4.2532e-2
		P7	—	—	—	—
		P8	9.0404e-2	8.4654e-2	4.5186e-2	9.9962e-2
		P9	3.1743e-2	2.7732e-2	4.0475e-2	4.5661e-2

Table D.10.: IQR values of IGD and HV for Ref_2 with MDG2 and the four Transfer Strategies for the WFG test suite and both 100 and 200 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
n=100	IGD	P1	8.2953e-2	8.9964e-2	1.1656e-1	9.7134e-2
		P2	2.7117e-3	4.2677e-3	5.9042e-3	2.1761e-3
		P3	3.9656e-3	9.2117e-4	2.7193e-3	1.8369e-3
		P4	2.7119e-4	2.8673e-4	1.6408e-4	2.7163e-4
		P5	5.4402e-3	8.0282e-4	5.7376e-3	1.9404e-3
		P6	6.5688e-4	5.8841e-4	5.6989e-4	4.4237e-4
		P7	7.3119e-4	8.9917e-4	8.6754e-4	9.7041e-4
		P8	1.1453e-2	1.4310e-2	1.5184e-2	1.5048e-2
		P9	1.3473e-2	1.0772e-2	1.1843e-2	1.3894e-2
	HV	P1	3.4255e-1	3.4155e-1	4.3923e-1	3.9853e-1
		P2	2.3495e-2	3.5722e-2	6.2046e-2	2.8448e-2
		P3	4.2775e-2	1.8141e-2	2.9158e-2	2.8246e-2
		P4	1.4935e-3	7.1476e-4	9.8322e-4	1.3894e-3
		P5	4.4852e-2	1.1499e-2	4.5019e-2	2.7733e-2
		P6	3.8926e-3	6.2005e-3	5.7485e-3	4.9638e-3
		P7	1.3814e-2	1.4536e-2	7.8942e-3	1.3856e-2
		P8	5.9973e-2	7.3035e-2	7.9908e-2	6.9911e-2
		P9	1.0303e-1	4.5464e-2	8.2197e-2	1.0172e-1
n=200	IGD	P1	7.6443e-2	5.3478e-2	1.0172e-1	6.9163e-2
		P2	2.5014e-3	2.0342e-3	2.2168e-3	2.5268e-3
		P3	8.1989e-4	1.7685e-3	1.0561e-3	1.1698e-3
		P4	1.8608e-4	3.0353e-4	2.9088e-4	3.9811e-4
		P5	5.3754e-3	5.4121e-3	5.4862e-3	5.4000e-3
		P6	2.6434e-4	3.0515e-4	2.8180e-4	3.5619e-4
		P7	9.4340e-3	1.0383e-2	1.0025e-2	8.3006e-3
		P8	8.6605e-3	7.9266e-3	1.0548e-2	8.2434e-3
		P9	1.1614e-2	1.0335e-2	1.3803e-2	8.7757e-3
	HV	P1	2.9296e-1	2.2773e-1	3.9625e-1	2.8598e-1
		P2	1.7788e-2	1.7084e-2	1.6324e-2	1.6470e-2
		P3	1.5504e-2	2.1355e-2	1.3973e-2	1.7354e-2
		P4	1.1605e-3	1.2658e-3	1.5476e-3	1.3331e-3
		P5	3.9504e-2	4.3269e-2	4.5296e-2	4.2290e-2
		P6	2.8068e-3	1.4336e-3	2.0267e-3	1.9765e-3
		P7	5.8698e-2	6.3895e-2	5.2691e-2	4.3319e-2
		P8	4.3585e-2	4.4632e-2	5.5753e-2	3.8365e-2
		P9	6.2907e-2	4.7073e-2	5.5506e-2	7.4716e-2

Table D.11.: IQR values of IGD and HV for Ref_1 and Ref_2 with MDG2 and the four Transfer Strategies for the LSMOP test suite and 500 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
Ref_1	IGD	P1	2.3065e-3	2.1234e-3	4.7668e-3	3.1667e-3
		P2	3.1850e-4	6.5299e-4	4.2215e-4	5.8278e-4
		P3	6.5066e-2	8.7684e-2	6.0485e-2	3.4589e-1
		P4	2.7429e-3	4.8991e-3	3.7170e-3	3.3369e-3
		P5	2.5540e-1	4.0973e-1	7.0233e-1	1.8249e-1
		P6	4.7214e-1	5.0635e-1	1.4684e-1	1.8861e-1
		P7	4.0779e-1	1.5088e-1	2.0520e-1	1.1740e-1
		P8	4.6667e-3	1.3979e-2	4.1175e-3	1.1885e-2
		P9	1.2664e-2	2.3342e-2	8.7956e-3	1.0642e-2
	HV	P1	5.0301e-3	6.4213e-3	7.3332e-3	6.8601e-3
		P2	5.1551e-4	1.2341e-3	5.7299e-4	1.0419e-3
		P3	6.1040e-2	5.9927e-2	8.2072e-2	3.9510e-2
		P4	3.5655e-3	7.3559e-3	6.3486e-3	6.0025e-3
		P5	2.7468e-4	6.5314e-2	2.7966e-1	2.2334e-4
		P6	—	1.0560e-1	7.0103e-3	1.0702e-1
		P7	—	—	—	—
		P8	3.1159e-3	8.2690e-3	3.6082e-3	7.5121e-3
		P9	2.2886e-2	1.9803e-2	8.0093e-3	6.3304e-3
Ref_2	IGD	P1	1.4707e-2	2.2536e-2	1.5939e-2	1.2921e-2
		P2	6.1501e-4	1.1518e-3	1.4270e-3	1.3378e-3
		P3	5.3380e-4	2.4182e-1	3.2744e-2	4.6450e-2
		P4	3.0867e-3	3.5483e-3	7.4156e-4	7.8481e-4
		P5	1.8334e-5	2.0009e-1	1.8563e-1	1.6695e-5
		P6	8.9522e-3	7.9924e-2	8.6826e-2	6.8488e-2
		P7	1.7155e-1	2.8015e-1	1.5817e-1	1.9006e-1
		P8	5.3220e-2	7.4886e-2	3.7071e-2	5.5904e-2
		P9	1.6167e-2	6.9964e-3	1.0999e-2	3.3127e-2
	HV	P1	2.0821e-2	3.0840e-2	2.2720e-2	1.7281e-2
		P2	1.0418e-3	1.8282e-3	2.3298e-3	2.2477e-3
		P3	8.2643e-4	6.9133e-2	4.6532e-2	4.8250e-2
		P4	4.3310e-3	5.2913e-3	1.3634e-3	1.0089e-3
		P5	5.0329e-5	3.1600e-2	5.0781e-5	4.5391e-5
		P6	9.5111e-3	1.5013e-2	2.8350e-2	2.5374e-2
		P7	—	—	—	—
		P8	4.6673e-2	6.7540e-2	4.1029e-2	4.9366e-2
		P9	1.5654e-2	8.3401e-3	1.6573e-2	4.2614e-2

Table D.12.: IQR values of IGD and HV for Ref_1 and Ref_2 with MDG2 and the four Transfer Strategies for the WFG test suite and 500 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA
Ref_1	IGD	P1	1.5110e-1	7.3884e-2	6.0976e-2	6.1493e-2
		P2	6.7684e-4	2.5471e-3	7.0254e-4	1.4271e-3
		P3	4.6573e-4	3.3324e-4	2.7318e-4	2.9615e-4
		P4	3.9857e-4	2.9317e-4	5.3241e-4	8.7831e-4
		P5	6.9287e-4	1.3071e-3	7.2749e-4	8.2467e-4
		P6	2.9216e-4	2.1812e-4	2.4190e-4	3.6443e-4
		P7	5.9530e-2	6.1333e-2	7.1024e-2	6.1592e-2
		P8	5.0247e-3	2.9245e-3	2.0347e-3	4.1133e-3
		P9	7.2624e-2	8.2160e-2	3.2411e-2	4.0718e-2
	HV	P1	6.1426e-1	3.2206e-1	2.7252e-1	2.6479e-1
		P2	1.1263e-3	2.5086e-3	1.7205e-3	1.2632e-3
		P3	1.4506e-3	9.5070e-4	7.9875e-4	1.1211e-3
		P4	2.0825e-3	1.8191e-3	2.0083e-3	1.1616e-3
		P5	2.9482e-3	6.8278e-3	3.5170e-3	3.4748e-3
		P6	1.1113e-3	1.1134e-3	1.3999e-3	7.5782e-4
		P7	3.1436e-1	3.6068e-1	3.6016e-1	3.2123e-1
		P8	2.4668e-2	1.6498e-2	1.2807e-2	1.4565e-2
		P9	3.5106e-1	4.0333e-1	1.6374e-1	2.0118e-1
Ref_2	IGD	P1	6.8481e-2	4.7449e-2	7.7773e-2	6.6049e-2
		P2	2.0402e-3	1.8413e-3	2.2845e-3	2.0123e-3
		P3	7.2358e-4	4.4124e-4	6.9303e-4	3.3719e-4
		P4	3.0466e-4	2.5866e-4	2.2954e-4	1.7028e-4
		P5	6.3026e-4	4.5441e-4	2.1301e-3	7.2464e-4
		P6	3.2934e-4	4.0203e-4	2.9268e-4	3.5299e-4
		P7	1.0538e-2	1.3218e-2	1.0226e-2	9.2438e-3
		P8	3.6210e-3	2.5213e-3	2.9509e-3	5.9225e-3
		P9	2.4217e-3	1.3874e-2	4.0925e-3	4.4237e-3
	HV	P1	2.7146e-1	1.9220e-1	3.1042e-1	2.6203e-1
		P2	7.3485e-3	9.5928e-3	5.8639e-3	5.0901e-3
		P3	5.5907e-3	7.8192e-3	5.8239e-3	6.4371e-3
		P4	1.2762e-3	1.5244e-3	1.2065e-3	8.0595e-4
		P5	1.2117e-2	9.1285e-3	2.6951e-2	7.3108e-3
		P6	1.0285e-3	9.6187e-4	1.6606e-3	1.0671e-3
		P7	6.8691e-2	8.2443e-2	6.8165e-2	5.2813e-2
		P8	1.8584e-2	1.7728e-2	1.9023e-2	3.5387e-2
		P9	2.2217e-2	1.0471e-1	2.2352e-2	2.5678e-2

E. General Comparison

In this section the tables to compare nine different Grouping Methods against each other are depicted. The nine algorithms are: MDG2 with the four Transfer Strategies, MRAND+S2, MDELTA+S2, GN, G1 and Correlation Analysis from LMEA. The results are displayed in 12 tables.

Table E.1.: Median IGD and HV values for Ref_1 with the nine grouping methods for the LSMOP and WFG test suite and 100 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	1	1.3749e-2$^{(o)}$	**1.3550e-2**$^{(r)}$	1.4594e-2$^{(o)}$	1.5101e-2$^{(o)}$	3.5644e-2$^{(-)}$	3.5625e-2$^{(-)}$	1.5099e-2$^{(o)}$	*2.5577e-1*$^{(-)}$	1.5880e-2$^{(-)}$
		2	4.2302e-2$^{(-)}$	4.2361e-2$^{(-)}$	**2.7792e-2**$^{(r)}$	2.8625e-2$^{(o)}$	3.5003e-2$^{(-)}$	3.1553e-2$^{(-)}$	2.9119e-2$^{(o)}$	*1.0503e-1*$^{(-)}$	3.5910e-2$^{(-)}$
		3	9.5886e-1$^{(-)}$	8.9326e-1$^{(-)}$	**5.9298e-1**$^{(r)}$	6.0404e-1$^{(o)}$	9.6557e-1$^{(-)}$	1.0098e+0$^{(-)}$	6.1923e-1$^{(o)}$	*1.0556e+0*$^{(-)}$	8.4584e-1$^{(-)}$
		4	1.4990e-1$^{(-)}$	1.4602e-1$^{(-)}$	5.7010e-2$^{(-)}$	5.4147e-2$^{(-)}$	**4.9708e-2**$^{(r)}$	6.0935e-2$^{(-)}$	5.4329e-2$^{(-)}$	*2.1042e-1*$^{(-)}$	8.2985e-2$^{(-)}$
		5	4.7878e-1$^{(o)}$	6.5130e-1$^{(-)}$	**4.3436e-1**$^{(r)}$	5.1016e-1$^{(o)}$	4.7698e-1$^{(-)}$	6.2532e-1$^{(-)}$	4.6764e-1$^{(o)}$	*7.4098e-1*$^{(-)}$	4.4094e-1$^{(o)}$
		6	7.3982e-1$^{(-)}$	**5.9514e-1**$^{(r)}$	6.8864e-1$^{(o)}$	7.2588e-1$^{(o)}$	7.3005e-1$^{(o)}$	6.9817e-1$^{(o)}$	6.4219e-1$^{(o)}$	*1.6872e+0*$^{(-)}$	6.5262e-1$^{(o)}$
		7	**1.2873e+0**$^{(r)}$	1.3063e+0$^{(o)}$	1.5162e+0$^{(o)}$	1.3345e+0$^{(o)}$	1.3473e+0$^{(o)}$	1.3495e+0$^{(o)}$	1.3281e+0$^{(o)}$	*1.7755e+0*$^{(-)}$	1.3041e+0$^{(o)}$
		8	9.3913e-2$^{(o)}$	**9.2113e-2**$^{(r)}$	9.4772e-2$^{(o)}$	9.3089e-2$^{(o)}$	1.3515e-1$^{(-)}$	1.2810e-1$^{(-)}$	9.5050e-2$^{(o)}$	*3.7221e-1*$^{(-)}$	9.3932e-2$^{(o)}$
		9	5.9746e-1$^{(-)}$	6.3058e-1$^{(-)}$	4.8033e-1$^{(o)}$	4.7855e-1$^{(o)}$	4.8592e-1$^{(o)}$	5.1701e-1$^{(o)}$	**4.7701e-1**$^{(r)}$	*7.1598e-1*$^{(-)}$	5.3288e-1$^{(-)}$
	HV	1	**6.8238e-1**$^{(r)}$	6.8209e-1$^{(o)}$	6.8195e-1$^{(o)}$	6.8021e-1$^{(o)}$	6.4324e-1$^{(-)}$	6.4317e-1$^{(-)}$	6.7857e-1$^{(-)}$	*3.6549e-1*$^{(-)}$	6.7816e-1$^{(-)}$
		2	6.3565e-1$^{(-)}$	6.3544e-1$^{(-)}$	**6.5762e-1**$^{(r)}$	6.5673e-1$^{(o)}$	6.4725e-1$^{(-)}$	6.5229e-1$^{(-)}$	6.5598e-1$^{(o)}$	*5.4672e-1*$^{(-)}$	6.4461e-1$^{(-)}$
		3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		4	4.8260e-1$^{(-)}$	4.8871e-1$^{(-)}$	6.0909e-1$^{(-)}$	6.1136e-1$^{(-)}$	**6.1720e-1**$^{(r)}$	5.9980e-1$^{(-)}$	6.1091e-1$^{(-)}$	*4.0789e-1*$^{(-)}$	5.6832e-1$^{(-)}$
		5	1.0999e-1$^{(o)}$	*1.0995e-1*$^{(-)}$	1.1003e-1$^{(o)}$	1.0998e-1$^{(o)}$	1.0997e-1$^{(o)}$	1.0998e-1$^{(o)}$	1.0998e-1$^{(o)}$	1.0999e-1$^{(-)}$	**1.1006e-1**$^{(r)}$
		6	—$^{(-)}$	2.4308e-2$^{(-)}$	1.0155e-1$^{(o)}$	**1.0459e-1**$^{(r)}$	8.9504e-2$^{(-)}$	8.8915e-2$^{(-)}$	7.5398e-2$^{(-)}$	—$^{(-)}$	—$^{(-)}$
		7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		8	2.8500e-1$^{(o)}$	2.8626e-1$^{(o)}$	**2.8763e-1**$^{(r)}$	2.8597e-1$^{(o)}$	2.5430e-1$^{(-)}$	2.5369e-1$^{(-)}$	2.8494e-1$^{(o)}$	*1.1001e-1*$^{(-)}$	2.8488e-1$^{(o)}$
		9	5.4732e-1$^{(-)}$	5.2678e-1$^{(-)}$	6.8928e-1$^{(o)}$	6.9437e-1$^{(o)}$	6.8276e-1$^{(o)}$	6.5125e-1$^{(-)}$	**6.9696e-1**$^{(r)}$	*4.3371e-1*$^{(-)}$	6.2771e-1$^{(-)}$
WFG	IGD	1	1.2721e+0$^{(-)}$	1.2675e+0$^{(-)}$	1.2755e+0$^{(-)}$	1.2939e+0$^{(-)}$	1.1869e+0$^{(-)}$	1.1881e+0$^{(-)}$	1.2994e+0$^{(-)}$	**1.1227e+0**$^{(r)}$	*1.3137e+0*$^{(-)}$
		2	1.3555e-2$^{(o)}$	1.3527e-2$^{(o)}$	1.3273e-2$^{(o)}$	**1.3163e-2**$^{(r)}$	3.5706e-2$^{(-)}$	2.8966e-2$^{(-)}$	*1.4499e-1*$^{(-)}$	5.7365e-2$^{(-)}$	1.4715e-2$^{(-)}$
		3	1.3041e-2$^{(o)}$	1.3092e-2$^{(o)}$	**1.3008e-2**$^{(r)}$	1.3170e-2$^{(o)}$	3.4219e-2$^{(-)}$	2.6337e-2$^{(-)}$	*1.5986e-1*$^{(-)}$	6.1300e-2$^{(-)}$	1.3461e-2$^{(-)}$
		4	1.4055e-2$^{(o)}$	1.3956e-2$^{(o)}$	*1.4252e-2*$^{(-)}$	1.3932e-2$^{(o)}$	1.4053e-2$^{(o)}$	**1.3888e-2**$^{(r)}$	1.3916e-2$^{(o)}$	1.3893e-2$^{(o)}$	1.4246e-2$^{(o)}$
		5	6.8977e-2$^{(o)}$	6.9257e-2$^{(o)}$	6.9116e-2$^{(-)}$	**6.8790e-2**$^{(r)}$	6.9494e-2$^{(-)}$	6.9754e-2$^{(-)}$	6.9202e-2$^{(o)}$	*6.9866e-2*$^{(-)}$	6.9034e-2$^{(o)}$
		6	*1.9710e-2*$^{(-)}$	1.9228e-2$^{(o)}$	1.9333e-2$^{(o)}$	1.9534e-2$^{(o)}$	**1.9226e-2**$^{(r)}$	1.9432e-2$^{(-)}$	1.9305e-2$^{(o)}$	1.9330e-2$^{(o)}$	1.9383e-2$^{(o)}$
		7	*8.6450e-2*$^{(-)}$	8.1438e-2$^{(-)}$	7.7506e-2$^{(o)}$	7.4459e-2$^{(o)}$	7.1881e-2$^{(o)}$	7.1800e-2$^{(o)}$	**6.5809e-2**$^{(r)}$	8.3792e-2$^{(o)}$	8.2432e-2$^{(o)}$
		8	6.2279e-2$^{(-)}$	5.5432e-2$^{(-)}$	5.7599e-2$^{(-)}$	5.5533e-2$^{(-)}$	5.5949e-2$^{(-)}$	5.3835e-2$^{(-)}$	**4.3376e-2**$^{(r)}$	7.1771e-2$^{(-)}$	*7.2698e-2*$^{(-)}$
		9	1.3954e-1$^{(-)}$	1.3254e-1$^{(-)}$	1.4439e-1$^{(-)}$	1.3619e-1$^{(-)}$	**7.7393e-2**$^{(r)}$	8.5152e-2$^{(o)}$	9.3118e-2$^{(o)}$	1.4331e-1$^{(-)}$	*1.5019e-1*$^{(-)}$
	HV	1	1.2346e+0$^{(-)}$	1.2224e+0$^{(-)}$	1.2427e+0$^{(-)}$	1.1351e+0$^{(-)}$	1.8236e+0$^{(-)}$	1.9389e+0$^{(-)}$	1.1137e+0$^{(-)}$	**2.5721e+0**$^{(r)}$	1.0564e+0$^{(-)}$
		2	**6.1306e+0**$^{(r)}$	6.1300e+0$^{(o)}$	6.1305e+0$^{(o)}$	6.1306e+0$^{(o)}$	5.9602e+0$^{(-)}$	6.0019e+0$^{(-)}$	5.3442e+0$^{(-)}$	5.8317e+0$^{(-)}$	6.1174e+0$^{(-)}$
		3	5.6297e+0$^{(o)}$	5.6296e+0$^{(o)}$	**5.6299e+0**$^{(r)}$	5.6291e+0$^{(o)}$	5.4836e+0$^{(-)}$	5.5272e+0$^{(-)}$	4.8634e+0$^{(-)}$	5.3442e+0$^{(-)}$	5.6218e+0$^{(-)}$
		4	3.3615e+0$^{(-)}$	3.3613e+0$^{(o)}$	3.3616e+0$^{(o)}$	3.3612e+0$^{(o)}$	**3.3620e+0**$^{(r)}$	3.3612e+0$^{(o)}$	3.3612e+0$^{(o)}$	3.3611e+0$^{(o)}$	3.3613e+0$^{(-)}$
		5	2.9851e+0$^{(o)}$	2.9838e+0$^{(-)}$	2.9841e+0$^{(o)}$	**2.9861e+0**$^{(r)}$	2.9825e+0$^{(-)}$	2.9811e+0$^{(-)}$	2.9834e+0$^{(o)}$	2.9803e+0$^{(-)}$	2.9849e+0$^{(o)}$
		6	3.2941e+0$^{(-)}$	**3.2980e+0**$^{(r)}$	3.2971e+0$^{(o)}$	3.2971e+0$^{(o)}$	3.2959e+0$^{(o)}$	3.2953e+0$^{(o)}$	3.2975e+0$^{(o)}$	3.2957e+0$^{(o)}$	3.2954e+0$^{(o)}$
		7	2.9509e+0$^{(-)}$	3.0040e+0$^{(-)}$	3.0132e+0$^{(-)}$	3.0268e+0$^{(o)}$	3.0374e+0$^{(o)}$	3.0350e+0$^{(o)}$	**3.0478e+0**$^{(r)}$	2.9713e+0$^{(o)}$	2.9859e+0$^{(-)}$
		8	3.0612e+0$^{(-)}$	3.0985e+0$^{(-)}$	3.0824e+0$^{(-)}$	3.0954e+0$^{(-)}$	3.1133e+0$^{(-)}$	3.1265e+0$^{(-)}$	**3.1845e+0**$^{(r)}$	3.0145e+0$^{(-)}$	3.0024e+0$^{(-)}$
		9	2.6351e+0$^{(-)}$	2.6634e+0$^{(-)}$	2.6016e+0$^{(-)}$	2.6515e+0$^{(-)}$	**2.9488e+0**$^{(r)}$	2.9118e+0$^{(o)}$	2.8777e+0$^{(o)}$	2.6035e+0$^{(-)}$	2.5871e+0$^{(-)}$

Table E.2.: Median IGD and HV values for Ref_2 with the nine grouping methods for the LSMOP and WFG test suite and $n = 100$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	1	4.9400e-2$^{(o)}$	**4.2126e-2**$^{(r)}$	4.2778e-2$^{(o)}$	4.3647e-2$^{(o)}$	1.4588e-1$^{(-)}$	1.4202e-1$^{(-)}$	4.4802e-2$^{(o)}$	*3.6720e-1*$^{(-)}$	5.2469e-2$^{(o)}$
		2	2.5206e-2$^{(-)}$	2.5082e-2$^{(-)}$	2.2858e-2$^{(o)}$	2.2836e-2$^{(o)}$	2.3937e-2$^{(-)}$	2.2768e-2$^{(o)}$	**2.1950e-2**$^{(r)}$	*1.2450e-1*$^{(-)}$	3.5211e-2$^{(-)}$
		3	6.5317e-1$^{(-)}$	5.0935e-1$^{(o)}$	4.9420e-1$^{(-)}$	**4.5374e-1**$^{(r)}$	7.0735e-1$^{(-)}$	7.0762e-1$^{(-)}$	4.6519e-1$^{(o)}$	*1.1393e+0*$^{(-)}$	6.4168e-1$^{(-)}$
		4	1.3994e-1$^{(-)}$	1.3877e-1$^{(-)}$	2.5655e-2$^{(-)}$	2.5683e-2$^{(o)}$	**2.4374e-2**$^{(r)}$	2.9015e-2$^{(-)}$	2.5113e-2$^{(o)}$	*1.6794e-1*$^{(-)}$	4.8197e-2$^{(-)}$
		5	3.4246e-1$^{(o)}$	3.8915e-1$^{(-)}$	3.6967e-1$^{(-)}$	4.1671e-1$^{(-)}$	**3.4110e-1**$^{(r)}$	3.4177e-1$^{(o)}$	*4.2182e-1*$^{(-)}$	3.4251e-1$^{(-)}$	4.0153e-1$^{(-)}$
		6	6.6970e-1$^{(-)}$	5.3033e-1$^{(o)}$	4.3171e-1$^{(o)}$	**4.0620e-1**$^{(r)}$	6.3628e-1$^{(-)}$	6.7058e-1$^{(-)}$	4.3662e-1$^{(o)}$	*7.5486e-1*$^{(-)}$	7.0649e-1$^{(-)}$
		7	**1.2893e+0**$^{(r)}$	1.5628e+0$^{(-)}$	1.2999e+0$^{(o)}$	*2.0325e+0*$^{(-)}$	1.3645e+0$^{(-)}$	1.7244e+0$^{(-)}$	1.3824e+0$^{(-)}$	1.3036e+0$^{(o)}$	1.3431e+0$^{(o)}$
		8	1.9835e-1$^{(o)}$	**1.7292e-1**$^{(r)}$	2.0641e-1$^{(o)}$	1.9979e-1$^{(o)}$	3.7289e-1$^{(-)}$	3.7049e-1$^{(-)}$	1.9593e-1$^{(o)}$	*3.8365e-1*$^{(-)}$	2.0024e-1$^{(o)}$
		9	3.3452e-1$^{(-)}$	3.5736e-1$^{(-)}$	1.0933e-1$^{(o)}$	8.1321e-2$^{(o)}$	1.5321e-1$^{(-)}$	1.7731e-1$^{(-)}$	**5.8147e-2**$^{(r)}$	*9.8096e-1*$^{(-)}$	1.0297e-1$^{(-)}$
	HV	1	6.3865e-1$^{(o)}$	**6.4800e-1**$^{(r)}$	6.4748e-1$^{(o)}$	6.4533e-1$^{(o)}$	5.0182e-1$^{(-)}$	5.0794e-1$^{(-)}$	6.4368e-1$^{(o)}$	*3.4257e-1*$^{(-)}$	6.3373e-1$^{(o)}$
		2	6.6903e-1$^{(-)}$	6.6907e-1$^{(-)}$	6.6755e-1$^{(-)}$	6.6792e-1$^{(-)}$	6.6844e-1$^{(-)}$	**6.7071e-1**$^{(r)}$	6.6934e-1$^{(o)}$	*5.2467e-1*$^{(-)}$	6.5057e-1$^{(-)}$
		3	1.0998e-1$^{(o)}$	**1.3265e-1**$^{(r)}$	6.5100e-2$^{(-)}$	1.2505e-1$^{(o)}$	1.0962e-1$^{(-)}$	1.0901e-1$^{(-)}$	9.2051e-2$^{(o)}$	—$^{(-)}$	1.1008e-1$^{(o)}$
		4	4.8785e-1$^{(-)}$	4.8872e-1$^{(-)}$	6.6220e-1$^{(-)}$	6.6206e-1$^{(-)}$	**6.6468e-1**$^{(r)}$	6.5500e-1$^{(-)}$	6.6314e-1$^{(o)}$	*4.6284e-1*$^{(-)}$	6.1715e-1$^{(-)}$
		5	1.4439e-1$^{(o)}$	*1.0999e-1*$^{(-)}$	1.0999e-1$^{(-)}$	1.1045e-1$^{(-)}$	2.0730e-1$^{(o)}$	**2.0935e-1**$^{(r)}$	1.0999e-1$^{(-)}$	2.0840e-1$^{(o)}$	1.1138e-1$^{(o)}$
		6	**1.0776e-1**$^{(r)}$	9.9507e-2$^{(-)}$	4.4427e-2$^{(-)}$	6.1107e-2$^{(-)}$	1.0528e-1$^{(-)}$	1.0676e-1$^{(o)}$	8.9634e-2$^{(-)}$	*3.4635e-2*$^{(-)}$	9.3517e-2$^{(-)}$
		7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		8	1.9189e-1$^{(o)}$	**2.1949e-1**$^{(r)}$	1.9985e-1$^{(o)}$	2.0983e-1$^{(o)}$	1.1000e-1$^{(-)}$	1.1000e-1$^{(-)}$	2.0002e-1$^{(o)}$	*1.1000e-1*$^{(-)}$	1.9766e-1$^{(o)}$
		9	5.0462e-1$^{(-)}$	5.1033e-1$^{(-)}$	8.7179e-1$^{(-)}$	8.9882e-1$^{(-)}$	7.6316e-1$^{(-)}$	7.2486e-1$^{(-)}$	**9.2526e-1**$^{(r)}$	*7.4402e-2*$^{(-)}$	8.0714e-1$^{(-)}$
WFG	IGD	1	8.4828e-1$^{(-)}$	8.5021e-1$^{(-)}$	8.4947e-1$^{(-)}$	8.5100e-1$^{(-)}$	4.8609e-1$^{(-)}$	5.1361e-1$^{(-)}$	8.4791e-1$^{(-)}$	**3.1013e-1**$^{(r)}$	*9.1067e-1*$^{(-)}$
		2	1.5417e-2$^{(o)}$	1.5432e-2$^{(o)}$	1.5574e-2$^{(o)}$	**1.4874e-2**$^{(r)}$	1.0999e-1$^{(-)}$	1.0905e-1$^{(-)}$	*1.9879e-1*$^{(-)}$	6.3862e-2$^{(-)}$	2.0958e-2$^{(-)}$
		3	1.4968e-2$^{(-)}$	**1.3652e-2**$^{(r)}$	1.3916e-2$^{(o)}$	1.3762e-2$^{(o)}$	1.1817e-1$^{(-)}$	1.1043e-1$^{(-)}$	*2.1542e-1*$^{(-)}$	6.7320e-2$^{(-)}$	2.1563e-2$^{(-)}$
		4	**1.3745e-2**$^{(r)}$	1.3926e-2$^{(-)}$	1.3850e-2$^{(o)}$	1.3801e-2$^{(o)}$	1.3899e-2$^{(-)}$	1.3877e-2$^{(o)}$	1.3806e-2$^{(o)}$	*1.3934e-2*$^{(-)}$	1.3806e-2$^{(o)}$
		5	*6.4712e-2*$^{(-)}$	6.3867e-2$^{(-)}$	6.4624e-2$^{(-)}$	6.3988e-2$^{(-)}$	6.3712e-2$^{(o)}$	6.3731e-2$^{(o)}$	6.4435e-2$^{(-)}$	**6.3700e-2**$^{(r)}$	6.4276e-2$^{(-)}$
		6	1.6132e-2$^{(o)}$	1.6130e-2$^{(-)}$	1.6017e-2$^{(o)}$	1.6104e-2$^{(o)}$	1.9132e-2$^{(-)}$	1.9354e-2$^{(-)}$	*1.9685e-2*$^{(-)}$	**1.5952e-2**$^{(r)}$	1.6004e-2$^{(o)}$
		7	1.4261e-2$^{(-)}$	1.4253e-2$^{(-)}$	1.4801e-2$^{(-)}$	1.4884e-2$^{(-)}$	1.4207e-2$^{(-)}$	1.4198e-2$^{(-)}$	*1.4964e-2*$^{(-)}$	1.3767e-2$^{(o)}$	**1.3723e-2**$^{(r)}$
		8	1.1370e-1$^{(-)}$	1.0370e-1$^{(-)}$	1.1213e-1$^{(-)}$	1.0097e-1$^{(-)}$	5.5198e-2$^{(o)}$	5.6867e-2$^{(o)}$	**5.4694e-2**$^{(r)}$	*1.5291e-1*$^{(-)}$	1.5071e-1$^{(-)}$
		9	2.5763e-2$^{(o)}$	2.4425e-2$^{(o)}$	2.4310e-2$^{(-)}$	2.3788e-2$^{(o)}$	3.4599e-2$^{(-)}$	3.4366e-2$^{(-)}$	3.4925e-2$^{(-)}$	*3.8730e-2*$^{(-)}$	**2.1425e-2**$^{(r)}$
	HV	1	2.7027e+0$^{(-)}$	2.6923e+0$^{(-)}$	2.7039e+0$^{(-)}$	2.6978e+0$^{(-)}$	4.2942e+0$^{(-)}$	4.1373e+0$^{(-)}$	2.6899e+0$^{(-)}$	**5.1980e+0**$^{(r)}$	2.4647e+0$^{(-)}$
		2	6.1187e+0$^{(o)}$	**6.1190e+0**$^{(r)}$	6.1008e+0$^{(o)}$	6.1133e+0$^{(o)}$	5.5364e+0$^{(-)}$	5.5402e+0$^{(-)}$	5.0613e+0$^{(-)}$	5.7986e+0$^{(-)}$	6.0633e+0$^{(-)}$
		3	5.6063e+0$^{(-)}$	**5.6221e+0**$^{(r)}$	5.6143e+0$^{(-)}$	5.6220e+0$^{(o)}$	5.0641e+0$^{(-)}$	5.1023e+0$^{(-)}$	4.6032e+0$^{(-)}$	5.3143e+0$^{(-)}$	5.5574e+0$^{(-)}$
		4	3.3597e+0$^{(-)}$	3.3597e+0$^{(-)}$	3.3598e+0$^{(-)}$	3.3597e+0$^{(-)}$	3.3599e+0$^{(-)}$	3.3594e+0$^{(-)}$	3.3597e+0$^{(-)}$	**3.3611e+0**$^{(r)}$	3.3593e+0$^{(-)}$
		5	3.0154e+0$^{(-)}$	3.0313e+0$^{(-)}$	3.0187e+0$^{(-)}$	3.0275e+0$^{(-)}$	3.0329e+0$^{(-)}$	3.0321e+0$^{(-)}$	3.0227e+0$^{(-)}$	**3.0340e+0**$^{(r)}$	3.0259e+0$^{(-)}$
		6	3.3268e+0$^{(o)}$	3.3263e+0$^{(o)}$	3.3258e+0$^{(o)}$	**3.3285e+0**$^{(r)}$	3.2998e+0$^{(-)}$	3.2964e+0$^{(-)}$	3.2955e+0$^{(-)}$	3.3275e+0$^{(o)}$	3.3274e+0$^{(o)}$
		7	3.3552e+0$^{(-)}$	3.3550e+0$^{(-)}$	3.3452e+0$^{(-)}$	3.3471e+0$^{(-)}$	3.3592e+0$^{(-)}$	3.3599e+0$^{(-)}$	3.3434e+0$^{(-)}$	**3.3621e+0**$^{(r)}$	3.3611e+0$^{(-)}$
		8	2.7791e+0$^{(-)}$	2.8354e+0$^{(-)}$	2.7866e+0$^{(-)}$	2.8399e+0$^{(-)}$	3.1068e+0$^{(-)}$	3.1042e+0$^{(-)}$	**3.1324e+0**$^{(r)}$	2.5733e+0$^{(-)}$	2.5790e+0$^{(-)}$
		9	3.2451e+0$^{(o)}$	3.2596e+0$^{(o)}$	3.2586e+0$^{(-)}$	3.2562e+0$^{(-)}$	3.1717e+0$^{(-)}$	3.1717e+0$^{(-)}$	3.1712e+0$^{(-)}$	3.2080e+0$^{(-)}$	**3.2755e+0**$^{(r)}$

Table E.3.: Median IGD and HV values for Ref_1 with the nine grouping methods for the LSMOP and WFG test suite and $n = 200$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	1	2.1061e-2$^{(o)}$	2.2916e-2$^{(-)}$	2.1635e-2$^{(o)}$	2.1415e-2$^{(o)}$	5.3233e-2$^{(-)}$	5.2145e-2$^{(-)}$	**2.0925e-2**$^{(r)}$	*3.7022e-1*$^{(-)}$	2.9193e-2$^{(-)}$
		2	4.4804e-2$^{(-)}$	4.3915e-2$^{(-)}$	3.1111e-2$^{(o)}$	**3.0500e-2**$^{(r)}$	3.4332e-2$^{(-)}$	3.3090e-2$^{(-)}$	3.1268e-2$^{(o)}$	*8.7044e-2*$^{(-)}$	4.5050e-2$^{(-)}$
		3	1.0426e+0$^{(-)}$	**9.1020e-1**$^{(r)}$	1.0107e+0$^{(-)}$	9.4873e-1$^{(o)}$	9.6753e-1$^{(o)}$	1.0080e+0$^{(-)}$	9.4467e-1$^{(o)}$	1.1034e+0$^{(-)}$	*1.4855e+0*$^{(-)}$
		4	1.1519e-1$^{(-)}$	1.1641e-1$^{(-)}$	4.8278e-2$^{(o)}$	4.7211e-2$^{(o)}$	**4.6468e-2**$^{(r)}$	5.2774e-2$^{(-)}$	4.7474e-2$^{(o)}$	*1.3984e-1*$^{(-)}$	6.6628e-2$^{(-)}$
		5	4.7051e-1$^{(o)}$	4.8380e-1$^{(o)}$	4.4314e-1$^{(o)}$	**4.4286e-1**$^{(r)}$	4.8913e-1$^{(o)}$	4.8445e-1$^{(o)}$	4.4931e-1$^{(o)}$	*7.3983e-1*$^{(-)}$	4.4508e-1$^{(o)}$
		6	1.0156e+0$^{(-)}$	6.1255e-1$^{(-)}$	**4.6935e-1**$^{(r)}$	5.0772e-1$^{(o)}$	6.5753e-1$^{(-)}$	5.4734e-1$^{(-)}$	4.8295e-1$^{(o)}$	*1.5101e+0*$^{(-)}$	7.3566e-1$^{(-)}$
		7	1.8124e+0$^{(-)}$	1.4297e+0$^{(o)}$	*1.9499e+0*$^{(-)}$	1.4340e+0$^{(o)}$	1.4339e+0$^{(o)}$	1.4328e+0$^{(o)}$	1.4447e+0$^{(-)}$	1.9334e+0$^{(-)}$	**1.4174e+0**$^{(r)}$
		8	6.6225e-2$^{(o)}$	6.7041e-2$^{(o)}$	6.6891e-2$^{(o)}$	6.6119e-2$^{(o)}$	1.1585e-1$^{(-)}$	1.3432e-1$^{(-)}$	**6.5745e-2**$^{(r)}$	*3.5500e-1*$^{(-)}$	6.9678e-2$^{(-)}$
		9	5.6868e-1$^{(-)}$	5.7630e-1$^{(-)}$	4.8185e-1$^{(o)}$	4.8098e-1$^{(o)}$	4.9482e-1$^{(o)}$	4.9035e-1$^{(-)}$	**4.7329e-1**$^{(r)}$	*8.6025e-1*$^{(-)}$	5.1684e-1$^{(-)}$
	HV	1	6.6684e-1$^{(o)}$	6.6465e-1$^{(-)}$	6.6580e-1$^{(o)}$	6.6653e-1$^{(o)}$	6.1394e-1$^{(-)}$	6.1458e-1$^{(-)}$	**6.6723e-1**$^{(r)}$	*2.8803e-1*$^{(-)}$	6.5224e-1$^{(-)}$
		2	6.3375e-1$^{(-)}$	6.3435e-1$^{(-)}$	6.5377e-1$^{(o)}$	**6.5478e-1**$^{(r)}$	6.4892e-1$^{(-)}$	6.5026e-1$^{(-)}$	6.5304e-1$^{(o)}$	*5.7362e-1*$^{(-)}$	6.3097e-1$^{(-)}$
		3	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		4	5.3563e-1$^{(-)}$	5.3299e-1$^{(-)}$	6.2532e-1$^{(o)}$	6.2689e-1$^{(o)}$	**6.2738e-1**$^{(r)}$	6.1816e-1$^{(-)}$	6.2652e-1$^{(o)}$	*5.0098e-1*$^{(-)}$	5.9941e-1$^{(-)}$
		5	1.0897e-1$^{(-)}$	1.0909e-1$^{(-)}$	1.0868e-1$^{(-)}$	1.0901e-1$^{(-)}$	1.0929e-1$^{(-)}$	1.0920e-1$^{(-)}$	1.0912e-1$^{(-)}$	**1.0998e-1**$^{(r)}$	*1.0853e-1*$^{(-)}$
		6	—$^{(-)}$	—$^{(-)}$	3.2601e-3$^{(o)}$	—$^{(o)}$	1.1120e-2$^{(o)}$	2.1413e-3$^{(o)}$	**2.4416e-2**$^{(r)}$	—$^{(-)}$	—$^{(-)}$
		7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		8	**3.1538e-1**$^{(r)}$	3.1448e-1$^{(o)}$	3.1351e-1$^{(o)}$	3.1470e-1$^{(o)}$	2.5841e-1$^{(-)}$	2.4977e-1$^{(-)}$	3.1335e-1$^{(o)}$	*1.1000e-1*$^{(-)}$	3.0907e-1$^{(-)}$
		9	5.8259e-1$^{(-)}$	5.6038e-1$^{(-)}$	6.8503e-1$^{(o)}$	6.8915e-1$^{(o)}$	6.7104e-1$^{(o)}$	6.7764e-1$^{(o)}$	**6.9623e-1**$^{(r)}$	*2.4044e-1*$^{(-)}$	6.4859e-1$^{(-)}$
WFG	IGD	1	1.4132e+0$^{(-)}$	1.4108e+0$^{(-)}$	1.3999e+0$^{(-)}$	1.4033e+0$^{(-)}$	1.2919e+0$^{(-)}$	1.2502e+0$^{(o)}$	1.3987e+0$^{(-)}$	**1.1812e+0**$^{(r)}$	*1.4413e+0*$^{(-)}$
		2	1.4584e-2$^{(o)}$	**1.4137e-2**$^{(r)}$	1.5517e-2$^{(-)}$	1.5806e-2$^{(o)}$	5.4538e-2$^{(-)}$	4.7517e-2$^{(-)}$	*1.7989e-1*$^{(-)}$	1.3665e-1$^{(-)}$	2.1766e-2$^{(-)}$
		3	1.3360e-2$^{(o)}$	1.3404e-2$^{(o)}$	**1.3300e-2**$^{(r)}$	1.3643e-2$^{(o)}$	5.5815e-2$^{(-)}$	5.0570e-2$^{(-)}$	*1.9475e-1*$^{(-)}$	1.5257e-1$^{(-)}$	1.8619e-2$^{(-)}$
		4	1.5181e-2$^{(o)}$	*1.6167e-2*$^{(-)}$	1.4592e-2$^{(o)}$	1.4681e-2$^{(o)}$	1.5040e-2$^{(o)}$	1.4754e-2$^{(o)}$	**1.4560e-2**$^{(r)}$	1.4570e-2$^{(o)}$	1.5926e-2$^{(-)}$
		5	6.9294e-2$^{(o)}$	6.9300e-2$^{(o)}$	6.9400e-2$^{(o)}$	**6.9288e-2**$^{(r)}$	6.9703e-2$^{(-)}$	6.9742e-2$^{(-)}$	6.9355e-2$^{(o)}$	*6.9910e-2*$^{(-)}$	6.9395e-2$^{(o)}$
		6	1.6041e-2$^{(o)}$	*1.6194e-2*$^{(o)}$	1.5790e-2$^{(o)}$	1.5872e-2$^{(o)}$	1.5865e-2$^{(o)}$	**1.5759e-2**$^{(r)}$	1.6169e-2$^{(-)}$	1.5900e-2$^{(o)}$	1.5916e-2$^{(o)}$
		7	1.7435e-1$^{(o)}$	1.7658e-1$^{(o)}$	*1.8999e-1*$^{(-)}$	1.7425e-1$^{(o)}$	1.4802e-1$^{(o)}$	**1.3922e-1**$^{(r)}$	1.7540e-1$^{(-)}$	1.7290e-1$^{(o)}$	1.8339e-1$^{(o)}$
		8	5.7467e-2$^{(-)}$	4.2103e-2$^{(-)}$	5.7425e-2$^{(-)}$	4.1838e-2$^{(-)}$	5.1535e-2$^{(-)}$	5.1552e-2$^{(-)}$	**3.8002e-2**$^{(r)}$	*8.6498e-2*$^{(-)}$	8.4118e-2$^{(-)}$
		9	2.6891e-1$^{(-)}$	2.5994e-1$^{(-)}$	*2.9208e-1*$^{(-)}$	2.6427e-1$^{(-)}$	1.5105e-1$^{(-)}$	1.2064e-1$^{(o)}$	**1.0863e-1**$^{(r)}$	2.6791e-1$^{(-)}$	2.5811e-1$^{(-)}$
	HV	1	6.0500e-1$^{(-)}$	6.2606e-1$^{(-)}$	6.5210e-1$^{(-)}$	6.4523e-1$^{(-)}$	1.3132e+0$^{(-)}$	1.5267e+0$^{(-)}$	6.5965e-1$^{(-)}$	**2.1428e+0**$^{(r)}$	*4.8345e-1*$^{(-)}$
		2	6.1291e+0$^{(o)}$	**6.1292e+0**$^{(r)}$	6.1275e+0$^{(o)}$	6.1277e+0$^{(o)}$	5.8528e+0$^{(-)}$	5.8955e+0$^{(-)}$	5.1615e+0$^{(-)}$	5.3884e+0$^{(-)}$	6.0764e+0$^{(-)}$
		3	5.6272e+0$^{(o)}$	5.6275e+0$^{(o)}$	**5.6283e+0**$^{(r)}$	5.6264e+0$^{(-)}$	5.3696e+0$^{(-)}$	5.3971e+0$^{(-)}$	4.6879e+0$^{(-)}$	4.8996e+0$^{(-)}$	5.5792e+0$^{(-)}$
		4	3.3604e+0$^{(-)}$	3.3613e+0$^{(o)}$	3.3605e+0$^{(-)}$	3.3604e+0$^{(-)}$	**3.3616e+0**$^{(r)}$	3.3612e+0$^{(o)}$	3.3610e+0$^{(o)}$	3.3613e+0$^{(o)}$	3.3612e+0$^{(o)}$
		5	**2.9833e+0**$^{(r)}$	2.9826e+0$^{(o)}$	2.9827e+0$^{(o)}$	2.9826e+0$^{(o)}$	2.9801e+0$^{(-)}$	2.9812e+0$^{(o)}$	2.9830e+0$^{(o)}$	2.9798e+0$^{(-)}$	2.9808e+0$^{(-)}$
		6	3.3268e+0$^{(o)}$	3.3252e+0$^{(-)}$	3.3252e+0$^{(o)}$	3.3270e+0$^{(o)}$	3.3268e+0$^{(o)}$	**3.3282e+0**$^{(r)}$	3.3275e+0$^{(o)}$	3.3267e+0$^{(o)}$	3.3254e+0$^{(-)}$
		7	2.4635e+0$^{(o)}$	2.4575e+0$^{(o)}$	2.4186e+0$^{(-)}$	2.4857e+0$^{(o)}$	2.6022e+0$^{(o)}$	**2.6686e+0**$^{(r)}$	2.4639e+0$^{(-)}$	2.4911e+0$^{(o)}$	2.4343e+0$^{(o)}$
		8	3.0750e+0$^{(-)}$	3.1760e+0$^{(-)}$	3.0814e+0$^{(-)}$	3.1725e+0$^{(-)}$	3.1166e+0$^{(-)}$	3.1231e+0$^{(-)}$	**3.2060e+0**$^{(r)}$	2.9180e+0$^{(-)}$	2.9277e+0$^{(-)}$
		9	1.9976e+0$^{(-)}$	2.0367e+0$^{(-)}$	1.8903e+0$^{(-)}$	2.0011e+0$^{(-)}$	2.5635e+0$^{(-)}$	2.7313e+0$^{(o)}$	**2.8428e+0**$^{(r)}$	1.9892e+0$^{(-)}$	2.0643e+0$^{(-)}$

Table E.4.: Median IGD and HV values for Ref_2 with the nine grouping methods for the LSMOP and WFG test suite and $n = 200$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	1	4.1708e-2$^{(o)}$	4.0746e-2$^{(o)}$	4.9521e-2$^{(o)}$	**3.9865e-2**$^{(r)}$	1.3970e-1$^{(-)}$	1.4154e-1$^{(-)}$	4.6014e-2$^{(o)}$	*3.8801e-1*$^{(-)}$	4.3492e-2$^{(o)}$
		2	**2.0698e-2**$^{(r)}$	2.1550e-2$^{(-)}$	5.0271e-2$^{(-)}$	5.0365e-2$^{(-)}$	3.2586e-2$^{(-)}$	3.2788e-2$^{(-)}$	5.0533e-2$^{(-)}$	*9.0668e-2*$^{(-)}$	7.8313e-2$^{(-)}$
		3	7.2281e-1$^{(-)}$	**7.0383e-1**$^{(r)}$	7.8059e-1$^{(-)}$	7.8600e-1$^{(-)}$	9.5222e-1$^{(-)}$	8.5948e-1$^{(-)}$	7.9055e-1$^{(-)}$	*1.3117e+0*$^{(-)}$	8.4702e-1$^{(-)}$
		4	1.1392e-1$^{(-)}$	1.2347e-1$^{(-)}$	1.9582e-2$^{(-)}$	1.9792e-2$^{(-)}$	**1.7460e-2**$^{(r)}$	2.0585e-2$^{(-)}$	1.8771e-2$^{(-)}$	*1.3823e-1*$^{(-)}$	4.7629e-2$^{(-)}$
		5	7.4214e-1$^{(-)}$	7.4214e-1$^{(-)}$	7.4215e-1$^{(-)}$	7.4212e-1$^{(o)}$	*7.4235e-1*$^{(-)}$	7.4234e-1$^{(-)}$	7.4214e-1$^{(-)}$	**5.5845e-1**$^{(r)}$	7.4216e-1$^{(-)}$
		6	**4.0873e-1**$^{(r)}$	4.1677e-1$^{(o)}$	4.4172e-1$^{(o)}$	4.7271e-1$^{(o)}$	*7.5622e-1*$^{(-)}$	7.2720e-1$^{(-)}$	5.0653e-1$^{(-)}$	5.2750e-1$^{(-)}$	5.6681e-1$^{(-)}$
		7	1.3835e+0$^{(o)}$	2.1501e+0$^{(-)}$	**1.3587e+0**$^{(r)}$	1.8201e+0$^{(-)}$	2.1555e+0$^{(-)}$	2.2251e+0$^{(-)}$	*2.2936e+0*$^{(-)}$	1.7477e+0$^{(-)}$	2.0903e+0$^{(-)}$
		8	2.0250e-1$^{(o)}$	2.0920e-1$^{(o)}$	1.9654e-1$^{(o)}$	1.9737e-1$^{(o)}$	3.4422e-1$^{(-)}$	3.6138e-1$^{(-)}$	**1.8643e-1**$^{(r)}$	*3.6432e-1*$^{(-)}$	1.9354e-1$^{(o)}$
		9	3.9139e-1$^{(-)}$	3.9608e-1$^{(-)}$	4.7749e-2$^{(o)}$	**4.0729e-2**$^{(r)}$	6.5119e-2$^{(-)}$	8.9466e-2$^{(-)}$	4.3810e-2$^{(o)}$	*6.4610e-1*$^{(-)}$	1.0785e-1$^{(-)}$
	HV	1	6.4795e-1$^{(o)}$	6.4736e-1$^{(o)}$	6.3661e-1$^{(o)}$	**6.4972e-1**$^{(r)}$	5.1006e-1$^{(-)}$	5.0702e-1$^{(-)}$	6.4254e-1$^{(o)}$	*3.2784e-1*$^{(-)}$	6.4289e-1$^{(o)}$
		2	**6.7571e-1**$^{(r)}$	6.7447e-1$^{(-)}$	6.2626e-1$^{(-)}$	6.2651e-1$^{(-)}$	6.5570e-1$^{(-)}$	6.5493e-1$^{(-)}$	6.2628e-1$^{(-)}$	*5.6878e-1*$^{(-)}$	5.8530e-1$^{(-)}$
		3	**8.5679e-2**$^{(r)}$	7.7373e-2$^{(o)}$	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$	—$^{(-)}$
		4	5.3159e-1$^{(-)}$	5.1939e-1$^{(-)}$	6.7333e-1$^{(-)}$	6.7272e-1$^{(-)}$	**6.7795e-1**$^{(r)}$	6.7100e-1$^{(-)}$	6.7466e-1$^{(-)}$	*5.0469e-1*$^{(-)}$	6.2374e-1$^{(-)}$
		5	1.0980e-1$^{(-)}$	1.0983e-1$^{(-)}$	1.0983e-1$^{(-)}$	1.0989e-1$^{(o)}$	*1.0918e-1*$^{(-)}$	1.0920e-1$^{(-)}$	1.0983e-1$^{(-)}$	**1.1371e-1**$^{(r)}$	1.0976e-1$^{(-)}$
		6	**8.0354e-2**$^{(r)}$	6.1649e-2$^{(-)}$	2.1618e-2$^{(-)}$	8.8879e-3$^{(-)}$	6.5976e-2$^{(-)}$	5.3938e-2$^{(-)}$	2.8176e-4$^{(-)}$	—$^{(-)}$	—$^{(-)}$
		7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		8	1.8843e-1$^{(o)}$	1.9095e-1$^{(o)}$	1.9988e-1$^{(o)}$	1.8020e-1$^{(o)}$	1.3437e-1$^{(o)}$	1.1006e-1$^{(-)}$	2.0271e-1$^{(o)}$	*1.1000e-1*$^{(-)}$	**2.0966e-1**$^{(r)}$
		9	3.8412e-1$^{(-)}$	3.7923e-1$^{(-)}$	9.3436e-1$^{(o)}$	**9.4300e-1**$^{(r)}$	8.8806e-1$^{(-)}$	8.6941e-1$^{(-)}$	9.3694e-1$^{(o)}$	*2.8095e-1*$^{(-)}$	8.2008e-1$^{(-)}$
WFG	IGD	1	1.1425e+0$^{(-)}$	1.1575e+0$^{(-)}$	1.1671e+0$^{(-)}$	1.1567e+0$^{(-)}$	8.2091e-1$^{(-)}$	8.1476e-1$^{(-)}$	1.1713e+0$^{(-)}$	**4.3943e-1**$^{(r)}$	*1.2759e+0*$^{(-)}$
		2	1.4582e-2$^{(o)}$	1.4743e-2$^{(o)}$	1.5088e-2$^{(o)}$	**1.4394e-2**$^{(r)}$	1.7438e-1$^{(-)}$	1.6831e-1$^{(-)}$	*2.3048e-1*$^{(-)}$	1.0794e-1$^{(-)}$	2.3249e-2$^{(-)}$
		3	1.3518e-2$^{(o)}$	1.3446e-2$^{(o)}$	**1.3374e-2**$^{(r)}$	1.3532e-2$^{(o)}$	1.8317e-1$^{(-)}$	1.7431e-1$^{(-)}$	*2.5753e-1*$^{(-)}$	1.2146e-1$^{(-)}$	2.8209e-2$^{(-)}$
		4	1.3984e-2$^{(o)}$	1.4061e-2$^{(o)}$	**1.3927e-2**$^{(r)}$	1.4109e-2$^{(o)}$	1.4190e-2$^{(-)}$	1.4192e-2$^{(-)}$	1.3993e-2$^{(o)}$	1.4168e-2$^{(-)}$	*1.4405e-2*$^{(-)}$
		5	6.6503e-2$^{(-)}$	6.6671e-2$^{(-)}$	6.7780e-2$^{(-)}$	6.9691e-2$^{(-)}$	6.4049e-2$^{(-)}$	6.3903e-2$^{(-)}$	6.5164e-2$^{(-)}$	**6.3752e-2**$^{(r)}$	*6.9715e-2*$^{(-)}$
		6	1.4851e-2$^{(-)}$	1.4900e-2$^{(-)}$	1.4998e-2$^{(-)}$	1.4828e-2$^{(-)}$	1.5716e-2$^{(-)}$	1.5823e-2$^{(-)}$	*1.5985e-2*$^{(-)}$	**1.4721e-2**$^{(r)}$	1.5636e-2$^{(-)}$
		7	*2.9674e-2*$^{(-)}$	2.8874e-2$^{(-)}$	2.7075e-2$^{(-)}$	2.8966e-2$^{(-)}$	1.7377e-2$^{(-)}$	1.7883e-2$^{(-)}$	2.7258e-2$^{(-)}$	**1.3616e-2**$^{(r)}$	1.3751e-2$^{(o)}$
		8	1.7617e-1$^{(-)}$	1.5651e-1$^{(-)}$	1.7815e-1$^{(-)}$	1.5515e-1$^{(-)}$	5.4534e-2$^{(-)}$	5.4638e-2$^{(-)}$	**5.0576e-2**$^{(r)}$	1.8382e-1$^{(-)}$	*1.8552e-1*$^{(-)}$
		9	2.8400e-2$^{(-)}$	2.6177e-2$^{(o)}$	2.5588e-2$^{(o)}$	**2.3841e-2**$^{(r)}$	2.8217e-2$^{(-)}$	2.8471e-2$^{(-)}$	3.5766e-2$^{(-)}$	*3.9913e-2*$^{(-)}$	3.0428e-2$^{(-)}$
	HV	1	1.6298e+0$^{(-)}$	1.5749e+0$^{(-)}$	1.5350e+0$^{(-)}$	1.5648e+0$^{(-)}$	2.8247e+0$^{(-)}$	2.8440e+0$^{(-)}$	1.4786e+0$^{(-)}$	**4.5919e+0**$^{(r)}$	1.1392e+0$^{(-)}$
		2	6.1183e+0$^{(o)}$	6.1182e+0$^{(o)}$	**6.1207e+0**$^{(r)}$	6.1128e+0$^{(o)}$	5.1906e+0$^{(-)}$	5.2220e+0$^{(-)}$	4.8978e+0$^{(-)}$	5.5453e+0$^{(-)}$	6.0434e+0$^{(-)}$
		3	5.6245e+0$^{(o)}$	5.6210e+0$^{(o)}$	**5.6271e+0**$^{(r)}$	5.6218e+0$^{(o)}$	4.7534e+0$^{(-)}$	4.7961e+0$^{(-)}$	4.4063e+0$^{(-)}$	5.0490e+0$^{(-)}$	5.5168e+0$^{(-)}$
		4	3.3547e+0$^{(-)}$	3.3546e+0$^{(-)}$	3.3543e+0$^{(-)}$	3.3545e+0$^{(-)}$	3.3522e+0$^{(-)}$	3.3524e+0$^{(-)}$	3.3548e+0$^{(o)}$	**3.3554e+0**$^{(r)}$	3.3475e+0$^{(-)}$
		5	3.0029e+0$^{(-)}$	2.9988e+0$^{(-)}$	2.9904e+0$^{(-)}$	2.9819e+0$^{(-)}$	3.0292e+0$^{(-)}$	3.0280e+0$^{(-)}$	3.0125e+0$^{(-)}$	**3.0341e+0**$^{(r)}$	2.9826e+0$^{(-)}$
		6	3.3419e+0$^{(-)}$	3.3412e+0$^{(-)}$	3.3415e+0$^{(-)}$	3.3413e+0$^{(-)}$	3.3285e+0$^{(-)}$	3.3289e+0$^{(-)}$	3.3274e+0$^{(-)}$	**3.3442e+0**$^{(r)}$	3.3339e+0$^{(-)}$
		7	3.2363e+0$^{(-)}$	3.2412e+0$^{(-)}$	3.2517e+0$^{(-)}$	3.2431e+0$^{(-)}$	3.3214e+0$^{(-)}$	3.3174e+0$^{(-)}$	3.2575e+0$^{(-)}$	**3.3612e+0**$^{(r)}$	3.3612e+0$^{(o)}$
		8	2.4512e+0$^{(-)}$	2.5524e+0$^{(-)}$	2.4400e+0$^{(-)}$	2.5638e+0$^{(-)}$	3.1003e+0$^{(-)}$	3.1008e+0$^{(-)}$	**3.1370e+0**$^{(r)}$	2.4123e+0$^{(-)}$	2.4059e+0$^{(-)}$
		9	3.2562e+0$^{(o)}$	**3.2640e+0**$^{(r)}$	3.2586e+0$^{(o)}$	3.2527e+0$^{(o)}$	3.2218e+0$^{(-)}$	3.2199e+0$^{(-)}$	3.2037e+0$^{(-)}$	3.2200e+0$^{(-)}$	3.2445e+0$^{(o)}$

Table E.5.: Median IGD and HV values for Ref_1 with the nine grouping methods for the LSMOP and WFG test suite and $n = 500$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	1	**1.1531e-2**$^{(r)}$	1.3105e-2$^{(o)}$	1.2651e-2$^{(o)}$	1.2684e-2$^{(o)}$	4.1822e-2$^{(-)}$	3.9152e-2$^{(-)}$	1.2696e-2$^{(o)}$	*3.4528e-1*$^{(-)}$	1.5292e-2$^{(-)}$
		2	2.0292e-2$^{(-)}$	2.0549e-2$^{(-)}$	1.4511e-2$^{(o)}$	**1.4505e-2**$^{(r)}$	1.6756e-2$^{(-)}$	1.5745e-2$^{(-)}$	1.4761e-2$^{(o)}$	*4.7136e-2*$^{(-)}$	1.6949e-2$^{(-)}$
		3	*7.5657e-1*$^{(-)}$	7.3110e-1$^{(-)}$	**3.8550e-1**$^{(r)}$	7.2370e-1$^{(-)}$	7.4425e-1$^{(-)}$	7.3945e-1$^{(-)}$	4.0118e-1$^{(o)}$	7.5549e-1$^{(-)}$	6.8197e-1$^{(-)}$
		4	5.6787e-2$^{(-)}$	5.6332e-2$^{(-)}$	2.0417e-2$^{(-)}$	2.0623e-2$^{(-)}$	**1.8695e-2**$^{(r)}$	2.2789e-2$^{(-)}$	1.9743e-2$^{(-)}$	*7.1289e-2*$^{(-)}$	3.0729e-2$^{(-)}$
		5	6.4406e-1$^{(o)}$	5.4559e-1$^{(o)}$	**4.5672e-1**$^{(r)}$	4.7360e-1$^{(o)}$	*7.4213e-1*$^{(-)}$	7.4211e-1$^{(-)}$	5.5545e-1$^{(o)}$	7.4205e-1$^{(o)}$	5.1019e-1$^{(o)}$
		6	4.9515e-1$^{(-)}$	4.1686e-1$^{(o)}$	4.0106e-1$^{(o)}$	4.0887e-1$^{(o)}$	3.9131e-1$^{(o)}$	**3.4556e-1**$^{(r)}$	3.7258e-1$^{(o)}$	*1.0770e+0*$^{(-)}$	3.8880e-1$^{(o)}$
		7	*1.6219e+0*$^{(-)}$	1.4748e+0$^{(o)}$	1.5072e+0$^{(-)}$	1.4704e+0$^{(o)}$	1.4781e+0$^{(o)}$	1.4813e+0$^{(o)}$	**1.4544e+0**$^{(r)}$	1.6017e+0$^{(-)}$	1.4727e+0$^{(o)}$
		8	**3.4902e-2**$^{(r)}$	3.8289e-2$^{(-)}$	3.5635e-2$^{(o)}$	3.6743e-2$^{(-)}$	8.9130e-2$^{(-)}$	8.6469e-2$^{(-)}$	3.6153e-2$^{(o)}$	*3.5200e-1*$^{(-)}$	3.5673e-2$^{(o)}$
		9	5.0278e-1$^{(-)}$	5.0335e-1$^{(-)}$	4.5686e-1$^{(o)}$	4.5647e-1$^{(o)}$	**3.2566e-1**$^{(r)}$	4.7365e-1$^{(o)}$	4.5668e-1$^{(o)}$	*6.5042e-1*$^{(-)}$	4.7511e-1$^{(o)}$
	HV	1	**6.8504e-1**$^{(r)}$	6.8333e-1$^{(o)}$	6.8377e-1$^{(o)}$	6.8413e-1$^{(o)}$	6.3328e-1$^{(-)}$	6.4046e-1$^{(-)}$	6.8483e-1$^{(o)}$	*2.4915e-1*$^{(-)}$	6.7847e-1$^{(-)}$
		2	6.7431e-1$^{(-)}$	6.7393e-1$^{(-)}$	6.8406e-1$^{(o)}$	**6.8420e-1**$^{(r)}$	6.8021e-1$^{(-)}$	6.8185e-1$^{(-)}$	6.8362e-1$^{(o)}$	*6.3276e-1*$^{(-)}$	6.7975e-1$^{(-)}$
		3	1.9245e-2$^{(-)}$	6.3999e-2$^{(o)}$	7.8136e-2$^{(o)}$	6.0408e-2$^{(-)}$	3.6190e-2$^{(-)}$	4.5345e-2$^{(-)}$	**8.3427e-2**$^{(r)}$	*1.2354e-2*$^{(-)}$	5.1583e-2$^{(-)}$
		4	6.1697e-1$^{(-)}$	6.1776e-1$^{(-)}$	6.7140e-1$^{(o)}$	6.7108e-1$^{(-)}$	**6.7419e-1**$^{(r)}$	6.6692e-1$^{(-)}$	6.7210e-1$^{(-)}$	*5.9548e-1*$^{(-)}$	6.5430e-1$^{(-)}$
		5	1.0979e-1$^{(-)}$	1.0980e-1$^{(-)}$	1.0994e-1$^{(o)}$	1.0988e-1$^{(-)}$	*1.0977e-1*$^{(-)}$	1.0983e-1$^{(-)}$	1.0989e-1$^{(-)}$	**1.1000e-1**$^{(r)}$	1.0984e-1$^{(-)}$
		6	—$^{(-)}$	1.7152e-2$^{(-)}$	**1.0814e-1**$^{(r)}$	1.0188e-1$^{(-)}$	5.8659e-2$^{(-)}$	5.9639e-2$^{(-)}$	6.4274e-2$^{(-)}$	—$^{(-)}$	3.7661e-2$^{(-)}$
		7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		8	**3.6704e-1**$^{(r)}$	3.6156e-1$^{(-)}$	3.6638e-1$^{(o)}$	3.6335e-1$^{(-)}$	3.2081e-1$^{(-)}$	3.2488e-1$^{(-)}$	3.6619e-1$^{(o)}$	*1.4758e-1*$^{(-)}$	3.6592e-1$^{(o)}$
		9	6.5834e-1$^{(-)}$	6.5596e-1$^{(-)}$	7.1868e-1$^{(o)}$	7.1994e-1$^{(o)}$	**7.2159e-1**$^{(r)}$	7.0162e-1$^{(o)}$	7.1975e-1$^{(o)}$	*5.0020e-1*$^{(-)}$	6.9877e-1$^{(o)}$
WFG	IGD	1	1.2568e+0$^{(-)}$	1.2832e+0$^{(-)}$	1.2790e+0$^{(-)}$	1.2265e+0$^{(-)}$	1.1700e+0$^{(-)}$	1.1210e+0$^{(-)}$	1.2795e+0$^{(-)}$	**1.0809e+0**$^{(r)}$	*1.3287e+0*$^{(-)}$
		2	1.3092e-2$^{(o)}$	1.3753e-2$^{(-)}$	**1.3024e-2**$^{(r)}$	1.3234e-2$^{(o)}$	6.1446e-2$^{(-)}$	5.7126e-2$^{(-)}$	*1.8934e-1*$^{(-)}$	1.6729e-1$^{(-)}$	1.8657e-2$^{(-)}$
		3	1.2923e-2$^{(o)}$	1.3065e-2$^{(o)}$	1.3049e-2$^{(o)}$	**1.2857e-2**$^{(r)}$	6.1244e-2$^{(-)}$	6.0081e-2$^{(-)}$	*2.0929e-1*$^{(-)}$	1.8900e-1$^{(-)}$	1.7713e-2$^{(-)}$
		4	1.3747e-2$^{(o)}$	1.3755e-2$^{(o)}$	1.3740e-2$^{(o)}$	1.3893e-2$^{(o)}$	**1.3672e-2**$^{(r)}$	1.3854e-2$^{(o)}$	1.3899e-2$^{(o)}$	1.3711e-2$^{(o)}$	*1.4119e-2*$^{(-)}$
		5	6.8932e-2$^{(o)}$	6.8918e-2$^{(o)}$	6.9105e-2$^{(o)}$	**6.8696e-2**$^{(r)}$	6.9594e-2$^{(-)}$	*6.9721e-2*$^{(-)}$	6.9158e-2$^{(o)}$	6.9585e-2$^{(-)}$	6.8938e-2$^{(o)}$
		6	1.4054e-2$^{(o)}$	1.3943e-2$^{(o)}$	1.4014e-2$^{(o)}$	1.4008e-2$^{(o)}$	1.4006e-2$^{(o)}$	**1.3888e-2**$^{(r)}$	1.4056e-2$^{(-)}$	*1.4088e-2*$^{(o)}$	1.3956e-2$^{(o)}$
		7	1.4630e-1$^{(o)}$	1.5038e-1$^{(o)}$	*1.6931e-1*$^{(o)}$	1.6852e-1$^{(o)}$	1.6736e-1$^{(o)}$	**1.3793e-1**$^{(r)}$	1.5211e-1$^{(o)}$	1.5712e-1$^{(o)}$	1.4661e-1$^{(o)}$
		8	3.1999e-2$^{(-)}$	**2.8175e-2**$^{(r)}$	3.1873e-2$^{(-)}$	2.9418e-2$^{(o)}$	4.2751e-2$^{(-)}$	3.9090e-2$^{(-)}$	2.8557e-2$^{(o)}$	*7.2780e-2*$^{(-)}$	7.2773e-2$^{(-)}$
		9	2.4502e-1$^{(-)}$	2.5224e-1$^{(-)}$	2.5035e-1$^{(-)}$	2.5278e-1$^{(-)}$	1.0848e-1$^{(o)}$	**7.9867e-2**$^{(r)}$	8.8713e-2$^{(o)}$	2.5177e-1$^{(-)}$	*2.5363e-1*$^{(-)}$
	HV	1	1.3098e+0$^{(-)}$	1.1857e+0$^{(-)}$	1.1894e+0$^{(-)}$	1.4029e+0$^{(-)}$	2.0144e+0$^{(-)}$	2.1959e+0$^{(-)}$	1.2116e+0$^{(-)}$	**2.9766e+0**$^{(r)}$	*9.8491e-1*$^{(-)}$
		2	6.1315e+0$^{(o)}$	6.1305e+0$^{(-)}$	**6.1315e+0**$^{(r)}$	6.1311e+0$^{(o)}$	5.8083e+0$^{(-)}$	5.8318e+0$^{(-)}$	5.1103e+0$^{(-)}$	5.2278e+0$^{(-)}$	6.0736e+0$^{(-)}$
		3	5.6309e+0$^{(o)}$	5.6303e+0$^{(o)}$	5.6302e+0$^{(o)}$	**5.6309e+0**$^{(r)}$	5.3439e+0$^{(-)}$	5.3506e+0$^{(-)}$	4.6314e+0$^{(-)}$	4.7276e+0$^{(-)}$	5.5811e+0$^{(-)}$
		4	3.3607e+0$^{(-)}$	3.3613e+0$^{(-)}$	3.3613e+0$^{(-)}$	3.3617e+0$^{(o)}$	3.3611e+0$^{(-)}$	3.3614e+0$^{(o)}$	3.3615e+0$^{(-)}$	**3.3621e+0**$^{(r)}$	3.3620e+0$^{(o)}$
		5	2.9847e+0$^{(o)}$	2.9854e+0$^{(o)}$	2.9849e+0$^{(o)}$	**2.9858e+0**$^{(r)}$	2.9814e+0$^{(-)}$	2.9814e+0$^{(-)}$	2.9842e+0$^{(o)}$	2.9813e+0$^{(-)}$	2.9847e+0$^{(o)}$
		6	3.3485e+0$^{(o)}$	**3.3489e+0**$^{(r)}$	3.3482e+0$^{(o)}$	3.3482e+0$^{(o)}$	3.3478e+0$^{(-)}$	3.3478e+0$^{(-)}$	3.3484e+0$^{(o)}$	3.3479e+0$^{(o)}$	3.3482e+0$^{(-)}$
		7	2.6160e+0$^{(o)}$	2.5812e+0$^{(o)}$	2.4917e+0$^{(o)}$	2.4887e+0$^{(o)}$	2.4956e+0$^{(o)}$	**2.6559e+0**$^{(r)}$	2.5688e+0$^{(o)}$	2.5648e+0$^{(o)}$	2.6062e+0$^{(o)}$
		8	3.2220e+0$^{(-)}$	3.2480e+0$^{(o)}$	3.2293e+0$^{(-)}$	3.2481e+0$^{(o)}$	3.1681e+0$^{(-)}$	3.1906e+0$^{(-)}$	**3.2501e+0**$^{(r)}$	2.9930e+0$^{(-)}$	2.9871e+0$^{(-)}$
		9	2.1008e+0$^{(-)}$	2.0464e+0$^{(-)}$	2.0722e+0$^{(-)}$	2.0631e+0$^{(-)}$	2.8769e+0$^{(o)}$	**3.0129e+0**$^{(r)}$	2.8709e+0$^{(o)}$	2.0606e+0$^{(-)}$	2.0525e+0$^{(-)}$

Table E.6.: Median IGD and HV values for Ref_2 with the nine grouping methods for the LSMOP and WFG test suite and $n = 500$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	1	3.3420e-2$^{(o)}$	3.1687e-2$^{(o)}$	3.7860e-2$^{(o)}$	**2.8595e-2**$^{(r)}$	1.2934e-1$^{(-)}$	1.0240e-1$^{(-)}$	3.2538e-2$^{(o)}$	*3.6788e-1*$^{(-)}$	3.1826e-2$^{(o)}$
		2	2.7682e-2$^{(-)}$	2.7397e-2$^{(-)}$	2.6139e-2$^{(-)}$	2.6369e-2$^{(-)}$	**1.7149e-2**$^{(r)}$	1.7937e-2$^{(o)}$	2.6274e-2$^{(-)}$	*4.2169e-2*$^{(-)}$	3.0264e-2$^{(-)}$
		3	7.0807e-1$^{(-)}$	5.0581e-1$^{(-)}$	4.6396e-1$^{(o)}$	4.7960e-1$^{(o)}$	7.6976e-1$^{(-)}$	7.1116e-1$^{(-)}$	**4.4937e-1**$^{(r)}$	*1.6800e+0*$^{(-)}$	5.9330e-1$^{(-)}$
		4	6.2552e-2$^{(-)}$	6.3717e-2$^{(-)}$	6.1712e-3$^{(-)}$	6.4966e-3$^{(-)}$	**5.9408e-3**$^{(r)}$	6.3570e-3$^{(-)}$	6.4036e-3$^{(-)}$	*7.9941e-2*$^{(-)}$	1.4496e-2$^{(-)}$
		5	7.4212e-1$^{(-)}$	7.4211e-1$^{(-)}$	7.4212e-1$^{(-)}$	7.4212e-1$^{(-)}$	*7.4224e-1*$^{(-)}$	7.4222e-1$^{(-)}$	7.4212e-1$^{(-)}$	**3.4246e-1**$^{(r)}$	7.4212e-1$^{(-)}$
		6	3.9854e-1$^{(o)}$	4.3118e-1$^{(-)}$	**3.8878e-1**$^{(r)}$	4.0529e-1$^{(o)}$	4.2839e-1$^{(-)}$	4.2722e-1$^{(-)}$	4.1407e-1$^{(o)}$	*4.3914e-1*$^{(-)}$	4.2938e-1$^{(-)}$
		7	1.4452e+0$^{(o)}$	1.6987e+0$^{(-)}$	**1.3643e+0**$^{(r)}$	1.7426e+0$^{(-)}$	1.9120e+0$^{(-)}$	2.0266e+0$^{(-)}$	*2.0646e+0*$^{(-)}$	1.5802e+0$^{(-)}$	1.9091e+0$^{(-)}$
		8	2.0016e-1$^{(o)}$	1.8425e-1$^{(o)}$	**1.7696e-1**$^{(r)}$	1.8504e-1$^{(o)}$	*3.5241e-1*$^{(-)}$	3.5185e-1$^{(-)}$	1.8183e-1$^{(o)}$	3.5210e-1$^{(-)}$	2.0610e-1$^{(o)}$
		9	1.7462e-1$^{(-)}$	1.7179e-1$^{(-)}$	2.5060e-2$^{(o)}$	3.3474e-2$^{(-)}$	**2.2770e-2**$^{(r)}$	4.9701e-2$^{(-)}$	2.7901e-2$^{(o)}$	*3.4693e-1*$^{(-)}$	2.9751e-2$^{(o)}$
	HV	1	6.6175e-1$^{(o)}$	6.6393e-1$^{(o)}$	6.5529e-1$^{(o)}$	**6.6711e-1**$^{(r)}$	5.2626e-1$^{(-)}$	5.6286e-1$^{(-)}$	6.6332e-1$^{(o)}$	*3.4174e-1*$^{(-)}$	6.6365e-1$^{(o)}$
		2	6.6225e-1$^{(-)}$	6.6269e-1$^{(-)}$	6.6490e-1$^{(-)}$	6.6438e-1$^{(-)}$	**6.8068e-1**$^{(r)}$	6.7914e-1$^{(-)}$	6.6460e-1$^{(-)}$	*6.4041e-1*$^{(-)}$	6.5826e-1$^{(-)}$
		3	1.0851e-1$^{(-)}$	**1.6409e-1**$^{(r)}$	1.1706e-1$^{(-)}$	1.0933e-1$^{(-)}$	1.8470e-2$^{(-)}$	1.0374e-1$^{(-)}$	1.2731e-1$^{(-)}$	—$^{(-)}$	1.0098e-1$^{(-)}$
		4	6.0987e-1$^{(-)}$	6.0831e-1$^{(-)}$	6.9937e-1$^{(-)}$	6.9878e-1$^{(-)}$	**6.9994e-1**$^{(r)}$	6.9885e-1$^{(-)}$	6.9906e-1$^{(-)}$	*5.8443e-1*$^{(-)}$	6.8325e-1$^{(-)}$
		5	1.0993e-1$^{(-)}$	1.0993e-1$^{(-)}$	1.0992e-1$^{(-)}$	1.0992e-1$^{(-)}$	*1.0956e-1*$^{(-)}$	1.0961e-1$^{(-)}$	1.0993e-1$^{(-)}$	**2.0950e-1**$^{(r)}$	1.0990e-1$^{(-)}$
		6	**8.1130e-2**$^{(r)}$	8.0274e-2$^{(o)}$	6.6705e-2$^{(-)}$	6.8552e-2$^{(-)}$	7.7108e-2$^{(-)}$	7.0148e-2$^{(-)}$	6.2921e-2$^{(-)}$	*4.7903e-2*$^{(-)}$	5.3675e-2$^{(-)}$
		7	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$	—$^{(*)}$
		8	2.2788e-1$^{(o)}$	2.3132e-1$^{(o)}$	**2.4040e-1**$^{(r)}$	2.2723e-1$^{(o)}$	*1.5022e-1*$^{(-)}$	1.5295e-1$^{(-)}$	2.2727e-1$^{(o)}$	1.5221e-1$^{(-)}$	2.0927e-1$^{(o)}$
		9	6.8592e-1$^{(-)}$	6.8558e-1$^{(-)}$	**9.8438e-1**$^{(r)}$	9.7266e-1$^{(-)}$	9.7847e-1$^{(o)}$	9.5634e-1$^{(-)}$	9.8236e-1$^{(o)}$	*4.3898e-1*$^{(-)}$	9.5590e-1$^{(-)}$
WFG	IGD	1	8.5918e-1$^{(-)}$	8.6530e-1$^{(-)}$	8.8230e-1$^{(-)}$	8.7295e-1$^{(-)}$	4.6989e-1$^{(-)}$	4.7081e-1$^{(-)}$	8.4001e-1$^{(-)}$	**2.9792e-1**$^{(r)}$	*9.7811e-1*$^{(-)}$
		2	**1.4088e-2**$^{(r)}$	1.4574e-2$^{(o)}$	1.4433e-2$^{(o)}$	1.4640e-2$^{(o)}$	2.1057e-1$^{(-)}$	1.9872e-1$^{(-)}$	*2.6049e-1*$^{(-)}$	1.1689e-1$^{(-)}$	2.9858e-2$^{(-)}$
		3	1.3566e-2$^{(o)}$	**1.3219e-2**$^{(r)}$	1.3266e-2$^{(o)}$	1.3349e-2$^{(o)}$	2.1588e-1$^{(-)}$	2.0209e-1$^{(-)}$	*2.8401e-1*$^{(-)}$	1.2892e-1$^{(-)}$	2.8697e-2$^{(-)}$
		4	1.3894e-2$^{(o)}$	1.3784e-2$^{(o)}$	1.3862e-2$^{(o)}$	1.3793e-2$^{(o)}$	*1.3921e-2*$^{(o)}$	**1.3782e-2**$^{(r)}$	1.3791e-2$^{(o)}$	1.3895e-2$^{(o)}$	1.3856e-2$^{(o)}$
		5	6.3978e-2$^{(-)}$	6.3921e-2$^{(-)}$	6.3769e-2$^{(-)}$	6.3706e-2$^{(-)}$	**6.3579e-2**$^{(r)}$	6.3597e-2$^{(o)}$	6.3922e-2$^{(-)}$	6.3787e-2$^{(-)}$	*6.4231e-2*$^{(-)}$
		6	1.4144e-2$^{(-)}$	1.4184e-2$^{(-)}$	1.4181e-2$^{(-)}$	1.4215e-2$^{(-)}$	1.4175e-2$^{(-)}$	1.4103e-2$^{(o)}$	1.4173e-2$^{(-)}$	**1.3965e-2**$^{(r)}$	*1.4532e-2*$^{(-)}$
		7	3.0626e-2$^{(-)}$	2.9983e-2$^{(-)}$	2.9010e-2$^{(-)}$	2.9821e-2$^{(-)}$	1.9377e-2$^{(-)}$	1.8572e-2$^{(-)}$	*3.0836e-2*$^{(-)}$	**1.3533e-2**$^{(r)}$	1.3638e-2$^{(o)}$
		8	6.1838e-2$^{(-)}$	5.8367e-2$^{(-)}$	6.2505e-2$^{(-)}$	5.6590e-2$^{(-)}$	4.3141e-2$^{(-)}$	4.3097e-2$^{(-)}$	**3.8266e-2**$^{(r)}$	1.4367e-1$^{(-)}$	*1.4397e-1*$^{(-)}$
		9	1.6196e-2$^{(o)}$	1.7316e-2$^{(o)}$	1.6602e-2$^{(o)}$	1.7215e-2$^{(-)}$	2.0973e-2$^{(-)}$	3.8276e-2$^{(-)}$	*4.9783e-2*$^{(-)}$	**1.5760e-2**$^{(r)}$	1.5910e-2$^{(o)}$
	HV	1	2.6630e+0$^{(-)}$	2.6340e+0$^{(-)}$	2.5714e+0$^{(-)}$	2.5994e+0$^{(-)}$	4.3511e+0$^{(-)}$	4.3402e+0$^{(-)}$	2.7188e+0$^{(-)}$	**5.2553e+0**$^{(r)}$	2.2025e+0$^{(-)}$
		2	6.1254e+0$^{(o)}$	**6.1292e+0**$^{(r)}$	6.1283e+0$^{(o)}$	6.1266e+0$^{(o)}$	5.0008e+0$^{(-)}$	5.0621e+0$^{(-)}$	4.7467e+0$^{(-)}$	5.5003e+0$^{(-)}$	6.0010e+0$^{(-)}$
		3	5.6270e+0$^{(o)}$	5.6278e+0$^{(o)}$	**5.6285e+0**$^{(r)}$	5.6265e+0$^{(o)}$	4.6005e+0$^{(-)}$	4.6651e+0$^{(-)}$	4.2803e+0$^{(-)}$	5.0133e+0$^{(-)}$	5.5143e+0$^{(-)}$
		4	3.3598e+0$^{(-)}$	3.3589e+0$^{(-)}$	3.3594e+0$^{(-)}$	3.3587e+0$^{(-)}$	3.3589e+0$^{(-)}$	3.3583e+0$^{(-)}$	3.3594e+0$^{(-)}$	**3.3616e+0**$^{(r)}$	3.3583e+0$^{(-)}$
		5	3.0311e+0$^{(-)}$	3.0307e+0$^{(-)}$	3.0309e+0$^{(-)}$	3.0296e+0$^{(-)}$	3.0333e+0$^{(-)}$	3.0334e+0$^{(-)}$	3.0286e+0$^{(-)}$	**3.0346e+0**$^{(r)}$	3.0264e+0$^{(-)}$
		6	3.3480e+0$^{(-)}$	3.3474e+0$^{(-)}$	3.3480e+0$^{(-)}$	3.3485e+0$^{(-)}$	3.3483e+0$^{(-)}$	3.3477e+0$^{(-)}$	3.3478e+0$^{(-)}$	**3.3553e+0**$^{(r)}$	3.3497e+0$^{(-)}$
		7	3.2265e+0$^{(-)}$	3.2325e+0$^{(-)}$	3.2412e+0$^{(-)}$	3.2373e+0$^{(-)}$	3.3024e+0$^{(-)}$	3.3099e+0$^{(-)}$	3.2248e+0$^{(-)}$	**3.3613e+0**$^{(r)}$	3.3611e+0$^{(o)}$
		8	3.0483e+0$^{(-)}$	3.0685e+0$^{(-)}$	3.0467e+0$^{(-)}$	3.0791e+0$^{(-)}$	3.1499e+0$^{(-)}$	3.1535e+0$^{(-)}$	**3.1861e+0**$^{(r)}$	2.6178e+0$^{(-)}$	2.6144e+0$^{(-)}$
		9	3.3367e+0$^{(o)}$	3.3290e+0$^{(o)}$	3.3397e+0$^{(o)}$	3.3331e+0$^{(o)}$	3.2671e+0$^{(-)}$	3.1927e+0$^{(-)}$	3.1193e+0$^{(-)}$	**3.3410e+0**$^{(r)}$	3.3399e+0$^{(o)}$

Table E.7.: IQR values of IGD and HV for Ref_1 with the nine grouping methods for the LSMOP and WFG test suite and 100 decision variables

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	P1	1.8613e-3	2.4253e-3	3.9458e-3	2.9737e-3	1.1298e-2	1.5432e-2	3.0890e-3	1.3357e-1	4.3827e-3
		P2	4.3432e-3	3.0429e-3	1.9557e-3	2.6788e-3	3.4569e-3	3.2974e-3	3.0211e-3	1.3088e-2	2.3382e-3
		P3	1.4010e-1	4.4587e-1	2.6642e-1	3.5120e-1	1.1979e-1	1.7298e-1	3.6490e-1	2.3399e-1	2.5870e-1
		P4	1.1085e-2	1.7123e-2	8.2148e-3	9.4498e-3	8.0815e-3	8.0478e-3	8.2477e-3	9.4183e-3	1.5218e-2
		P5	6.5918e-1	5.7146e-1	6.8840e-1	5.5120e-1	5.6419e-1	3.8186e-1	6.8079e-1	1.7726e-1	6.6883e-1
		P6	6.7556e-1	3.7882e-1	2.2332e-1	1.0120e-1	2.2477e-1	2.3844e-1	2.5830e-1	1.0288e+0	3.7356e-1
		P7	7.8980e-1	1.3994e-1	8.1471e-1	6.6176e-2	4.9723e-2	6.3112e-2	1.1607e-1	9.3581e-1	1.3998e-1
		P8	7.2256e-3	7.1406e-3	6.9977e-3	7.7912e-3	4.6373e-2	4.4984e-2	7.1416e-3	1.1240e-1	9.0458e-3
		P9	1.5762e-1	1.0197e-1	4.3721e-2	3.4156e-2	4.5985e-1	4.2288e-1	3.0010e-1	1.6786e-1	5.2411e-2
	HV	P1	2.5573e-3	5.7678e-3	8.7444e-3	7.0675e-3	2.0338e-2	2.3727e-2	6.5311e-3	1.2907e-1	7.1322e-3
		P2	5.5160e-3	4.7434e-3	2.9796e-3	5.0624e-3	6.1579e-3	5.7789e-3	5.9190e-3	1.7641e-2	3.3211e-3
		P3	—	3.2781e-2	4.5748e-2	9.4785e-2	—	—	5.9214e-2	—	—
		P4	1.0154e-2	1.7578e-2	1.0849e-2	1.3136e-2	1.3618e-2	1.2270e-2	1.2226e-2	1.1964e-2	2.4138e-2
		P5	2.4242e-1	1.6439e-1	2.6651e-1	1.5835e-1	1.6250e-1	7.5305e-2	2.6440e-1	5.5448e-4	2.6214e-1
		P6	—	1.0466e-1	2.2497e-2	6.2108e-3	6.4742e-2	5.4578e-2	9.6433e-2	—	9.0229e-2
		P7	—	—	—	—	—	—	—	—	—
		P8	8.2768e-3	1.0031e-2	6.8291e-3	5.9609e-3	2.8838e-2	4.1542e-2	7.6762e-3	3.6580e-2	6.4895e-3
		P9	1.5739e-1	9.9542e-2	4.6644e-2	3.8073e-2	1.9061e-1	1.7514e-1	1.2176e-1	1.3824e-1	5.8943e-2
WFG	IGD	P1	8.8250e-2	1.0165e-1	8.4489e-2	7.4186e-2	8.0765e-2	7.5147e-2	1.2366e-1	9.2393e-2	1.1168e-1
		P2	1.3458e-3	1.9117e-3	1.0359e-3	8.3219e-4	6.1030e-3	6.0070e-3	1.3190e-2	9.3043e-3	2.1209e-3
		P3	5.5295e-4	3.2943e-4	3.0162e-4	3.9280e-4	5.8151e-3	6.8316e-3	1.6562e-2	1.6000e-2	8.6068e-4
		P4	7.5777e-4	7.9474e-4	1.1924e-3	1.0510e-3	8.8021e-4	7.7411e-4	9.2226e-4	7.3066e-4	1.1067e-3
		P5	7.0009e-4	1.1999e-3	7.2489e-4	1.3992e-3	1.0172e-3	8.0231e-4	1.0020e-3	2.0822e-4	1.6713e-3
		P6	8.7077e-4	9.5905e-4	9.2686e-4	8.8502e-4	4.8900e-4	3.8462e-4	5.5717e-4	1.2668e-3	7.6518e-4
		P7	2.0016e-2	2.1667e-2	2.2801e-2	2.8894e-2	3.7697e-2	5.2842e-2	2.6189e-2	4.8222e-2	4.3551e-2
		P8	8.2576e-3	5.8549e-3	1.0481e-2	7.7915e-3	4.5300e-3	4.6666e-3	3.4742e-3	8.0886e-3	9.3204e-3
		P9	4.3259e-2	3.7597e-2	4.2718e-2	5.7988e-2	6.3710e-2	8.9771e-2	9.9887e-2	4.1419e-2	3.0449e-2
	HV	P1	3.9655e-1	3.8303e-1	3.6004e-1	3.4033e-1	3.9577e-1	5.3074e-1	4.8473e-1	4.7863e-1	5.1647e-1
		P2	1.7402e-3	2.1507e-3	1.4420e-3	1.5907e-3	3.8215e-2	3.8549e-2	8.1758e-2	5.1420e-2	2.4845e-2
		P3	2.6750e-3	1.4926e-3	1.0726e-3	1.8807e-3	3.3412e-2	3.8171e-2	7.8214e-2	8.0116e-2	1.3118e-2
		P4	1.8394e-3	1.8643e-3	2.0315e-3	1.8102e-3	1.5616e-3	1.2446e-3	1.4814e-3	1.6447e-3	2.4127e-3
		P5	5.0724e-3	6.7068e-3	4.9154e-3	7.0010e-3	5.4331e-3	4.4490e-3	5.4311e-3	1.5768e-3	8.4552e-3
		P6	6.2229e-3	7.5333e-3	6.0786e-3	6.1926e-3	2.3123e-3	3.6753e-3	4.1506e-3	8.3561e-3	6.4191e-3
		P7	1.0373e-1	1.2713e-1	1.2515e-1	1.8992e-1	2.3757e-1	2.9201e-1	1.3017e-1	2.5614e-1	2.1680e-1
		P8	5.3875e-2	3.4205e-2	5.1794e-2	5.2636e-2	1.4208e-2	3.1949e-2	1.5869e-2	5.3822e-2	4.8214e-2
		P9	2.1286e-1	2.0903e-1	2.2259e-1	3.0265e-1	3.7514e-1	4.6277e-1	5.5025e-1	2.0899e-1	1.7554e-1

Table E.8.: IQR values of IGD and HV for Ref_2 with the nine grouping methods for the LSMOP and WFG test suite and $n = 100$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	P1	2.4716e-2	2.7195e-2	2.3402e-2	2.4576e-2	5.0217e-2	5.6562e-2	2.9923e-2	3.2063e-2	2.1265e-2
		P2	2.0455e-3	1.9692e-3	4.2971e-3	4.5869e-3	1.9681e-3	1.8252e-3	4.6348e-3	7.0355e-3	2.7327e-3
		P3	1.3279e-1	2.5995e-1	1.4490e-1	1.2407e-1	2.0579e-2	1.1881e-1	1.3524e-1	7.9406e-1	2.0219e-1
		P4	1.1520e-2	8.4590e-3	3.5652e-3	3.8410e-3	2.7503e-3	3.7586e-3	3.0152e-3	1.4355e-2	1.6057e-2
		P5	3.2015e-1	1.8392e-1	1.1811e-1	1.5952e-1	4.3463e-3	1.8610e-3	2.3702e-1	3.9965e-1	3.2627e-1
		P6	1.4393e-1	2.1398e-1	2.4644e-1	1.5619e-1	2.5053e-1	2.7276e-1	2.0464e-1	4.0334e-2	1.3797e-1
		P7	1.0565e-1	7.8908e-1	7.4415e-1	1.0114e+0	7.2990e-1	7.1893e-1	7.4672e-1	9.8636e-2	7.7362e-1
		P8	1.1770e-1	1.4081e-1	6.7335e-2	9.3607e-2	4.6940e-2	7.4695e-2	1.7331e-1	3.6640e-2	1.3313e-1
		P9	1.0148e-1	1.0746e-1	1.2517e-1	1.0252e-1	4.3017e-2	1.8593e-1	5.4508e-2	1.2546e+0	9.3344e-2
	HV	P1	3.4125e-2	3.7521e-2	3.3149e-2	3.4260e-2	7.1223e-2	7.8852e-2	4.2600e-2	1.6115e-2	3.0633e-2
		P2	3.1269e-3	2.7620e-3	8.2898e-3	8.4030e-3	3.8812e-3	3.6089e-3	8.4290e-3	9.3953e-3	3.8593e-3
		P3	8.2115e-4	9.3720e-2	1.1137e-1	9.9279e-2	3.0899e-2	1.0959e-1	8.9910e-2	1.0900e-1	3.3678e-2
		P4	1.1872e-2	9.4848e-3	6.6323e-3	5.6650e-3	4.4806e-3	6.9457e-3	5.5458e-3	2.1744e-2	3.0707e-2
		P5	1.6713e-1	8.2215e-2	9.9789e-2	9.9675e-2	1.1440e-2	4.1625e-3	9.9642e-2	9.9999e-2	9.9888e-2
		P6	7.3189e-4	1.2337e-2	4.7879e-2	3.3281e-2	3.2733e-3	3.4064e-3	4.0472e-2	7.3145e-2	7.7494e-3
		P7	—	—	—	—	—	—	—	—	—
		P8	8.5551e-2	1.0486e-1	8.5741e-2	6.6029e-2	4.6898e-2	1.6638e-2	1.0718e-1	3.6388e-2	7.5822e-2
		P9	1.8123e-1	1.4013e-1	1.5055e-1	7.2189e-2	5.8805e-2	7.6467e-2	5.0275e-2	1.9662e-1	1.1788e-1
WFG	IGD	P1	8.2953e-2	8.9964e-2	1.1656e-1	9.7134e-2	1.0665e-1	8.5114e-2	1.1809e-1	6.8677e-2	6.5430e-2
		P2	2.7117e-3	4.2677e-3	5.9042e-3	2.1761e-3	2.1771e-2	1.7077e-2	2.7366e-2	2.2640e-2	1.0336e-2
		P3	3.9656e-3	9.2117e-4	2.7193e-3	1.8369e-3	2.6574e-2	1.8367e-2	2.5522e-2	2.4177e-2	1.1790e-2
		P4	2.7119e-4	2.8673e-4	1.6408e-4	2.7163e-4	2.7481e-4	2.9051e-4	3.6912e-4	2.2637e-4	3.7905e-4
		P5	5.4402e-3	8.0282e-4	5.7376e-3	1.9404e-3	2.6819e-4	2.4574e-4	4.2359e-3	1.0333e-4	3.3001e-3
		P6	6.5688e-4	5.8841e-4	5.6989e-4	4.4237e-4	1.3228e-3	1.3861e-3	8.1072e-4	4.1516e-4	4.0923e-4
		P7	7.3119e-4	8.9917e-4	8.6754e-4	9.7041e-4	2.7125e-4	3.6623e-4	1.3946e-3	4.1844e-4	2.8387e-4
		P8	1.1453e-2	1.4310e-2	1.5184e-2	1.5048e-2	6.2206e-3	8.2821e-3	6.9996e-3	1.9742e-2	2.2691e-2
		P9	1.3473e-2	1.0772e-2	1.1843e-2	1.3894e-2	7.2134e-3	8.3935e-3	3.2133e-3	1.3323e-2	1.7221e-2
	HV	P1	3.4255e-1	3.4155e-1	4.3923e-1	3.9853e-1	4.2555e-1	3.9344e-1	4.2913e-1	3.1455e-1	2.6177e-1
		P2	2.3495e-2	3.5722e-2	6.2046e-2	2.8448e-2	1.2210e-1	9.0587e-2	1.4228e-1	1.3124e-1	7.7624e-2
		P3	4.2775e-2	1.8141e-2	2.9158e-2	2.8246e-2	1.2820e-1	8.9947e-2	1.2015e-1	1.2077e-1	7.9295e-2
		P4	1.4935e-3	7.1476e-4	9.8322e-4	1.3894e-3	9.5274e-4	1.3005e-3	1.4317e-3	9.5913e-4	9.3985e-4
		P5	4.4852e-2	1.1499e-2	4.5019e-2	2.7733e-2	1.9558e-3	4.1108e-3	3.7716e-2	1.2156e-3	3.3381e-2
		P6	3.8926e-3	6.2005e-3	5.7485e-3	4.9638e-3	1.1142e-2	7.6507e-3	5.6193e-3	4.4950e-3	4.0558e-3
		P7	1.3814e-2	1.4536e-2	7.8942e-3	1.3856e-2	2.3340e-3	3.5992e-3	1.2710e-2	1.7571e-3	2.1586e-3
		P8	5.9973e-2	7.3035e-2	7.9908e-2	6.9911e-2	4.8952e-2	5.1483e-2	3.7061e-2	9.9280e-2	1.0781e-1
		P9	1.0303e-1	4.5464e-2	8.2197e-2	1.0172e-1	5.6164e-2	6.4974e-2	3.5327e-2	1.0836e-1	8.8573e-2

Table E.9.: IQR values of IGD and HV for Ref_1 with the nine grouping methods for the LSMOP and WFG test suite and $n = 200$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	P1	2.5600e-3	3.8882e-3	3.5342e-3	3.9961e-3	1.0519e-2	1.1718e-2	2.2252e-3	2.8988e-1	4.0582e-3
		P2	4.5289e-3	3.9684e-3	2.0798e-3	2.1503e-3	2.5660e-3	1.9423e-3	2.1379e-3	9.3533e-3	2.7346e-3
		P3	7.7251e-1	1.8457e-1	2.0055e-1	2.0722e-1	1.2787e-1	2.5384e-1	1.6775e-1	6.9089e-1	9.7201e-1
		P4	6.5701e-3	4.5970e-3	3.8023e-3	5.5915e-3	5.5735e-3	5.9824e-3	5.7918e-3	6.4463e-3	7.9760e-3
		P5	2.6497e-1	2.7565e-1	1.6311e-1	2.9381e-1	3.3469e-1	3.9280e-1	5.6315e-1	3.8924e-2	1.5265e-1
		P6	6.0724e-1	4.0605e-1	1.9193e-1	4.0968e-1	2.2826e-1	2.0323e-1	2.3660e-1	8.8547e-1	6.0561e-1
		P7	6.9418e-1	4.1829e-2	6.3931e-1	7.2794e-2	5.1441e-2	8.0327e-2	2.6285e-2	9.3838e-1	1.2606e-1
		P8	8.1867e-3	5.5514e-3	6.9728e-3	9.2413e-3	3.7113e-2	4.4884e-2	5.0862e-3	1.6522e-1	8.7653e-3
		P9	4.8871e-2	9.7377e-2	3.4103e-2	2.5956e-2	4.0808e-1	4.2684e-1	2.2632e-1	4.8576e-1	5.6486e-2
	HV	P1	4.8027e-3	6.9843e-3	4.9484e-3	9.3427e-3	1.4437e-2	2.7549e-2	3.9487e-3	1.8315e-1	8.2956e-3
		P2	7.1724e-3	5.2882e-3	3.2458e-3	3.9288e-3	4.2992e-3	3.0989e-3	3.3028e-3	1.2898e-2	4.2852e-3
		P3	—	—	—	—	—	—	—	—	—
		P4	9.7512e-3	7.0657e-3	5.5019e-3	8.3446e-3	8.2085e-3	9.9004e-3	9.2736e-3	6.9266e-3	1.1020e-2
		P5	1.5926e-3	9.9169e-4	1.4347e-3	1.0818e-1	2.1203e-3	7.8709e-2	1.6542e-1	7.7569e-5	2.2055e-3
		P6	—	—	6.1330e-2	3.1782e-2	2.9039e-2	1.7781e-2	4.1706e-2	—	—
		P7	—	—	—	—	—	—	—	—	—
		P8	6.7344e-3	6.8945e-3	8.6944e-3	8.9399e-3	3.5613e-2	6.0578e-2	9.6648e-3	1.8484e-2	1.0063e-2
		P9	6.2620e-2	9.8578e-2	3.7667e-2	2.6677e-2	1.5883e-1	1.8939e-1	7.5207e-2	3.7812e-1	5.2370e-2
WFG	IGD	P1	9.5721e-2	7.8695e-2	7.1195e-2	1.5983e-1	1.2501e-1	6.2751e-2	7.4665e-2	1.9400e-1	1.2200e-1
		P2	2.6700e-3	2.3664e-3	3.9659e-3	9.9178e-3	5.2635e-3	8.5608e-3	1.3649e-2	2.7757e-2	1.2217e-2
		P3	5.6617e-4	6.1676e-4	5.9175e-4	8.1797e-4	4.4613e-3	5.6135e-3	8.2058e-3	2.7465e-2	3.8124e-3
		P4	1.8992e-3	3.8132e-3	2.0154e-3	3.3569e-3	1.5756e-3	1.7109e-3	1.6980e-3	1.9178e-3	1.1598e-3
		P5	6.5755e-4	6.1590e-4	7.0646e-4	3.6951e-4	6.7631e-4	4.4957e-4	5.7955e-4	2.3679e-4	7.4390e-4
		P6	5.5133e-4	8.9858e-4	6.7557e-4	6.5443e-4	5.2930e-4	6.9801e-4	1.0766e-3	6.5424e-4	9.3779e-4
		P7	8.1382e-2	9.3314e-2	5.4763e-2	8.2631e-2	8.8726e-2	7.8728e-2	6.9493e-2	7.9216e-2	8.8713e-2
		P8	7.3597e-3	4.1999e-3	8.9005e-3	5.8692e-3	5.1147e-3	3.9563e-3	2.8941e-3	4.3868e-3	1.2413e-2
		P9	4.9990e-2	1.2068e-1	4.8643e-2	8.9410e-2	1.4474e-1	2.1565e-1	7.9406e-2	4.5604e-2	9.7765e-2
	HV	P1	3.9968e-1	3.3592e-1	3.1030e-1	6.6297e-1	6.6897e-1	4.7891e-1	3.2765e-1	7.6022e-1	4.8999e-1
		P2	1.9227e-3	3.8951e-3	3.0780e-3	5.9553e-3	3.2893e-2	4.0734e-2	7.3082e-2	1.4478e-1	3.9151e-2
		P3	4.8875e-3	3.3569e-3	2.3135e-3	3.7129e-3	2.3674e-2	3.1800e-2	3.7048e-2	1.2775e-1	3.3279e-2
		P4	2.3352e-3	3.0762e-3	2.5680e-3	3.8913e-3	1.8192e-3	2.4352e-3	2.8526e-3	3.2971e-3	3.9526e-3
		P5	4.3675e-3	4.6717e-3	3.8726e-3	3.3184e-3	3.3059e-3	3.8374e-3	2.4187e-3	2.2312e-3	6.1668e-3
		P6	4.0832e-3	4.3136e-3	5.6832e-3	5.2600e-3	2.0258e-3	2.7819e-3	3.5724e-3	3.4572e-3	4.8488e-3
		P7	4.1282e-1	4.3987e-1	3.1657e-1	4.5374e-1	4.7514e-1	4.4821e-1	3.9286e-1	4.3758e-1	4.6681e-1
		P8	4.5584e-2	2.3056e-2	5.1805e-2	3.2545e-2	1.7798e-2	2.3561e-2	1.0804e-2	3.4415e-2	7.1206e-2
		P9	2.3736e-1	5.9867e-1	2.1817e-1	4.1494e-1	7.5701e-1	1.1297e+0	4.2904e-1	2.1740e-1	4.7278e-1

Table E.10.: IQR values of IGD and HV for Ref_2 with the nine grouping methods for the LSMOP and WFG test suite and $n = 200$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	P1	1.9306e-2	1.8576e-2	2.6087e-2	2.8266e-2	5.2049e-2	6.4369e-2	2.5188e-2	1.0870e-2	1.5695e-2
		P2	3.2731e-3	4.1592e-3	2.9313e-3	2.9255e-3	3.3349e-3	2.3313e-3	3.1859e-3	4.5835e-3	5.3575e-3
		P3	1.3441e-2	1.6172e-1	1.1171e-1	1.1835e-1	2.1605e-1	1.1488e-1	1.0074e-1	7.6869e-1	2.2618e-1
		P4	3.7333e-3	3.8119e-3	2.5578e-3	2.9662e-3	1.9469e-3	2.7747e-3	2.7771e-3	1.4930e-2	1.4788e-2
		P5	2.6317e-1	2.1947e-1	1.8829e-2	5.2152e-1	1.0635e-1	4.1209e-2	8.2276e-2	3.9920e-1	5.9772e-2
		P6	5.0323e-2	3.4828e-2	4.6421e-2	8.8476e-2	1.2482e-1	2.2195e-1	3.8525e-2	5.9291e-2	2.5068e-1
		P7	3.6262e-1	8.3599e-1	3.5372e-1	4.3513e-1	6.7505e-1	9.6207e-1	8.6649e-1	5.4554e-1	7.1146e-1
		P8	1.0359e-1	9.3096e-2	5.3288e-2	1.2685e-1	1.5845e-1	1.1173e-1	1.5095e-1	8.6616e-2	6.7559e-2
		P9	3.6775e-2	2.7768e-2	2.3512e-2	3.2908e-2	2.3575e-2	7.9304e-2	3.2080e-2	2.7636e-1	7.1015e-2
	HV	P1	2.7165e-2	2.4975e-2	3.7542e-2	3.7493e-2	7.0608e-2	8.8870e-2	3.3833e-2	8.0936e-3	2.1375e-2
		P2	4.7797e-3	6.2796e-3	4.0022e-3	4.3445e-3	5.1677e-3	4.4093e-3	4.5028e-3	6.3836e-3	6.9786e-3
		P3	1.9578e-2	9.8720e-2	2.2833e-5	1.1084e-3	—	—	—	—	—
		P4	6.1002e-3	4.9176e-3	4.5139e-3	5.1194e-3	3.2974e-3	5.1134e-3	4.6625e-3	2.0639e-2	2.3368e-2
		P5	9.9752e-4	3.6754e-4	1.6282e-4	1.3768e-1	4.6124e-4	4.0106e-4	1.9183e-4	9.6023e-2	3.4412e-4
		P6	2.2227e-2	2.5260e-2	3.0822e-2	4.2532e-2	1.5944e-2	2.6949e-2	8.0243e-3	1.2897e-3	—
		P7	—	—	—	—	—	—	—	—	—
		P8	9.0404e-2	8.4654e-2	4.5186e-2	9.9962e-2	8.9770e-2	5.8067e-2	9.1424e-2	2.1521e-6	6.7261e-2
		P9	3.1743e-2	2.7732e-2	4.0475e-2	4.5661e-2	4.0290e-2	1.0173e-1	3.6993e-2	8.9960e-2	5.8214e-2
WFG	IGD	P1	7.6443e-2	5.3478e-2	1.0172e-1	6.9163e-2	8.1821e-2	9.2293e-2	8.2710e-2	4.3861e-2	9.0398e-2
		P2	2.5014e-3	2.0342e-3	2.2168e-3	2.5268e-3	1.8963e-2	3.1555e-2	1.9024e-2	7.8579e-3	1.1587e-2
		P3	8.1989e-4	1.7685e-3	1.0561e-3	1.1698e-3	2.8239e-2	3.0430e-2	1.9536e-2	1.3588e-2	1.3259e-2
		P4	1.8608e-4	3.0353e-4	2.9088e-4	3.9811e-4	2.2017e-4	2.2510e-4	2.5615e-4	1.9598e-4	4.1227e-4
		P5	5.3754e-3	5.4121e-3	5.4862e-3	5.4000e-3	6.0198e-3	4.6430e-3	5.6877e-3	1.2045e-4	5.2601e-3
		P6	2.6434e-4	3.0515e-4	2.8180e-4	3.5619e-4	5.4978e-4	3.6751e-4	5.2370e-4	3.1077e-4	1.1661e-3
		P7	9.4340e-3	1.0383e-2	1.0025e-2	8.3006e-3	2.1127e-3	2.6470e-3	1.0603e-2	3.3157e-4	3.9015e-4
		P8	8.6605e-3	7.9266e-3	1.0548e-2	8.2434e-3	5.4670e-3	5.1368e-3	6.6648e-3	1.6331e-2	1.6273e-2
		P9	1.1614e-2	1.0335e-2	1.3803e-2	8.7757e-3	1.3219e-2	1.4691e-2	1.8320e-2	9.7948e-3	8.5406e-3
	HV	P1	2.9296e-1	2.2773e-1	3.9625e-1	2.8598e-1	3.1511e-1	3.6740e-1	3.5098e-1	1.8778e-1	3.5735e-1
		P2	1.7788e-2	1.7084e-2	1.6324e-2	1.6470e-2	9.9503e-2	1.5559e-1	9.5252e-2	4.3730e-2	7.5052e-2
		P3	1.5504e-2	2.1355e-2	1.3973e-2	1.7354e-2	1.3409e-1	1.4600e-1	8.8945e-2	6.4739e-2	7.8366e-2
		P4	1.1605e-3	1.2658e-3	1.5476e-3	1.3331e-3	1.9942e-3	1.2102e-3	1.5400e-3	1.7080e-3	1.5380e-3
		P5	3.9504e-2	4.3269e-2	4.5296e-2	4.2290e-2	5.0729e-2	4.5603e-2	4.8749e-2	1.3865e-3	4.0309e-2
		P6	2.8068e-3	1.4336e-3	2.0267e-3	1.9765e-3	2.2200e-3	2.9084e-3	3.8193e-3	1.2210e-3	1.1788e-2
		P7	5.8698e-2	6.3895e-2	5.2691e-2	4.3319e-2	1.7378e-2	2.2705e-2	7.6208e-2	1.7642e-3	1.7526e-3
		P8	4.3585e-2	4.4632e-2	5.5753e-2	3.8365e-2	2.4459e-2	2.8484e-2	2.8214e-2	8.2389e-2	7.9139e-2
		P9	6.2907e-2	4.7073e-2	5.5506e-2	7.4716e-2	7.1992e-2	7.5350e-2	3.2691e-2	5.5105e-2	5.7479e-2

Table E.11.: IQR values of IGD and HV for Ref_1 with the nine grouping methods for the LSMOP and WFG test suite and $n = 500$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	P1	2.3065e-3	2.1234e-3	4.7668e-3	3.1667e-3	2.1238e-2	3.0540e-2	3.0574e-3	1.9965e-1	5.7430e-3
		P2	3.1850e-4	6.5299e-4	4.2215e-4	5.8278e-4	7.1997e-4	9.0084e-4	6.1115e-4	1.7808e-3	5.7619e-4
		P3	6.5066e-2	8.7684e-2	6.0485e-2	3.4589e-1	3.6749e-2	2.6991e-2	3.3975e-1	1.0164e-1	2.2453e-1
		P4	2.7429e-3	4.8991e-3	3.7170e-3	3.3369e-3	2.5246e-3	3.4955e-3	3.5049e-3	4.6136e-3	4.9744e-3
		P5	2.5540e-1	4.0973e-1	7.0233e-1	1.8249e-1	1.3809e-1	1.9352e-1	2.8534e-1	3.1908e-1	2.8575e-1
		P6	4.7214e-1	5.0635e-1	1.4684e-1	1.8861e-1	1.0635e-1	1.4153e-1	1.1625e-1	3.0815e-1	1.5990e-1
		P7	4.0779e-1	1.5088e-1	2.0520e-1	1.1740e-1	3.6376e-2	3.2618e-2	1.3246e-1	2.5612e-1	1.2783e-1
		P8	4.6667e-3	1.3979e-2	4.1175e-3	1.1885e-2	4.7364e-2	4.8571e-2	1.0088e-2	1.9306e-1	3.9195e-3
		P9	1.2664e-2	2.3342e-2	8.7956e-3	1.0642e-2	4.4570e-1	4.4709e-1	9.5315e-3	1.3528e-1	2.1943e-2
	HV	P1	5.0301e-3	6.4213e-3	7.3332e-3	6.8601e-3	3.9108e-2	5.3313e-2	5.9641e-3	1.0599e-1	1.0067e-2
		P2	5.1551e-4	1.2341e-3	5.7299e-4	1.0419e-3	1.2448e-3	1.6314e-3	1.2381e-3	3.1899e-3	1.0403e-3
		P3	6.1040e-2	5.9927e-2	8.2072e-2	3.9510e-2	3.5142e-2	2.4952e-2	9.0092e-2	5.4670e-2	3.0559e-2
		P4	3.5655e-3	7.3559e-3	6.3486e-3	6.0025e-3	4.0295e-3	5.7169e-3	5.6773e-3	6.0859e-3	9.0736e-3
		P5	2.7468e-4	6.5314e-2	2.7966e-1	2.2334e-4	2.6860e-4	3.3931e-4	2.4587e-4	2.5017e-2	3.9563e-4
		P6	—	1.0560e-1	7.0103e-3	1.0702e-1	3.3313e-2	5.9405e-2	7.6324e-2	—	6.3790e-2
		P7	—	—	—	—	—	—	—	—	—
		P8	3.1159e-3	8.2690e-3	3.6082e-3	7.5121e-3	3.7841e-2	5.5685e-2	7.6804e-3	1.6728e-2	4.2101e-3
		P9	2.2886e-2	1.9803e-2	8.0093e-3	6.3304e-3	2.4325e-1	2.3840e-1	8.5394e-3	1.0281e-1	2.0676e-2
WFG	IGD	P1	1.5110e-1	7.3884e-2	6.0976e-2	6.1493e-2	9.2408e-2	8.6162e-2	9.2012e-2	5.6886e-2	7.6827e-2
		P2	6.7684e-4	2.5471e-3	7.0254e-4	1.4271e-3	5.0393e-3	6.0971e-3	5.3209e-3	1.2425e-2	4.8301e-3
		P3	4.6573e-4	3.3324e-4	2.7318e-4	2.9615e-4	5.7215e-3	4.5660e-3	4.4216e-3	2.0596e-2	2.5194e-3
		P4	3.9857e-4	2.9317e-4	5.3241e-4	8.7831e-4	2.9975e-4	6.1922e-4	5.0438e-4	4.9662e-4	9.8254e-4
		P5	6.9287e-4	1.3071e-3	7.2749e-4	8.2467e-4	1.2456e-3	3.7921e-4	5.3346e-4	3.5984e-4	4.1380e-4
		P6	2.9216e-4	2.1812e-4	2.4190e-4	3.6443e-4	3.8130e-4	3.7504e-4	3.5846e-4	4.7628e-4	2.0820e-4
		P7	5.9530e-2	6.1333e-2	7.1024e-2	6.1592e-2	5.6272e-2	6.2558e-2	4.6755e-2	4.2865e-2	5.8793e-2
		P8	5.0247e-3	2.9245e-3	2.0347e-3	4.1133e-3	3.4535e-3	3.0978e-3	1.7944e-3	7.2583e-3	1.2529e-2
		P9	7.2624e-2	8.2160e-2	3.2411e-2	4.0718e-2	1.3365e-1	1.3580e-1	1.5323e-1	7.5915e-2	5.1310e-2
	HV	P1	6.1426e-1	3.2206e-1	2.7252e-1	2.6479e-1	3.4193e-1	2.8256e-1	3.8114e-1	3.1718e-1	3.4007e-1
		P2	1.1263e-3	2.5086e-3	1.7205e-3	1.2632e-3	3.6219e-2	3.6110e-2	2.6456e-2	6.8205e-2	3.2924e-2
		P3	1.4506e-3	9.5070e-4	7.9875e-4	1.1211e-3	2.9270e-2	2.2939e-2	2.2847e-2	9.7437e-2	1.7937e-2
		P4	2.0825e-3	1.8191e-3	2.0083e-3	1.1616e-3	1.4221e-3	2.8641e-3	8.9612e-4	9.7437e-4	2.0773e-3
		P5	2.9482e-3	6.8278e-3	3.5170e-3	3.4748e-3	5.7823e-3	1.6953e-3	3.0630e-3	2.4976e-3	2.0913e-3
		P6	1.1113e-3	1.1134e-3	1.3999e-3	7.5782e-4	1.6645e-3	1.5728e-3	1.2257e-3	2.3190e-3	1.2343e-3
		P7	3.1436e-1	3.6068e-1	3.6016e-1	3.2123e-1	2.8870e-1	3.2893e-1	2.5027e-1	2.4324e-1	3.0935e-1
		P8	2.4668e-2	1.6498e-2	1.2807e-2	1.4565e-2	9.6473e-3	1.9441e-2	6.4806e-3	3.4006e-2	6.9562e-2
		P9	3.5106e-1	4.0333e-1	1.6374e-1	2.0118e-1	7.1201e-1	7.0463e-1	8.4775e-1	3.8676e-1	2.5295e-1

Table E.12.: IQR values of IGD and HV for Ref_2 with the nine grouping methods for the LSMOP and WFG test suite and $n = 500$

			OS+VS	OS+VA	OA+VS	OA+VA	MRAND+S2	MDELTA+S2	GN	G1	CA
LSMOP	IGD	P1	1.4707e-2	2.2536e-2	1.5939e-2	1.2921e-2	2.9783e-2	3.2153e-2	2.0827e-2	2.0262e-2	2.6831e-2
		P2	6.1501e-4	1.1518e-3	1.4270e-3	1.3378e-3	7.5897e-4	1.8454e-3	1.4362e-3	1.6614e-3	1.0233e-3
		P3	5.3380e-4	2.4182e-1	3.2744e-2	4.6450e-2	9.0573e-2	1.2083e-2	4.2014e-2	9.7307e-1	2.2587e-1
		P4	3.0867e-3	3.5483e-3	7.4156e-4	7.8481e-4	4.0422e-4	5.0523e-4	5.9805e-4	3.7052e-3	4.2187e-3
		P5	1.8334e-5	2.0009e-1	1.8563e-1	1.6695e-5	5.3923e-5	4.7960e-5	2.3032e-2	3.0875e-4	1.0171e-1
		P6	8.9522e-3	7.9924e-2	8.6826e-2	6.8488e-2	9.1886e-2	7.7871e-2	1.7078e-2	2.6432e-2	1.1969e-2
		P7	1.7155e-1	2.8015e-1	1.5817e-1	1.9006e-1	3.9003e-1	2.8092e-1	3.8883e-1	1.9613e-1	5.1278e-1
		P8	5.3220e-2	7.4886e-2	3.7071e-2	5.5904e-2	1.5302e-1	1.4451e-1	5.0227e-2	5.6879e-4	7.0630e-2
		P9	1.6167e-2	6.9964e-3	1.0999e-2	3.3127e-2	1.6480e-2	3.2875e-2	1.4639e-2	4.9019e-1	1.1635e-2
	HV	P1	2.0821e-2	3.0840e-2	2.2720e-2	1.7281e-2	4.1733e-2	4.5147e-2	2.8953e-2	1.3435e-2	3.6668e-2
		P2	1.0418e-3	1.8282e-3	2.3298e-3	2.2477e-3	1.0992e-3	3.3739e-3	2.4805e-3	2.5066e-3	1.7096e-3
		P3	8.2643e-4	6.9133e-2	4.6532e-2	4.8250e-2	1.0359e-1	1.8265e-2	4.8965e-2	—	6.1105e-2
		P4	4.3310e-3	5.2913e-3	1.3634e-3	1.0089e-3	5.6939e-4	8.0167e-4	1.1298e-3	5.3220e-3	6.9529e-3
		P5	5.0329e-5	3.1600e-2	5.0781e-5	4.5391e-5	1.5857e-4	1.2477e-4	4.0961e-5	3.2453e-3	1.1171e-4
		P6	9.5111e-3	1.5013e-2	2.8350e-2	2.5374e-2	2.9538e-2	1.5325e-2	2.0787e-2	1.4555e-2	9.0807e-3
		P7	—	—	—	—	—	—	—	—	—
		P8	4.6673e-2	6.7540e-2	4.1029e-2	4.9366e-2	7.6561e-2	6.5841e-2	4.2349e-2	2.5867e-3	6.0579e-2
		P9	1.5654e-2	8.3401e-3	1.6573e-2	4.2614e-2	1.8566e-2	3.6989e-2	2.4141e-2	1.1985e-1	1.4595e-2
WFG	IGD	P1	6.8481e-2	4.7449e-2	7.7773e-2	6.6049e-2	3.6452e-2	6.0772e-2	9.2712e-2	5.2287e-2	6.4428e-2
		P2	2.0402e-3	1.8413e-3	2.2845e-3	2.0123e-3	1.7565e-2	1.9520e-2	1.2254e-2	1.2326e-2	9.2518e-3
		P3	7.2358e-4	4.4124e-4	6.9303e-4	3.3719e-4	2.3959e-2	1.6855e-2	8.1916e-3	7.9957e-3	7.7343e-3
		P4	3.0466e-4	2.5866e-4	2.2954e-4	1.7028e-4	2.9355e-4	3.2705e-4	3.6896e-4	2.2665e-4	2.4977e-4
		P5	6.3026e-4	4.5441e-4	2.1301e-3	7.2464e-4	1.7750e-4	1.6859e-4	6.1475e-3	2.0711e-4	2.1830e-3
		P6	3.2934e-4	4.0203e-4	2.9268e-4	3.5299e-4	2.4288e-4	3.4423e-4	2.4931e-4	2.8057e-4	4.9568e-4
		P7	1.0538e-2	1.3218e-2	1.0226e-2	9.2438e-3	2.4576e-3	4.7516e-3	1.5147e-2	3.3720e-4	3.3483e-4
		P8	3.6210e-3	2.5213e-3	2.9509e-3	5.9225e-3	5.9477e-3	3.7333e-3	3.7335e-3	1.5095e-2	3.0068e-2
		P9	2.4217e-3	1.3874e-2	4.0925e-3	4.4237e-3	1.9079e-2	2.6952e-2	6.0164e-2	2.9003e-3	3.6247e-3
	HV	P1	2.7146e-1	1.9220e-1	3.1042e-1	2.6203e-1	1.9466e-1	3.4603e-1	3.6983e-1	2.4413e-1	2.4701e-1
		P2	7.3485e-3	9.5928e-3	5.8639e-3	5.0901e-3	9.1262e-2	1.0096e-1	6.0597e-2	6.9206e-2	5.5799e-2
		P3	5.5907e-3	7.8192e-3	5.8239e-3	6.4371e-3	1.1301e-1	7.8940e-2	3.6860e-2	4.0944e-2	4.3507e-2
		P4	1.2762e-3	1.5244e-3	1.2065e-3	8.0595e-4	6.3579e-4	1.0656e-3	1.1963e-3	1.1656e-3	1.6919e-3
		P5	1.2117e-2	9.1285e-3	2.6951e-2	7.3108e-3	1.4698e-3	1.1280e-3	5.1774e-2	1.1034e-3	2.7081e-2
		P6	1.0285e-3	9.6187e-4	1.6606e-3	1.0671e-3	1.8236e-3	8.6796e-4	1.7275e-3	1.2059e-3	5.6671e-3
		P7	6.8691e-2	8.2443e-2	6.8165e-2	5.2813e-2	1.9010e-2	3.2310e-2	9.1554e-2	1.5905e-3	1.0759e-3
		P8	1.8584e-2	1.7728e-2	1.9023e-2	3.5387e-2	3.4045e-2	1.7218e-2	2.2365e-2	7.5608e-2	1.5320e-1
		P9	2.2217e-2	1.0471e-1	2.2352e-2	2.5678e-2	8.9617e-2	1.6205e-1	3.2215e-1	2.1369e-2	2.9734e-2

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Declaration of Independence

I hereby declare that this thesis was created by me and me alone using only the stated sources and tools.

Frederick Sander Magdeburg, April 9, 2018