Supplementary Material for the Article "A Framework for Large-scale Multi-objective Optimization based on Problem Transformation"

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1 Scaling the number of variables

In addition to the analysis that we gave for each of the test problems for $n = 1000$ variables, we examined how these results change with a change in the dimension of the search space. For each setting of $n \in \{40, 80, 200, 600, 1000, 2000\}$ we perform the same experiment as in the previous section. Tables 15 and 16 of this document give an overview about the experiments. For every problem we list the algorithm that achieved the highest median hypervolume value. An asterisk is used to indicate whether the respective value is significantly better than the second-best performance. Additionally, the respective second best algorithms are given in brackets together with an asterisk if the difference to the respective third best method is statistically significant. The analysis of these experiments is done in Subsection VI-C of the main article.

2 Further Plots of Experiments from Subsection VI-B

In addition to the plots in Figures 7, 8 and 9 in the main article, this document contains the plots for all tested benchmarks for $n = 1000$ variables in Figures 14 to 41. The description, parameter settings and analysis to those experiments can be found in Subsections VI-A and VI-B of the paper.

3 Sensitivity Analysis of WOFs' Parameters

This section shows the results of the sensitivity analysis of WOFs' parameters as described in the paper in Subsection VI-G. For each parameter that was tested a short explanation of the experiment setting is given and the tables for WOF-SMPSO and WOF-NSGA-II can be found in this supplement material.

3.1 Parameter q

The parameter q indicated the number of solutions $\vec{x}'_i$, $i = 1..q$ that are chosen from the current population. The following tables (Tables 1 and 2) show the results of the SMPSO and NSGA-II algorithms for values of $q \in \{1, 2, 3, 5, 8\}$. All other parameters remained unchanged.

3.2 Parameter γ

The parameter γ indicates the number of groups used (for example in the random grouping or the ordered grouping) and therefore also the number of decision variables of the transformed problem. Tables 3 and 4 show the results of the SMPSO and NSGA-II algorithms for values of $\gamma \in \{2, 4, 8, 20, 50\}$. All other parameters remained unchanged. The ordered grouping method is used in these experiments.

3.3 Portion of Function Evaluations used for Weight Optimization (δ)

The main experiments of the paper were conducted by using the alternation of normal optimization and weight optimization only during the first 50% of the total function evaluations (i.e. $\delta = 0.5$ in Algorithm 1 of the paper). After that, the normal optimization process took over to achieve greater diversity in the final solution population. In this analysis, we will alter this threshold and vary the value of δ. The following tables (Tables 5 and 6) show the results of the SMPSO and NSGA-II algorithms for values of $\delta \in \{0.1, 0.25, 0.5, 0.75, 1.0\}$. All other parameters remained unchanged.

3.4 Parameter p

The parameter p from the p-value transformation function (ψ_2) indicates the amount of maximum change that is possible with the weight optimization within each variables domain. A value of $p = 1.0$ is equivalent to the normal product tranformation function (ψ_1). The following tables (Tables 7 and 8) show the results of the SMPSO and NSGA-II algorithms for values of $p \in \{0.05, 0.1, 0.2, 0.3, 0.5\}$. All other parameters remained unchanged.

3.5 Parameter t_1

In each iteration of the WOF framework, first a normal optimization step is carried out, where the original problem is optimized for a fixed number t_1 of function evaluations. The parameter t_1 indicates the amount of function evaluations used in each optimization step of the normal optimizer (Line 4 in Algorithm 1 of the paper). The following tables (Tables 9 and 10) show the results of the SMPSO and NSGA-II algorithms for values of $t_1 \in \{400, 800, 1000, 1500, 2000\}$. All other parameters remained unchanged.

3.6 Parameter t_2

After each problem transformation, the optimization of the weight-variables that is carried out for a fixed number t_2 of function evaluations. The parameter t_2 indicates the amount of function evaluations used in each optimization step of the weight optimizer (Line 7 in Algorithm 1 and in Algorithm 2 of the paper). The following tables (Tables 11 and 12) show the results of the SMPSO and NSGA-II algorithms for values of $t_2 \in \{100, 300, 500, 700, 900\}$. All other parameters remained unchanged.

4 Comparison with MOEADVA

In this section we list the tables and figures that compare the performance of WOF-SMPSO with MOEA/DVA from the literature. The results of these experiments can be seen in Tables 13 and 14 as well as in Figures 1 to 13 of this document. Due to time and hardware limitations, these experiments were not performed on all benchmark problem that were used in the previous experiments in the article. The problems used were the UF1-10 and the 2-objective WFG1-5 and WFG7 and the 3-objective WFG1-5.

To ensure a fair comparison between the WOF-SMPSO, the MOEA/DVA and the original SMPSO, the experiments were performed with 10,000,000 function evaluations. Except the number of evaluations, all parameters used for the SMPSO and the WOF-SMPSO were the same as in the experiments performed in Subsections VI-A and VI-B of the main article. The parameters of the MOEA/DVA were the same as found in the original MOEA/DVA paper with the exception of adjusting the population sizes to the same sizes as the ones of SMPSO and WOF-SMPSO, to ensure a fair comparison using the hypervolume indicator. Furthermore, the distribution indexes of crossover and mutation are set to the same values as described in Subsection VI-B. No other changes were made to the original sourcecode provided by the authors.

The obtained relative hypervolume values by the end of the optimzation can be found in Tables 13 and 14. The solution sets and convergence plots for the 2-objective problems can be found in Figures 1 to 13 in this document.

5 Selection of the Solutions x'_k

For the selection of the solution x'_k (refer to Subsection IV-C of the main article), we compared a Crowding Distance-based selection as described in the article and a random selection mechanism, where we chose the x'_k randomly from the first non-dominated front of the current population. Tables 17 to 19 of this document list the median hypervolume values that are obtained by the original SMPSO, NSGA-II, GDE3 and their WOF-based versions using Crowding Distance and random selection. Figures 42 to 45 show the obtained solution sets for the 2-objective problems of these experiments using SMPSO.

Table 1: Median and IQR values of the WOF-SMPSO for different numbers of q. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-SMPSO ($q = 1$)	WOF-SMPSO ($q = 2$)	WOF-SMPSO ($q = 3$)	WOF-SMPSO ($q = 5$)	WOF-SMPSO ($q = 8$)
2	DTLZ1	**0.642949** (—)	0.642949 (—)	0.642949 (4.89E-10)	0.642949 (2.09E-9)	0.642949 (—)
2	DTLZ2	0.998546 (3.60E-4) *	0.998743 (3.65E-4)	0.998803 (1.25E-4)	0.998798 (7.61E-5)	**0.998822** (6.74E-5)
2	DTLZ3	**0.520072** (—)	0.520072 (2.39E-1)	0.520072 (—)	0.520072 (—)	0.520072 (—)
2	DTLZ4	0.998766 (1.27E-4)	0.998739 (3.01E-4)	0.998778 (1.40E-4)	**0.998794** (1.18E-4)	0.998769 (7.83E-5)
2	DTLZ5	0.999141 (2.85E-4)	0.999131 (3.35E-4)	0.999112 (2.45E-4)	0.999234 (1.44E-4)	**0.999241** (7.41E-5)
2	DTLZ6	0.998670 (1.31E-3)	**0.998922** (7.23E-4)	0.998841 (1.53E-3)	0.998622 (3.59E-1)	0.998094 (3.75E-1)
2	DTLZ7	0.798197 (2.94E-5) *	0.798205 (5.80E-5) *	0.798203 (2.03E-5) *	0.798204 (2.79E-5) *	**0.798234** (3.99E-5)
2	WFG1	0.641814 (8.85E-3) *	0.643834 (1.84E-2)	0.649253 (9.16E-3)	**0.651201** (1.66E-2)	0.642568 (9.91E-3)
2	WFG2	0.985050 (1.52E-2)	0.981406 (8.74E-3) *	**0.987061** (8.84E-3)	0.981110 (1.48E-2)	0.978464 (1.14E-2) *
2	WFG3	0.854031 (5.88E-3) *	0.854985 (3.44E-3)	0.857151 (2.15E-3)	**0.857156** (5.68E-3)	0.856299 (3.63E-3)
2	WFG4	0.978333 (1.70E-2)	**0.980147** (1.33E-2)	0.976262 (1.19E-2)	0.977570 (1.09E-2)	0.976353 (1.22E-2)
2	WFG5	0.931054 (2.66E-2) *	0.944171 (3.19E-2)	**0.950678** (2.17E-2)	0.943508 (2.42E-2)	0.937275 (1.73E-2)
2	WFG6	0.998430 (4.39E-4)	0.998423 (3.50E-4)	**0.998526** (3.22E-4)	0.998372 (3.46E-4)	0.998459 (4.02E-4)
2	WFG7	0.961736 (1.69E-2)	0.960444 (1.18E-2)	**0.962353** (1.80E-2)	0.961490 (9.55E-3)	0.960937 (1.05E-2)
2	WFG8	0.895947 (3.63E-2)	**0.899001** (2.99E-2)	0.884323 (1.94E-2)	0.888619 (4.17E-2)	0.881745 (2.19E-2)
2	WFG9	0.951472 (1.26E-2) *	0.959766 (1.88E-2) *	0.970001 (8.32E-3)	**0.970173** (1.38E-2)	0.968090 (9.63E-3)
2	UF1	0.784011 (4.64E-3)	0.782842 (4.34E-3) *	0.784867 (7.45E-3)	0.784732 (3.18E-2)	**0.788183** (3.74E-2)
2	UF2	0.931355 (1.02E-3) *	0.931269 (1.16E-3) *	0.931920 (2.25E-3)	**0.933351** (9.49E-3)	0.931852 (4.93E-3)
2	UF3	0.989121 (1.00E-3)	0.989070 (8.59E-4)	**0.989144** (1.31E-3)	0.988609 (1.10E-3)	0.988348 (1.45E-3)
2	UF4	0.899651 (1.49E-2) *	0.910014 (1.41E-2) *	0.918730 (1.85E-2)	0.917889 (1.34E-2)	**0.923918** (2.41E-2)
2	UF5	— (—)	— (—)	— (—)	— (—)	— (—)
2	UF6	0.260070 (2.20E-2)	0.257185 (1.88E-2)	0.254028 (1.96E-2)	**0.327658** (1.67E-1)	0.249822 (1.48E-1)
2	UF7	0.774940 (4.98E-3) *	0.776647 (5.99E-3) *	0.777838 (2.98E-2)	0.781441 (2.45E-2)	**0.797744** (5.21E-2)
2	ZDT1	0.998118 (4.26E-4)	0.998080 (4.56E-4)	**0.998166** (3.83E-4)	0.998043 (1.88E-4)	0.998158 (3.92E-4)
2	ZDT2	0.998010 (3.08E-4)	0.997918 (4.62E-4) *	0.997981 (2.95E-4)	0.997884 (3.87E-4)	**0.998165** (4.03E-4)
2	ZDT3	0.999126 (8.30E-5)	0.999127 (2.17E-4)	0.999122 (2.01E-4)	**0.999174** (1.42E-4)	0.999024 (2.06E-4)
2	ZDT4	0.998694 (1.82E-4)	0.998645 (3.49E-4)	0.998674 (1.19E-4)	0.998684 (1.93E-4)	**0.998712** (2.08E-4)
2	ZDT6	0.998132 (3.43E-4)	0.998158 (4.91E-4)	0.998183 (2.52E-4)	0.998142 (3.87E-4)	**0.998204** (2.91E-4)
3	DTLZ1	**0.587705** (—)	0.587705 (5.87E-1)	— (5.87E-1) *	— (—) *	— (—) *
3	DTLZ2	**0.979355** (4.03E-3)	0.978288 (1.89E-3)	0.977339 (4.85E-3)	0.978086 (4.27E-3)	0.973202 (7.02E-3) *
3	DTLZ3	**0.390379** (—)	0.390379 (3.90E-1)	— (3.90E-1) *	— (—) *	— (—) *
3	DTLZ4	1.001555 (6.37E-3)	**1.003125** (4.52E-3)	1.002027 (4.82E-3)	1.002939 (8.15E-3)	1.001143 (6.19E-3)
3	DTLZ5	0.998096 (8.84E-4)	**0.998260** (3.50E-4)	0.998211 (6.50E-4)	0.998184 (3.72E-4)	0.998224 (7.72E-4)
3	DTLZ6	0.998609 (4.48E-4) *	0.998789 (7.16E-4) *	0.998846 (5.89E-4) *	0.999748 (7.19E-4)	**0.999794** (3.70E-5)
3	DTLZ7	**0.983651** (7.86E-3)	0.983405 (3.73E-3)	0.981088 (5.01E-3)	0.979063 (6.63E-3)	0.977686 (7.89E-3) *
3	WFG1	0.596563 (5.68E-3) *	0.601458 (1.01E-2)	0.601991 (7.75E-3)	**0.603360** (6.61E-3)	0.599998 (6.08E-3)
3	WFG2	**0.967356** (1.68E-2)	0.961758 (1.55E-2)	0.956396 (1.42E-2)	0.961423 (1.53E-2)	0.960723 (1.96E-2)
3	WFG3	0.954661 (3.24E-2)	0.948524 (2.64E-2)	0.945946 (1.86E-2)	0.948792 (1.37E-2)	**0.957653** (2.29E-2)
3	WFG4	0.869199 (2.64E-2) *	0.887593 (2.61E-2)	0.891647 (2.95E-2)	**0.901305** (2.88E-2)	0.899976 (2.20E-2)
3	WFG5	0.807762 (3.80E-2) *	0.835129 (2.26E-2) *	0.833244 (2.73E-2) *	**0.857522** (2.92E-2)	0.839440 (1.52E-2) *
3	WFG6	0.963713 (1.48E-2)	0.960388 (1.25E-2) *	0.959280 (1.81E-2) *	**0.971238** (1.48E-2)	0.968675 (1.53E-2)
3	WFG7	0.856490 (3.46E-2)	**0.866678** (3.72E-2)	0.862995 (2.81E-2)	0.863982 (3.21E-2)	0.852158 (2.16E-2)
3	WFG8	0.804729 (2.57E-2)	0.816335 (2.80E-2)	**0.817947** (2.23E-2)	0.812730 (2.14E-2)	0.806158 (2.88E-2)
3	WFG9	0.887249 (3.17E-2) *	0.899238 (2.86E-2) *	0.904694 (2.52E-2)	**0.918614** (2.30E-2)	0.911698 (2.19E-2)
3	UF8	0.857957 (4.78E-4)	0.857928 (5.91E-4)	**0.858018** (5.59E-4)	0.857662 (4.03E-4)	0.857824 (5.43E-4)
3	UF9	0.615301 (3.79E-2) *	0.643631 (1.82E-2)	0.644259 (3.41E-2)	0.647270 (2.93E-2)	**0.656400** (5.34E-2)
3	UF10	0.854905 (3.41E-3)	**0.855473** (2.27E-3)	0.854761 (2.77E-3)	0.854865 (4.07E-3)	0.854786 (1.94E-3)

Table 2: Median and IQR values of the WOF-NSGA-II for different numbers of q. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-NSGA-II ($q = 1$)	WOF-NSGA-II ($q = 2$)	WOF-NSGA-II ($q = 3$)	WOF-NSGA-II ($q = 5$)	WOF-NSGA-II ($q = 8$)
2	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ2	**0.997601** (2.64E-4)	0.997418 (5.70E-4)	0.997480 (4.74E-4)	0.997417 (3.63E-4)	0.997093 (2.84E-4) *
2	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ4	**0.996971** (5.33E-4)	0.996396 (1.88E-1)	0.996067 (8.18E-4) *	0.995851 (1.36E-3) *	0.995214 (2.86E-3) *
2	DTLZ5	0.997946 (2.50E-4)	0.997959 (2.64E-4)	**0.998061** (6.45E-4)	0.997857 (3.84E-4)	0.997712 (5.62E-4) *
2	DTLZ6	0.999159 (7.13E-5)	**0.999165** (7.31E-5)	0.999129 (8.11E-5)	0.999113 (8.52E-5)	0.999124 (1.18E-4) *
2	DTLZ7	0.798231 (2.06E-5)	**0.798238** (1.68E-5)	0.798232 (1.83E-5)	0.798236 (1.18E-5)	0.798229 (2.02E-5)
2	WFG1	0.644332 (1.42E-2)	0.646255 (5.43E-3)	**0.646478** (8.82E-3)	0.644780 (4.42E-3)	0.643956 (9.66E-3)
2	WFG2	0.802035 (1.33E-2)	0.800923 (1.77E-2)	**0.802842** (1.91E-2)	0.800613 (5.22E-3)	0.798707 (1.19E-2)
2	WFG3	0.800700 (1.06E-2) *	**0.808753** (4.76E-3)	0.807231 (7.44E-3)	0.803227 (3.60E-3) *	0.796486 (5.45E-3) *
2	WFG4	0.851799 (2.49E-2)	0.842657 (3.15E-2)	0.856827 (1.57E-2)	**0.858775** (2.12E-2)	0.848662 (1.88E-2)
2	WFG5	0.873557 (2.49E-2)	0.871323 (4.42E-2)	0.865368 (8.09E-2)	0.877880 (7.59E-2)	**0.907891** (8.07E-2)
2	WFG6	0.829354 (3.40E-2) *	0.860678 (6.48E-2)	**0.876864** (7.32E-2)	0.835647 (6.00E-2)	0.830226 (5.47E-2) *
2	WFG7	0.860419 (4.70E-3)	**0.862006** (7.75E-3)	0.860684 (7.47E-3)	0.859763 (4.86E-3)	0.860528 (4.67E-3)
2	WFG8	0.804532 (2.17E-2)	**0.819398** (2.24E-2)	0.811861 (2.56E-2)	0.808399 (2.25E-2)	0.796978 (1.21E-2) *
2	WFG9	0.921250 (4.18E-2)	0.918818 (3.77E-2)	0.935157 (3.15E-2)	**0.935421** (3.12E-2)	0.931501 (2.07E-2)
2	UF1	0.901587 (1.16E-1)	0.864657 (1.60E-1)	**0.902002** (2.04E-2)	0.892736 (7.22E-3) *	0.883573 (6.15E-3) *
2	UF2	0.911294 (3.08E-3)	0.911111 (3.00E-3)	0.911595 (2.38E-3)	0.911524 (1.59E-3)	**0.911743** (1.63E-3)
2	UF3	0.979186 (3.70E-3) *	**0.981598** (4.45E-3)	0.980282 (3.16E-3)	0.979093 (3.17E-3) *	0.975321 (6.27E-3) *
2	UF4	0.884957 (2.32E-2) *	0.902977 (2.06E-2) *	0.906497 (5.19E-3)	0.888949 (2.39E-2) *	**0.910466** (2.06E-2)
2	UF5	**0.202595** (1.37E-1)	0.180071 (4.80E-2)	0.192806 (7.13E-2)	0.189315 (5.26E-2)	0.175859 (3.17E-2)
2	UF6	0.710128 (2.68E-1)	**0.712377** (1.45E-1)	0.694797 (6.93E-2)	0.659706 (5.03E-2)	0.626488 (4.57E-2) *
2	UF7	0.901126 (2.86E-2)	0.889958 (3.12E-2)	**0.908302** (2.33E-2)	0.902980 (3.43E-2)	0.890768 (2.11E-2) *
2	ZDT1	**0.998496** (1.53E-4)	0.998456 (8.30E-5)	0.998436 (1.55E-4)	0.998441 (1.34E-4)	0.998424 (1.86E-4)
2	ZDT2	0.998304 (1.16E-4)	0.998264 (1.27E-4)	**0.998315** (1.25E-4)	0.998304 (1.03E-4)	0.998311 (1.25E-4)
2	ZDT3	0.999292 (1.15E-4)	**0.999332** (6.04E-5)	0.999280 (9.21E-5)	0.999286 (9.94E-5)	0.999295 (7.64E-5)
2	ZDT4	— (—)	— (—)	— (—)	— (—)	— (—)
2	ZDT6	0.998640 (1.46E-4)	**0.998655** (1.11E-4)	0.998641 (8.69E-5)	0.998626 (1.56E-4)	0.998580 (2.19E-4) *
3	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ2	0.975414 (4.09E-3)	0.977036 (4.29E-3)	0.976089 (4.69E-3)	0.976742 (5.18E-3)	**0.978412** (8.52E-3)
3	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ4	0.973030 (2.00E-2)	**0.979217** (1.49E-2)	0.977994 (1.16E-2)	0.959289 (1.36E-2) *	0.949469 (1.87E-2) *
3	DTLZ5	0.979542 (1.30E-2)	**0.983113** (4.61E-3)	0.982375 (7.26E-3)	0.975713 (6.26E-3) *	0.974568 (1.93E-2)
3	DTLZ6	**0.999373** (1.12E-4)	0.999347 (2.12E-4)	0.999329 (1.80E-4)	0.999320 (1.22E-4)	0.999303 (1.35E-4)
3	DTLZ7	0.987866 (8.11E-2)	0.989221 (4.40E-2)	0.989626 (2.14E-3)	**0.990195** (1.62E-3)	0.989358 (3.27E-3)
3	WFG1	0.584165 (9.08E-3) *	0.589215 (7.33E-3)	0.589615 (9.74E-3)	0.590351 (5.86E-3)	**0.590974** (8.70E-3)
3	WFG2	0.803012 (5.58E-2) *	0.863410 (4.57E-2)	**0.867997** (1.79E-2)	0.864024 (1.34E-2)	0.850305 (5.79E-2) *
3	WFG3	0.789703 (2.26E-2) *	**0.804040** (2.47E-2)	0.802280 (3.91E-2)	0.802405 (2.11E-2)	0.777435 (1.64E-2) *
3	WFG4	0.785193 (3.49E-2) *	0.792066 (2.18E-2) *	0.805241 (3.25E-2)	**0.818848** (2.67E-2)	0.799452 (1.28E-2) *
3	WFG5	0.770073 (4.48E-2) *	**0.791207** (3.46E-2)	0.788228 (6.05E-2)	0.780974 (2.61E-2)	0.775045 (4.36E-2)
3	WFG6	0.746550 (6.42E-2)	0.748878 (8.47E-2)	0.791561 (9.82E-2)	**0.806926** (1.14E-1)	0.735521 (8.67E-2)
3	WFG7	0.780285 (2.38E-2) *	0.799950 (2.06E-2)	0.800210 (2.41E-2)	**0.808246** (2.13E-2)	0.803948 (8.53E-3)
3	WFG8	0.739168 (2.37E-2) *	0.747299 (2.42E-2)	**0.756687** (2.48E-2)	0.748587 (2.40E-2)	0.749643 (2.04E-2)
3	WFG9	0.824977 (2.29E-2) *	0.833897 (4.69E-2)	0.845746 (2.55E-2)	0.843011 (2.39E-2)	**0.846237** (2.16E-2)
3	UF8	0.856338 (2.64E-3) *	0.857164 (1.61E-3)	0.856790 (1.16E-3) *	**0.857423** (6.24E-4)	0.857334 (5.96E-4)
3	UF9	0.628111 (5.16E-2)	0.643891 (7.20E-2)	0.639850 (7.04E-2)	**0.649317** (5.56E-2)	0.643340 (2.41E-2)
3	UF10	**0.773924** (7.49E-2)	0.708974 (1.02E-1) *	0.689097 (9.12E-2) *	0.711341 (6.78E-2) *	0.724115 (3.74E-2) *

Table 3: Median and IQR values of the WOF-SMPSO for different numbers of γ. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-SMPSO ($\gamma = 2$)	WOF-SMPSO ($\gamma = 4$)	WOF-SMPSO ($\gamma = 8$)	WOF-SMPSO ($\gamma = 20$)	WOF-SMPSO ($\gamma = 50$)
2	DTLZ1	**0.642949** (5.90E-10)	0.642949 (—)	0.642949 (—)	0.642949 (—)	0.642949 (—)
2	DTLZ2	0.998753 (2.30E-4)	0.998765 (2.65E-4)	0.998741 (2.80E-4)	0.998808 (1.51E-4)	**0.998820** (1.60E-4)
2	DTLZ3	**0.520072** (9.33E-10)	0.520072 (—)	0.520072 (—)	0.520072 (—)	0.520072 (—)
2	DTLZ4	0.998765 (1.24E-4)	0.998758 (2.00E-4)	0.998787 (1.69E-4)	**0.998804** (8.35E-5)	0.998798 (1.23E-4)
2	DTLZ5	0.999158 (2.86E-4) *	0.999200 (1.65E-4) *	0.999235 (1.49E-4)	0.999241 (1.18E-4)	**0.999264** (5.50E-5)
2	DTLZ6	0.998901 (1.71E-1)	0.998893 (1.02E-3)	0.998719 (1.71E-1)	**0.998903** (1.01E-3)	— (—) *
2	DTLZ7	0.798206 (3.81E-5) *	0.798210 (2.84E-5) *	0.798209 (3.35E-5) *	**0.798227** (4.18E-5)	0.784392 (2.27E-2) *
2	WFG1	**0.694950** (5.90E-2)	0.648861 (1.54E-2) *	0.639921 (5.51E-3) *	0.634657 (4.02E-3) *	0.628998 (4.06E-3) *
2	WFG2	0.980668 (1.22E-2)	**0.982501** (9.73E-3)	0.975849 (1.17E-2) *	0.955752 (1.53E-2) *	0.884776 (1.21E-2) *
2	WFG3	0.853006 (8.83E-3) *	**0.856149** (2.48E-3)	0.849484 (5.78E-3) *	0.831407 (4.98E-3) *	0.797301 (8.20E-3) *
2	WFG4	0.974765 (5.46E-3) *	**0.979114** (6.07E-3)	0.963269 (1.07E-2) *	0.948314 (1.07E-2) *	0.920327 (1.21E-2) *
2	WFG5	0.909553 (1.17E-2) *	**0.947978** (1.83E-2)	0.934624 (1.91E-2)	0.901406 (1.67E-2) *	0.859790 (1.47E-2) *
2	WFG6	0.997796 (3.73E-4) *	**0.998502** (4.86E-4)	0.997785 (1.24E-3) *	0.996448 (3.36E-4) *	0.993412 (1.01E-2) *
2	WFG7	0.954256 (1.39E-2) *	**0.964590** (1.16E-2)	0.958233 (1.71E-2)	0.927240 (1.61E-2) *	0.883785 (1.04E-2) *
2	WFG8	0.877451 (2.34E-2) *	**0.891981** (2.11E-2)	0.881690 (1.60E-2)	0.855140 (1.28E-2) *	0.790396 (2.37E-2) *
2	WFG9	0.960199 (1.59E-2)	**0.969884** (1.84E-2)	0.968649 (1.65E-2)	0.953557 (2.05E-2) *	0.923713 (2.85E-2) *
2	UF1	0.781492 (3.19E-3) *	0.784139 (4.41E-3)	0.787625 (6.08E-3)	0.788363 (1.08E-2)	**0.791011** (9.81E-3)
2	UF2	0.931215 (5.52E-4)	0.931580 (1.10E-3)	**0.931655** (1.69E-3)	0.931332 (1.28E-3)	0.924573 (8.45E-3) *
2	UF3	0.981119 (4.86E-4) *	**0.988977** (8.76E-4)	0.988581 (5.12E-3)	0.977262 (6.12E-3) *	0.963826 (6.64E-3) *
2	UF4	0.910258 (2.92E-2)	0.913491 (2.12E-2)	**0.919816** (1.45E-2)	0.897540 (8.41E-3) *	0.889538 (2.64E-3) *
2	UF5	— (—)	— (—)	— (—)	— (—)	— (—)
2	UF6	0.247475 (1.54E-2)	0.262453 (3.26E-2)	0.271356 (1.40E-2)	0.274613 (5.16E-2)	**0.284470** (7.36E-2)
2	UF7	0.775884 (5.83E-3) *	0.777430 (7.56E-3)	0.780409 (2.58E-2)	**0.782123** (1.26E-2)	0.781782 (1.32E-2)
2	ZDT1	0.998114 (2.49E-4) *	0.998086 (3.37E-4) *	0.998037 (2.61E-4) *	**0.998473** (4.76E-4)	0.964800 (2.73E-2) *
2	ZDT2	0.997890 (4.33E-4)	0.997987 (3.59E-4)	0.998006 (4.32E-4)	0.998076 (4.16E-4)	**0.998670** (1.33E-2)
2	ZDT3	0.999106 (1.03E-4)	**0.999137** (2.67E-4)	0.999076 (1.62E-4)	0.998996 (8.40E-4)	0.928881 (5.97E-2) *
2	ZDT4	0.998702 (2.54E-4)	0.998695 (2.28E-4)	0.998681 (9.62E-5)	0.998680 (1.23E-4)	**0.998713** (8.14E-5)
2	ZDT6	0.998142 (3.25E-4) *	0.998214 (3.27E-4) *	0.998105 (3.07E-4) *	**0.998735** (5.95E-4)	— (—) *
3	DTLZ1	**0.587705** (—)	— (5.87E-1) *	— (—) *	— (—) *	— (—) *
3	DTLZ2	0.976746 (3.39E-3)	0.978836 (1.93E-3)	**0.979299** (4.55E-3)	0.977439 (3.87E-3)	0.974968 (5.24E-3) *
3	DTLZ3	**0.390379** (3.90E-1)	— (3.90E-1)	— (—) *	— (—) *	— (—) *
3	DTLZ4	1.001643 (3.89E-3)	1.001691 (4.87E-3)	**1.002836** (4.48E-3)	1.001086 (7.94E-3)	1.000844 (5.13E-3)
3	DTLZ5	0.998323 (3.41E-4)	**0.998366** (4.58E-4)	0.998162 (4.92E-4)	0.998314 (3.13E-4)	0.998251 (3.41E-4)
3	DTLZ6	0.998864 (4.59E-4)	0.998942 (5.81E-4)	**0.999776** (1.01E-3)	— (—) *	— (—) *
3	DTLZ7	0.979480 (6.89E-3)	**0.982147** (5.67E-3)	0.980456 (6.63E-3)	0.975697 (6.24E-3) *	0.892393 (3.68E-2) *
3	WFG1	**0.618637** (5.09E-2)	0.602072 (1.25E-2)	0.607194 (2.42E-3)	0.601980 (2.79E-3)	0.597314 (4.50E-3) *
3	WFG2	0.915173 (2.51E-2) *	**0.956949** (1.63E-2)	0.954588 (1.20E-2)	0.936699 (8.27E-3) *	0.893715 (2.98E-2) *
3	WFG3	0.955463 (2.42E-2)	**0.960519** (2.14E-2)	0.917918 (2.37E-2) *	0.868610 (2.12E-2) *	0.825592 (1.83E-2) *
3	WFG4	0.858388 (4.03E-2) *	0.901633 (3.59E-2)	**0.910815** (1.55E-2)	0.876940 (1.37E-2) *	0.823997 (1.73E-2) *
3	WFG5	0.778995 (1.30E-2) *	**0.845827** (3.66E-2)	0.823780 (2.85E-2) *	0.773733 (2.34E-2) *	0.737514 (1.79E-2) *
3	WFG6	0.945010 (1.56E-2) *	0.962727 (1.41E-2) *	**0.982320** (6.54E-3)	0.978942 (1.27E-2) *	0.920116 (4.83E-2) *
3	WFG7	0.831571 (4.23E-2) *	**0.863671** (2.73E-2)	0.862596 (1.47E-2)	0.839186 (1.53E-2) *	0.778312 (1.56E-2) *
3	WFG8	0.779011 (3.28E-2) *	0.813690 (2.91E-2) *	**0.836826** (2.82E-2)	0.778089 (2.33E-2) *	0.718953 (2.73E-2) *
3	WFG9	0.907784 (1.81E-2)	0.907874 (1.95E-2)	**0.913409** (1.93E-2)	0.890143 (2.33E-2) *	0.844928 (2.15E-2) *
3	UF8	**0.858872** (4.36E-3)	0.858001 (3.73E-4)	0.857777 (3.94E-4)	0.857858 (3.19E-4)	0.857484 (6.12E-4) *
3	UF9	0.640358 (6.24E-3)	0.641393 (8.89E-3)	**0.642703** (8.51E-3)	0.634103 (1.19E-2) *	0.600815 (2.61E-2) *
3	UF10	**0.855573** (3.08E-3)	0.855501 (3.07E-3)	0.855290 (1.88E-3)	0.854499 (1.97E-3)	0.855170 (3.44E-3)

Table 4: Median and IQR values of the WOF-NSGA-II for different numbers of γ. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-NSGA-II ($\gamma = 2$)	WOF-NSGA-II ($\gamma = 4$)	WOF-NSGA-II ($\gamma = 8$)	WOF-NSGA-II ($\gamma = 20$)	WOF-NSGA-II ($\gamma = 50$)
2	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ2	0.996843 (1.30E-3) *	**0.997676** (3.38E-4)	0.996420 (6.36E-4) *	0.954352 (1.39E-2) *	0.778339 (4.04E-2) *
2	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ4	0.992992 (1.75E-3) *	**0.996218** (1.25E-3)	0.995079 (1.47E-3) *	0.942645 (2.10E-2) *	0.743163 (4.24E-2) *
2	DTLZ5	0.997282 (1.23E-3) *	**0.998033** (4.93E-4)	0.996920 (8.76E-4) *	0.957879 (1.63E-2) *	0.765279 (6.02E-2) *
2	DTLZ6	0.999129 (7.88E-5)	**0.999145** (1.25E-4)	0.999136 (1.34E-4)	0.999108 (6.21E-3)	— (—) *
2	DTLZ7	0.645004 (1.53E-1) *	**0.798238** (1.92E-5)	0.798236 (1.31E-5)	0.798238 (9.97E-6)	0.797154 (7.42E-4) *
2	WFG1	**0.649799** (1.90E-2)	0.646938 (9.04E-3)	0.634147 (8.60E-3) *	0.610761 (2.05E-2) *	0.574902 (2.16E-2) *
2	WFG2	**0.831761** (6.63E-2)	0.801822 (9.76E-3)	0.805110 (8.33E-2) *	0.734301 (8.12E-2) *	0.773258 (8.10E-2) *
2	WFG3	0.785815 (5.90E-3) *	**0.806489** (8.98E-3)	0.805113 (5.92E-3)	0.781467 (1.18E-2) *	0.751727 (7.30E-3) *
2	WFG4	0.848078 (1.34E-2)	**0.860004** (1.46E-2)	0.858314 (1.66E-2)	0.830109 (2.29E-2) *	0.770953 (2.99E-2) *
2	WFG5	0.868532 (1.85E-2) *	0.873636 (9.98E-2)	**0.890709** (6.45E-2)	0.869424 (2.57E-2) *	0.805587 (1.98E-2) *
2	WFG6	**0.918005** (1.08E-1)	0.832192 (5.58E-2)	0.855965 (7.53E-3)	0.848273 (6.24E-3)	0.780020 (5.79E-2) *
2	WFG7	0.850291 (5.78E-3) *	**0.862764** (7.06E-3)	0.860227 (3.53E-3)	0.854059 (6.74E-3) *	0.828927 (1.19E-2) *
2	WFG8	0.784399 (1.98E-2) *	**0.819662** (1.68E-2)	0.794406 (2.54E-2) *	0.762321 (1.29E-2) *	0.716255 (1.74E-2) *
2	WFG9	**0.936475** (4.70E-2)	0.927312 (9.39E-3)	0.914824 (3.20E-2)	0.858273 (6.63E-2) *	0.694824 (1.00E-1) *
2	UF1	0.890988 (4.22E-2)	0.882098 (7.00E-2)	**0.906288** (5.08E-2)	0.893685 (8.58E-2)	0.895190 (6.34E-2)
2	UF2	0.911161 (1.75E-3)	**0.911494** (2.85E-3)	0.910565 (2.24E-3)	0.903514 (3.65E-3) *	0.890741 (8.52E-3) *
2	UF3	0.979569 (7.04E-4) *	**0.981905** (3.03E-3)	0.970614 (5.15E-3) *	0.926850 (2.44E-2) *	0.728335 (6.96E-2) *
2	UF4	**0.936498** (3.24E-3)	0.907145 (1.44E-2) *	0.898978 (9.51E-3) *	0.899406 (7.40E-3) *	0.891411 (8.02E-3) *
2	UF5	0.175332 (5.82E-2)	**0.198566** (4.32E-2)	0.184024 (1.20E-1)	0.184155 (7.02E-2)	0.143586 (8.79E-2) *
2	UF6	0.669223 (2.66E-2)	0.702190 (4.85E-2)	0.714327 (1.85E-1)	**0.724926** (1.19E-1)	0.680535 (4.90E-2)
2	UF7	0.895933 (2.74E-2)	**0.908850** (2.91E-2)	0.904017 (3.18E-2)	0.900073 (7.99E-2)	0.895479 (2.82E-2)
2	ZDT1	0.998440 (6.57E-5)	0.998443 (1.16E-4)	0.998415 (9.84E-5)	**0.998446** (1.14E-4)	0.997317 (7.07E-4) *
2	ZDT2	**0.998331** (1.08E-4)	0.998259 (1.22E-4) *	0.998263 (9.71E-5) *	0.998233 (1.39E-4) *	0.995387 (1.43E-3) *
2	ZDT3	0.841334 (6.72E-5) *	**0.999309** (5.70E-5)	0.999291 (9.98E-5)	0.999287 (1.29E-4)	0.997958 (1.33E-3) *
2	ZDT4	— (—)	— (—)	— (—)	— (—)	— (—)
2	ZDT6	0.998593 (1.37E-4)	0.998613 (9.64E-5)	0.998621 (9.82E-5)	**0.998632** (1.24E-4)	0.129794 (1.19E-1) *
3	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ2	0.945872 (1.53E-2) *	0.976586 (3.52E-3) *	**0.979442** (2.48E-3)	0.929577 (1.58E-2) *	0.597865 (1.12E-1) *
3	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ4	0.921198 (4.47E-2) *	0.979398 (9.93E-3) *	**0.988621** (8.22E-3)	0.928924 (3.50E-2) *	0.504794 (1.51E-1) *
3	DTLZ5	0.885355 (6.51E-2) *	0.982410 (7.79E-3) *	**0.986877** (3.22E-3)	0.897548 (3.76E-2) *	0.154356 (2.34E-1) *
3	DTLZ6	0.999341 (2.14E-4)	**0.999343** (1.65E-4)	0.999307 (1.92E-4)	0.998875 (9.99E-1) *	— (—) *
3	DTLZ7	0.823453 (1.24E-1) *	0.989339 (4.80E-3)	**0.989591** (2.58E-3)	0.989169 (2.45E-3)	0.981978 (4.03E-3) *
3	WFG1	0.578959 (1.07E-2) *	0.589692 (7.08E-3) *	**0.607215** (4.06E-3)	0.594066 (6.89E-3) *	0.584850 (4.89E-3) *
3	WFG2	0.808515 (1.62E-2) *	**0.863257** (2.93E-2)	0.811242 (7.56E-2) *	0.731805 (1.10E-2) *	0.707778 (7.40E-3) *
3	WFG3	0.722477 (3.12E-2) *	0.804676 (1.72E-2)	**0.805751** (3.18E-2)	0.755686 (3.53E-2) *	0.668887 (3.22E-2) *
3	WFG4	0.768438 (3.00E-2) *	**0.806999** (2.48E-2)	0.796184 (4.44E-2)	0.732222 (4.16E-2) *	0.646529 (4.32E-2) *
3	WFG5	0.777309 (3.90E-2) *	0.785732 (2.19E-2)	**0.825357** (7.82E-2)	0.738513 (5.03E-2) *	0.621502 (3.24E-2) *
3	WFG6	0.707678 (8.32E-2) *	0.736396 (3.58E-2) *	**0.756304** (9.53E-2)	0.711798 (7.06E-2) *	0.564885 (5.61E-2) *
3	WFG7	0.764515 (1.17E-2) *	0.804777 (1.82E-2) *	**0.830381** (2.41E-2)	0.779977 (1.45E-2) *	0.716761 (2.37E-2) *
3	WFG8	0.714908 (3.42E-2) *	0.752196 (1.89E-2) *	**0.777121** (3.13E-2)	0.733433 (1.49E-2) *	0.671263 (1.98E-2) *
3	WFG9	0.835715 (5.53E-2)	**0.846090** (1.84E-2)	0.805763 (5.38E-2) *	0.738342 (2.79E-2) *	0.668580 (4.32E-2) *
3	UF8	0.856182 (1.90E-3) *	0.857286 (1.04E-3)	**0.857444** (7.05E-4)	0.845619 (3.98E-3) *	0.782877 (1.16E-2) *
3	UF9	0.636536 (5.99E-2) *	**0.652847** (2.68E-2)	0.652518 (5.21E-2)	0.614633 (4.16E-2) *	0.562695 (1.30E-2) *
3	UF10	0.592852 (1.73E-1) *	**0.699435** (1.16E-1)	0.594050 (5.11E-2) *	0.251769 (9.40E-2) *	0.024859 (2.81E-2) *

Table 5: Median and IQR values of the WOF-SMPSO for different numbers of δ. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-SMPSO ($\delta = 0.10$)	WOF-SMPSO ($\delta = 0.25$)	WOF-SMPSO ($\delta = 0.50$)	WOF-SMPSO ($\delta = 0.75$)	WOF-SMPSO ($\delta = 1.00$)
2	DTLZ1	**0.642949** (—)	0.642949 (—)	0.642949 (—)	0.642949 (—)	0.642949 (2.81E-3)
2	DTLZ2	**0.998901** (3.89E-5)	0.998893 (7.39E-5)	0.998762 (1.51E-4) *	0.998323 (5.73E-4) *	0.997008 (6.24E-4) *
2	DTLZ3	**0.520072** (—)	0.520072 (—)	0.520072 (—)	0.520072 (—)	0.520072 (2.34E-1)
2	DTLZ4	**0.998881** (5.63E-5)	0.998869 (7.09E-5)	0.998756 (1.61E-4) *	0.998299 (5.38E-4) *	0.996101 (1.76E-3) *
2	DTLZ5	**0.999327** (5.73E-5)	0.999317 (7.86E-5)	0.999198 (1.30E-4) *	0.998772 (3.70E-4) *	0.997648 (5.30E-4) *
2	DTLZ6	— (6.24E-1) *	**0.999372** (1.88E-1)	0.999048 (1.10E-3)	0.998674 (1.71E-1)	0.998789 (3.58E-1)
2	DTLZ7	**0.798277** (6.58E-6)	0.798245 (3.21E-5) *	0.798211 (3.12E-5) *	0.798215 (2.40E-5) *	0.798233 (2.65E-5) *
2	WFG1	0.634431 (5.34E-3) *	0.639444 (3.28E-3) *	0.651233 (2.16E-2)	0.652642 (1.68E-2)	**0.657325** (2.15E-2)
2	WFG2	**0.983391** (1.65E-2)	0.982049 (1.38E-2)	0.982428 (1.27E-2)	0.981490 (8.01E-3)	0.977639 (1.24E-2)
2	WFG3	0.855541 (6.78E-3)	0.855701 (4.25E-3)	**0.856140** (3.86E-3)	0.855945 (4.53E-3)	0.854747 (4.03E-3)
2	WFG4	0.964853 (1.21E-2) *	0.970661 (1.45E-2) *	0.977649 (1.02E-2)	0.978465 (1.20E-2)	**0.984489** (1.27E-2)
2	WFG5	0.907148 (3.16E-2) *	0.931733 (2.77E-2) *	0.945941 (1.55E-2)	**0.961860** (2.91E-2)	0.956058 (2.33E-2)
2	WFG6	0.996823 (1.48E-3) *	0.998145 (1.29E-3) *	0.998457 (2.88E-4)	**0.998566** (4.22E-4)	0.998541 (2.80E-4)
2	WFG7	0.956010 (1.44E-2) *	0.958454 (1.16E-2)	**0.964229** (1.33E-2)	0.958906 (8.07E-3)	0.960700 (1.06E-2)
2	WFG8	**0.895063** (1.98E-2)	0.891250 (3.35E-2)	0.894340 (2.57E-2)	0.885006 (2.32E-2)	0.886898 (4.52E-2)
2	WFG9	0.956459 (1.48E-2) *	0.957507 (1.34E-2)	**0.966917** (1.06E-2)	0.961448 (1.12E-2)	0.966576 (1.14E-2)
2	UF1	0.784852 (1.90E-2)	0.785418 (3.79E-3)	0.785041 (4.88E-3)	0.786504 (4.74E-3)	**0.788618** (1.34E-2)
2	UF2	0.931702 (9.18E-4)	**0.931732** (7.65E-4)	0.931646 (5.88E-3)	0.931089 (2.60E-3)	0.931494 (4.87E-3)
2	UF3	0.987782 (3.79E-3) *	0.989193 (1.16E-3)	**0.989195** (1.39E-3)	0.989088 (1.39E-3)	0.989013 (7.05E-4)
2	UF4	0.898647 (6.51E-3) *	0.906905 (1.72E-2)	0.904990 (1.95E-2)	0.911543 (1.39E-2)	**0.918142** (2.45E-2)
2	UF5	— (—)	— (—)	— (—)	— (—)	— (—)
2	UF6	**0.265733** (1.32E-1)	0.254441 (4.95E-2)	0.254740 (7.48E-2)	0.259927 (1.38E-1)	0.257997 (7.02E-2)
2	UF7	0.779438 (5.20E-3)	**0.780202** (2.53E-3)	0.779529 (2.44E-2)	0.774977 (5.43E-3) *	0.777456 (1.15E-2)
2	ZDT1	**0.998864** (1.27E-5)	0.998269 (3.50E-4) *	0.998254 (3.48E-4) *	0.998294 (2.35E-4) *	0.998334 (3.07E-4) *
2	ZDT2	**0.998763** (1.45E-5)	0.998146 (5.26E-4) *	0.997935 (4.79E-4) *	0.997876 (3.66E-4) *	0.998213 (3.45E-4) *
2	ZDT3	**0.999347** (1.31E-4)	0.999152 (1.32E-4) *	0.999181 (1.76E-4) *	0.999176 (1.27E-4) *	0.999266 (1.59E-4) *
2	ZDT4	**0.998788** (5.24E-5)	0.998766 (4.76E-5)	0.998685 (3.32E-4) *	0.998209 (6.99E-4) *	— (1.86E-1) *
2	ZDT6	**0.998884** (3.33E-5)	0.998209 (3.23E-4) *	0.998223 (2.69E-4) *	0.998253 (1.97E-4) *	0.998452 (2.98E-4) *
3	DTLZ1	**0.587705** (—)	— (5.87E-1)	0.587705 (5.87E-1)	— (5.87E-1) *	— (—) *
3	DTLZ2	**0.981495** (1.79E-3)	0.980850 (2.41E-3)	0.978480 (3.02E-3) *	0.975750 (5.68E-3) *	0.974111 (4.69E-3) *
3	DTLZ3	**0.390379** (1.95E-1)	0.390379 (3.90E-1)	0.390379 (3.90E-1)	— (1.95E-1) *	— (—) *
3	DTLZ4	**1.004913** (3.35E-3)	1.004034 (3.01E-3)	1.001510 (4.20E-3) *	0.999960 (5.93E-3) *	0.996003 (4.21E-3) *
3	DTLZ5	**0.998471** (2.80E-4)	0.998437 (2.07E-4)	0.998215 (5.75E-4) *	0.997341 (1.80E-3) *	0.993916 (2.42E-3) *
3	DTLZ6	— (—) *	**0.999796** (9.39E-1)	0.998861 (4.53E-4)	0.998627 (4.87E-4)	0.999058 (6.15E-4)
3	DTLZ7	0.979700 (6.43E-3) *	0.978029 (4.75E-3) *	0.980351 (4.53E-3)	0.983162 (5.73E-3)	**0.985190** (6.96E-3)
3	WFG1	0.587214 (6.31E-3) *	0.595855 (9.24E-3) *	0.604991 (8.54E-3) *	0.602527 (6.40E-3) *	**0.621133** (1.90E-2)
3	WFG2	0.941857 (2.99E-2) *	0.950058 (1.49E-2)	0.959036 (1.21E-2)	0.956256 (1.66E-2)	**0.963586** (1.83E-2)
3	WFG3	0.947050 (1.71E-2)	0.950144 (2.97E-2)	**0.961685** (3.49E-2)	0.953640 (2.05E-2)	0.952776 (1.75E-2)
3	WFG4	0.829774 (2.76E-2) *	0.878313 (3.54E-2) *	0.890378 (3.71E-2)	**0.905355** (3.26E-2)	0.899048 (3.30E-2)
3	WFG5	0.770944 (2.82E-2) *	0.819386 (2.57E-2) *	0.846120 (4.26E-2)	0.852141 (3.74E-2)	**0.871365** (3.26E-2)
3	WFG6	0.946934 (2.73E-2) *	0.958477 (1.56E-2)	0.963425 (1.23E-2)	**0.966542** (1.38E-2)	0.958162 (1.58E-2)
3	WFG7	0.815287 (1.92E-2) *	0.854898 (1.56E-2) *	0.863750 (3.03E-2)	0.863018 (2.54E-2)	**0.880541** (2.86E-2)
3	WFG8	0.788923 (2.50E-2) *	0.803960 (3.11E-2) *	0.821105 (2.10E-2)	0.817186 (2.57E-2)	**0.823239** (2.64E-2)
3	WFG9	0.864874 (3.20E-2) *	0.895795 (2.48E-2) *	0.911907 (2.79E-2)	0.906436 (3.41E-2)	**0.926098** (2.64E-2)
3	UF8	0.857815 (5.39E-4)	0.857729 (3.73E-4)	**0.857925** (4.37E-4)	0.857877 (7.79E-4)	0.857511 (5.59E-4) *
3	UF9	0.639738 (4.10E-2)	0.642385 (2.17E-2)	0.641938 (1.55E-2)	**0.643934** (2.52E-2)	0.643115 (3.84E-2)
3	UF10	0.854543 (3.28E-3)	0.854622 (2.58E-3)	**0.854876** (3.11E-3)	0.854088 (3.56E-3)	0.816951 (6.39E-2) *

Table 6: Median and IQR values of the WOF-NSGA-II for different numbers of δ. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-NSGA-II ($\delta = 0.10$)	WOF-NSGA-II ($\delta = 0.25$)	WOF-NSGA-II ($\delta = 0.50$)	WOF-NSGA-II ($\delta = 0.75$)	WOF-NSGA-II ($\delta = 1.00$)
2	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ2	0.980641 (1.07E-2) *	0.995800 (2.91E-3) *	0.997516 (4.77E-4)	**0.997715** (3.19E-4)	0.996620 (6.91E-4) *
2	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ4	0.955032 (1.09E-2) *	0.987604 (5.36E-3) *	0.996433 (5.53E-4)	**0.996861** (7.73E-4)	0.996095 (9.94E-4) *
2	DTLZ5	0.979147 (2.37E-2) *	0.996901 (1.36E-3) *	**0.998039** (4.18E-4)	0.998023 (2.26E-4)	0.996993 (2.76E-4) *
2	DTLZ6	— (—) *	0.999084 (6.50E-5)	**0.999146** (8.48E-5)	0.999130 (9.03E-5)	0.998879 (2.12E-4) *
2	DTLZ7	0.796835 (4.00E-4) *	**0.798241** (1.03E-5)	0.798240 (1.49E-5)	0.798231 (1.69E-5)	0.798214 (1.72E-5) *
2	WFG1	0.612220 (5.06E-2) *	0.630640 (1.18E-2) *	0.646128 (5.92E-3)	**0.648453** (7.24E-3)	0.648406 (7.63E-3)
2	WFG2	0.769689 (7.73E-2) *	0.790205 (1.11E-2) *	0.805473 (1.68E-2)	0.807457 (4.50E-2)	**0.866753** (7.06E-2)
2	WFG3	0.750282 (1.61E-2) *	0.793083 (1.04E-2) *	0.808830 (1.08E-2) *	0.811576 (4.78E-3)	**0.816142** (7.85E-3)
2	WFG4	0.764010 (6.15E-2) *	0.834534 (3.71E-2) *	0.862971 (2.77E-2)	0.851615 (1.81E-2)	**0.866698** (1.88E-2)
2	WFG5	0.784259 (4.25E-2) *	0.857456 (3.15E-2) *	0.865101 (1.27E-1)	0.882043 (8.83E-2)	**0.907492** (9.20E-2)
2	WFG6	0.737752 (7.88E-2) *	0.826923 (1.70E-2)	0.835203 (6.69E-2)	0.844759 (7.23E-2)	**0.904695** (1.05E-1)
2	WFG7	0.823078 (2.24E-2) *	0.853331 (5.94E-3) *	**0.863234** (6.71E-3)	0.860579 (4.26E-3)	0.861290 (5.72E-3)
2	WFG8	0.732083 (2.11E-2) *	0.781766 (1.25E-2) *	0.814633 (1.68E-2) *	0.822770 (1.95E-2)	**0.831681** (1.71E-2)
2	WFG9	0.869612 (3.82E-2) *	0.915676 (3.30E-2) *	0.929706 (1.54E-2)	0.932066 (2.64E-2)	**0.936912** (2.02E-2)
2	UF1	**0.912757** (5.08E-2)	0.907185 (3.52E-2)	0.897132 (5.23E-2) *	0.888384 (7.34E-3) *	0.870400 (3.39E-2) *
2	UF2	0.901031 (1.43E-2) *	0.909399 (5.64E-3) *	0.911528 (1.79E-3)	**0.911553** (1.15E-3)	0.910488 (1.50E-3) *
2	UF3	0.869029 (3.86E-2) *	0.967095 (9.29E-3) *	0.981122 (2.42E-3) *	0.984516 (1.86E-3) *	**0.985395** (2.14E-3)
2	UF4	0.897508 (1.74E-2) *	0.906971 (2.44E-2)	**0.908138** (6.47E-3)	0.906254 (2.50E-2)	0.906224 (1.42E-2)
2	UF5	**0.248858** (9.43E-2)	0.245660 (4.42E-2)	0.212878 (8.91E-2)	0.109466 (7.75E-2) *	0.025811 (3.61E-2) *
2	UF6	**0.719705** (1.90E-1)	0.706889 (1.16E-1)	0.685373 (1.38E-1)	0.677654 (7.00E-2)	0.575743 (1.93E-1) *
2	UF7	0.906248 (3.37E-2)	0.889737 (4.05E-2)	**0.907899** (3.59E-2)	0.899180 (2.39E-2)	0.874864 (3.93E-2) *
2	ZDT1	0.997069 (5.25E-4) *	**0.998466** (1.12E-4)	0.998436 (1.23E-4)	0.998434 (1.54E-4)	0.998242 (1.87E-4) *
2	ZDT2	0.995955 (9.05E-4) *	**0.998265** (2.17E-4)	0.998186 (2.21E-4)	0.998241 (1.26E-4)	0.998134 (4.14E-4)
2	ZDT3	0.985127 (6.84E-2) *	0.999296 (8.23E-5)	0.999306 (7.38E-5)	**0.999317** (8.01E-5)	0.999139 (1.92E-4) *
2	ZDT4	— (—)	— (—)	— (—)	— (—)	— (—)
2	ZDT6	0.165030 (6.78E-2) *	0.998286 (3.78E-4) *	0.998646 (6.92E-5)	**0.998654** (5.98E-5)	0.998522 (3.45E-4) *
3	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ2	0.616122 (1.68E-1) *	0.936816 (2.05E-2) *	0.975454 (2.16E-3)	**0.977396** (2.90E-3)	0.971706 (3.00E-3) *
3	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ4	0.606573 (2.30E-1) *	0.906640 (4.11E-2) *	0.977484 (1.29E-2) *	**0.986115** (6.30E-3)	0.977653 (9.90E-3) *
3	DTLZ5	0.191279 (3.89E-1) *	0.870092 (4.57E-2) *	0.980470 (5.28E-3) *	**0.991457** (1.91E-3)	0.988631 (2.97E-3) *
3	DTLZ6	— (—) *	0.999069 (2.71E-4) *	**0.999399** (1.18E-4)	0.999391 (1.43E-4)	0.999151 (4.66E-4) *
3	DTLZ7	0.979096 (5.41E-3) *	0.988769 (3.93E-3)	**0.990659** (2.68E-3)	0.988506 (5.75E-3)	0.988615 (3.57E-3)
3	WFG1	0.573582 (9.25E-3) *	0.586031 (9.95E-3) *	0.590260 (7.07E-3)	**0.592624** (5.90E-3)	0.591687 (5.65E-3)
3	WFG2	0.767807 (6.32E-2) *	0.797423 (4.81E-2) *	0.861452 (5.28E-2) *	0.869774 (2.08E-2)	**0.876278** (1.39E-2)
3	WFG3	0.679926 (2.91E-2) *	0.755486 (3.93E-2) *	0.803246 (2.55E-2) *	0.827078 (2.41E-2)	**0.840874** (2.76E-2)
3	WFG4	0.664655 (4.11E-2) *	0.760034 (3.12E-2) *	0.806329 (1.87E-2) *	0.825743 (2.63E-2)	**0.836971** (1.87E-2)
3	WFG5	0.652192 (3.42E-2) *	0.754221 (2.78E-2) *	0.783480 (3.55E-2)	0.785222 (5.20E-2)	**0.806557** (4.59E-2)
3	WFG6	0.590996 (7.01E-2) *	0.722823 (4.38E-2) *	0.745883 (1.23E-1)	0.748959 (9.01E-2)	**0.811578** (1.44E-1)
3	WFG7	0.694004 (3.78E-2) *	0.773123 (1.58E-2) *	0.804930 (1.71E-2) *	0.812475 (1.70E-2)	**0.814716** (1.16E-2)
3	WFG8	0.634186 (2.27E-2) *	0.717715 (2.57E-2) *	0.755092 (2.00E-2) *	0.761284 (1.86E-2)	**0.768992** (3.20E-2)
3	WFG9	0.741291 (1.13E-1) *	0.826084 (3.44E-2) *	0.844207 (3.11E-2)	0.858806 (1.76E-2)	**0.867901** (2.69E-2)
3	UF8	0.853086 (2.49E-3) *	0.856683 (9.23E-4)	0.857201 (1.18E-3)	**0.857280** (1.47E-3)	0.856369 (1.41E-3) *
3	UF9	0.591604 (2.49E-2) *	0.622889 (5.36E-2) *	0.658849 (2.49E-2)	0.661696 (2.13E-2)	**0.664151** (1.88E-2)
3	UF10	0.485280 (1.27E-1) *	0.677565 (7.13E-2) *	**0.719782** (8.23E-2)	0.687488 (8.12E-2)	0.695152 (1.17E-1)

Table 7: Median and IQR values of the WOF-SMPSO for different numbers of p. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-SMPSO ($p = 0.05$)	WOF-SMPSO ($p = 0.10$)	WOF-SMPSO ($p = 0.20$)	WOF-SMPSO ($p = 0.30$)	WOF-SMPSO ($p = 0.50$)
2	DTLZ1	**0.642949** (—)	0.642949 (3.27E-10)	0.642949 (—)	0.642949 (—)	0.642949 (—)
2	DTLZ2	0.998676 (4.97E-4)	0.998677 (3.86E-4)	0.998719 (2.16E-4)	0.998775 (1.89E-4)	**0.998803** (2.09E-4)
2	DTLZ3	**0.520072** (2.32E-9)	0.520072 (—)	0.520072 (—)	0.520072 (—)	0.520072 (—)
2	DTLZ4	0.998790 (6.92E-5)	**0.998794** (1.87E-4)	0.998785 (1.96E-4)	0.998792 (1.03E-4)	0.998784 (1.41E-4)
2	DTLZ5	0.999151 (2.16E-4)	**0.999187** (2.21E-4)	0.999145 (1.48E-4)	0.999180 (1.99E-4)	0.999106 (2.63E-4)
2	DTLZ6	0.998726 (3.75E-1)	**0.998803** (1.88E-1)	0.998740 (3.40E-4)	0.998615 (5.50E-4)	0.998652 (5.62E-3)
2	DTLZ7	**0.798238** (3.32E-5)	0.798212 (2.56E-5) *	0.798204 (3.17E-5) *	0.798198 (1.87E-5) *	0.798191 (4.31E-5) *
2	WFG1	0.647699 (1.87E-2)	**0.651329** (2.07E-2)	0.644554 (1.49E-2)	0.645087 (1.04E-2)	0.649816 (1.34E-2)
2	WFG2	0.973361 (1.25E-2) *	0.979849 (1.18E-2)	**0.985077** (8.24E-3)	0.981384 (8.64E-3)	0.980233 (1.03E-2)
2	WFG3	0.854767 (5.65E-3)	0.855717 (8.20E-3)	0.855753 (2.57E-3)	0.854835 (5.05E-3)	**0.856893** (3.60E-3)
2	WFG4	0.971009 (1.11E-2) *	0.972524 (1.35E-2)	0.979782 (1.46E-2)	0.975561 (1.37E-2)	**0.980872** (1.16E-2)
2	WFG5	0.924229 (1.13E-2) *	0.932779 (2.46E-2) *	0.950748 (3.14E-2) *	0.950066 (3.18E-2) *	**0.978639** (3.14E-2)
2	WFG6	0.998194 (7.15E-4) *	0.998386 (4.93E-4)	**0.998590** (2.76E-4)	0.998534 (3.23E-4)	0.998466 (3.08E-4)
2	WFG7	0.956664 (9.89E-3) *	0.958325 (7.07E-3) *	0.961949 (1.19E-2) *	0.958925 (1.13E-2) *	**0.969344** (9.48E-3)
2	WFG8	0.873294 (3.62E-2) *	0.886360 (2.55E-2)	**0.888135** (3.11E-2)	0.887403 (2.44E-2)	0.884691 (1.28E-2)
2	WFG9	0.954797 (1.65E-2) *	0.963156 (1.63E-2)	0.967377 (1.85E-2)	0.965950 (1.35E-2)	**0.970429** (8.59E-3)
2	UF1	0.780871 (4.82E-3) *	0.783840 (1.50E-2)	**0.785007** (3.91E-3)	0.784361 (4.21E-3)	0.784617 (1.27E-2)
2	UF2	0.931608 (1.21E-3)	**0.931840** (1.98E-3)	0.931641 (8.75E-4)	0.931775 (1.08E-3)	0.931754 (1.60E-3)
2	UF3	0.988203 (1.34E-3) *	0.989103 (8.15E-4)	0.989076 (1.22E-3)	0.988424 (1.58E-3)	**0.989157** (9.98E-4)
2	UF4	0.893024 (7.84E-3) *	0.892034 (8.58E-3) *	0.911961 (1.31E-2) *	**0.956810** (3.66E-3)	0.954875 (3.98E-3) *
2	UF5	— (—)	— (—)	— (—)	— (—)	— (—)
2	UF6	0.242122 (1.07E-1) *	0.246394 (6.29E-2)	0.259255 (9.73E-2)	**0.260556** (8.25E-2)	0.257303 (2.77E-2)
2	UF7	0.774947 (6.99E-3) *	0.778893 (6.79E-3)	0.778292 (9.96E-3)	**0.779469** (1.29E-2)	0.777480 (7.18E-3)
2	ZDT1	**0.998798** (1.42E-4)	0.998020 (3.27E-4) *	0.998093 (2.27E-4) *	0.998158 (3.44E-4) *	0.998157 (3.73E-4) *
2	ZDT2	**0.998398** (8.56E-4)	0.998064 (4.55E-4)	0.997941 (2.44E-4) *	0.998031 (2.80E-4)	0.997979 (4.95E-4) *
2	ZDT3	0.997630 (3.21E-3) *	0.999077 (1.64E-4)	0.999091 (1.41E-4)	0.999048 (1.40E-4)	**0.999105** (1.93E-4)
2	ZDT4	0.998681 (2.54E-4)	0.998654 (4.04E-4)	**0.998693** (7.54E-5)	0.998651 (2.08E-4)	0.998662 (1.17E-4)
2	ZDT6	**0.998488** (5.17E-4)	0.998129 (2.78E-4) *	0.998248 (2.14E-4) *	0.998132 (6.21E-4) *	0.997997 (3.90E-4) *
3	DTLZ1	— (5.87E-1)	— (5.87E-1)	— (5.87E-1)	**0.587705** (5.87E-1)	0.587705 (3.82E-1)
3	DTLZ2	0.979506 (4.57E-3)	0.976088 (5.43E-3)	0.979270 (4.45E-3)	0.977669 (4.13E-3)	**0.980112** (5.11E-3)
3	DTLZ3	— (3.90E-1)	— (3.90E-1)	— (3.90E-1)	**0.390379** (3.90E-1)	0.390379 (1.95E-1)
3	DTLZ4	1.002438 (2.39E-3)	**1.002789** (2.76E-3)	1.000986 (5.76E-3)	1.002496 (4.61E-3)	1.002711 (3.98E-3)
3	DTLZ5	**0.998335** (4.71E-4)	0.998230 (4.37E-4)	0.998091 (7.78E-4)	0.998218 (4.52E-4)	0.998117 (7.69E-4)
3	DTLZ6	— (3.21E-2) *	0.121049 (9.99E-1)	0.998700 (9.12E-4)	**0.998773** (7.25E-4)	0.998712 (7.48E-4)
3	DTLZ7	0.968241 (6.44E-3) *	0.979740 (4.89E-3) *	0.981992 (5.67E-3)	0.982792 (5.23E-3)	**0.982946** (4.32E-3)
3	WFG1	0.595571 (1.65E-2) *	0.596238 (8.02E-3) *	0.602434 (1.01E-2)	0.607678 (1.05E-2)	**0.610790** (1.11E-2)
3	WFG2	0.933297 (2.08E-2) *	0.946023 (1.68E-2) *	**0.965311** (1.07E-2)	0.959160 (1.99E-2)	0.960408 (1.32E-2)
3	WFG3	0.942755 (2.26E-2)	0.948434 (1.55E-2)	0.949178 (2.53E-2)	**0.956257** (2.55E-2)	0.940223 (2.44E-2)
3	WFG4	0.846732 (3.51E-2) *	0.888461 (4.71E-2)	0.892551 (3.74E-2)	**0.906322** (2.75E-2)	0.905112 (3.22E-2)
3	WFG5	0.785504 (1.28E-2) *	0.821061 (2.26E-2) *	0.842773 (4.24E-2) *	0.864765 (3.47E-2)	**0.882125** (3.65E-2)
3	WFG6	0.952850 (9.48E-3) *	0.959607 (2.04E-2)	0.960581 (1.38E-2)	0.964774 (1.42E-2)	**0.970810** (1.57E-2)
3	WFG7	0.838669 (5.22E-2) *	0.851526 (1.90E-2) *	0.871164 (2.51E-2) *	0.869717 (2.00E-2) *	**0.889878** (1.24E-2)
3	WFG8	0.783287 (1.84E-2) *	0.805031 (1.91E-2)	**0.814287** (2.15E-2)	0.812454 (2.07E-2)	0.804684 (2.15E-2)
3	WFG9	0.857532 (2.05E-2) *	0.879754 (4.49E-2) *	0.907898 (2.30E-2)	0.910519 (2.18E-2)	**0.913804** (1.49E-2)
3	UF8	0.857696 (6.73E-4)	0.857940 (5.22E-4)	**0.857943** (2.44E-3)	0.857809 (8.31E-4)	0.857791 (4.35E-4)
3	UF9	**0.646317** (2.76E-2)	0.642435 (4.62E-3)	0.643393 (7.76E-3)	0.643177 (7.92E-3)	0.640801 (1.05E-2)
3	UF10	0.855283 (3.97E-3)	0.855110 (3.02E-3)	0.853991 (3.40E-3)	0.855612 (2.29E-3)	**0.855828** (2.85E-3)

Table 8: Median and IQR values of the WOF-NSGA-II for different numbers of p. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-NSGA-II ($p = 0.05$)	WOF-NSGA-II ($p = 0.10$)	WOF-NSGA-II ($p = 0.20$)	WOF-NSGA-II ($p = 0.30$)	WOF-NSGA-II ($p = 0.50$)
2	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ2	0.997004 (4.55E-4) *	**0.997524** (6.59E-4)	0.997363 (6.64E-4)	0.997393 (3.22E-4)	0.997058 (6.72E-4) *
2	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ4	0.995570 (1.43E-3) *	0.996082 (1.17E-3)	**0.996256** (1.34E-3)	0.996000 (8.82E-4)	0.995252 (1.43E-3) *
2	DTLZ5	0.997466 (4.59E-4) *	**0.998037** (3.73E-4)	0.997992 (5.84E-4)	0.997777 (3.50E-4) *	0.997464 (5.94E-4) *
2	DTLZ6	— (—) *	0.999128 (7.13E-5)	**0.999150** (9.63E-5)	0.999123 (1.17E-4)	0.999141 (9.69E-5)
2	DTLZ7	0.798123 (6.84E-5) *	0.798235 (1.60E-5)	**0.798239** (1.30E-5)	0.798224 (1.53E-1) *	0.645003 (1.69E-6) *
2	WFG1	0.614503 (1.61E-2) *	0.631913 (1.44E-3) *	0.646740 (9.30E-3) *	0.648727 (8.13E-3)	**0.654954** (1.13E-2)
2	WFG2	0.780470 (7.00E-2) *	0.773259 (7.30E-2) *	0.800406 (1.31E-2) *	0.885966 (5.05E-2) *	**0.936733** (8.55E-3)
2	WFG3	0.761214 (8.63E-3) *	0.792951 (1.24E-2) *	0.807126 (6.22E-3) *	0.809016 (8.31E-3)	**0.813139** (6.78E-3)
2	WFG4	0.747365 (4.23E-2) *	0.843665 (1.73E-2) *	0.866891 (2.36E-2) *	0.883719 (1.75E-2) *	**0.906159** (1.00E-2)
2	WFG5	0.803494 (3.32E-2) *	0.872709 (3.03E-2) *	0.926599 (9.16E-2) *	0.906759 (5.68E-2) *	**0.945576** (5.57E-2)
2	WFG6	0.745599 (2.53E-2) *	0.827927 (7.73E-3) *	0.834749 (5.08E-2) *	0.978617 (7.14E-2)	**0.987645** (1.26E-2)
2	WFG7	0.832956 (7.54E-3) *	**0.862512** (9.38E-3)	0.861105 (3.75E-3)	0.860461 (5.61E-3)	0.857720 (8.49E-3)
2	WFG8	0.750011 (9.02E-3) *	0.785470 (1.04E-2) *	0.820924 (1.65E-2)	0.816016 (3.08E-2)	**0.835987** (4.35E-2)
2	WFG9	0.679320 (1.43E-2) *	0.856551 (9.85E-2) *	0.934883 (2.12E-2) *	0.952356 (2.02E-2) *	**0.964272** (1.56E-2)
2	UF1	0.895596 (1.31E-1)	0.897264 (8.30E-2)	0.896877 (1.04E-1)	**0.900885** (5.23E-3)	0.899327 (1.56E-2)
2	UF2	0.912036 (1.53E-3)	0.911358 (1.70E-3)	0.911526 (2.32E-3)	0.910787 (2.29E-3)	**0.917761** (2.44E-2)
2	UF3	0.915212 (3.64E-2) *	0.978437 (4.14E-3) *	**0.981865** (1.76E-3)	0.981377 (3.46E-3)	0.977021 (5.63E-3) *
2	UF4	0.883050 (1.06E-2) *	0.884085 (8.56E-3) *	0.905257 (2.40E-2) *	0.928592 (1.80E-2) *	**0.941992** (4.97E-3)
2	UF5	0.124285 (1.49E-1) *	0.123479 (1.79E-1) *	0.186211 (8.21E-2) *	0.222767 (5.73E-2)	**0.254951** (5.11E-2)
2	UF6	0.657581 (2.42E-1)	0.675395 (2.19E-1)	0.688466 (1.55E-1)	**0.700089** (3.17E-2)	0.677217 (2.53E-2)
2	UF7	0.897700 (6.08E-2) *	0.902638 (3.48E-2)	0.895692 (2.34E-2) *	0.908360 (2.23E-2)	**0.912768** (7.25E-3)
2	ZDT1	0.997807 (7.73E-4) *	0.998457 (8.25E-5)	0.998416 (1.22E-4)	0.998420 (1.58E-4)	**0.998473** (1.02E-4)
2	ZDT2	0.997731 (3.60E-4) *	0.998265 (1.18E-4)	**0.998294** (1.04E-4)	0.998241 (1.20E-4) *	0.998271 (1.33E-4)
2	ZDT3	0.993365 (8.65E-3) *	0.999273 (1.26E-4)	**0.999321** (9.55E-5)	0.999307 (7.12E-5)	0.999310 (4.23E-5)
2	ZDT4	— (—)	— (—)	— (—)	— (—)	— (—)
2	ZDT6	0.196177 (3.44E-1) *	0.998625 (1.74E-1)	0.998620 (1.09E-4)	**0.998647** (9.68E-5)	0.998616 (8.32E-5)
3	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ2	0.928744 (3.30E-2) *	0.972007 (4.77E-3) *	**0.977184** (4.40E-3)	0.976264 (2.14E-3)	0.975899 (1.92E-3)
3	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ4	0.967296 (1.82E-2) *	0.972312 (1.52E-2)	0.976665 (1.36E-2)	0.976007 (1.00E-2)	**0.980364** (1.35E-2)
3	DTLZ5	0.335025 (8.04E-1) *	0.969894 (1.50E-2) *	0.982798 (3.59E-3)	**0.983785** (5.43E-3)	0.983204 (4.28E-3)
3	DTLZ6	— (—) *	0.999244 (2.03E-4)	0.999316 (1.62E-4)	**0.999350** (1.51E-4)	0.999295 (2.01E-4)
3	DTLZ7	0.986621 (2.68E-3) *	0.988920 (3.39E-3)	**0.990637** (3.83E-3)	0.990097 (8.19E-2)	0.823238 (2.49E-4) *
3	WFG1	0.575839 (7.91E-3) *	0.585748 (9.14E-3) *	0.588649 (7.98E-3) *	0.591747 (7.01E-3) *	**0.601565** (1.07E-2)
3	WFG2	0.711845 (4.78E-3) *	0.722352 (6.22E-2) *	0.862129 (2.00E-2) *	0.880369 (1.54E-2)	**0.882038** (1.72E-2)
3	WFG3	0.718937 (2.97E-2) *	0.765044 (2.82E-2) *	0.812316 (2.03E-2)	0.818314 (2.65E-2)	**0.823586** (2.39E-2)
3	WFG4	0.637640 (2.62E-2) *	0.753559 (3.81E-2) *	0.809127 (1.69E-2) *	0.820751 (1.78E-2) *	**0.853251** (2.19E-2)
3	WFG5	0.637027 (2.77E-2) *	0.733832 (2.98E-2) *	0.802614 (3.58E-2) *	0.833740 (5.08E-2) *	**0.878317** (4.03E-2)
3	WFG6	0.596926 (3.89E-2) *	0.703806 (7.15E-2) *	0.730915 (3.08E-2) *	0.865284 (9.56E-2) *	**0.945327** (1.78E-2)
3	WFG7	0.724924 (1.26E-2) *	0.785660 (2.07E-2) *	0.803748 (1.69E-2)	0.803317 (1.44E-2)	**0.813188** (2.10E-2)
3	WFG8	0.680447 (1.44E-2) *	0.732735 (2.11E-2) *	0.749929 (1.51E-2)	0.757326 (2.27E-2)	**0.762244** (2.64E-2)
3	WFG9	0.585370 (7.08E-2) *	0.765783 (9.25E-2) *	0.846195 (2.34E-2) *	0.878024 (2.63E-2) *	**0.909701** (1.64E-2)
3	UF8	0.856538 (1.22E-3) *	0.857170 (7.18E-4)	0.857226 (1.05E-3)	**0.857358** (6.50E-4)	0.857340 (8.80E-4)
3	UF9	0.642848 (6.14E-2)	0.654463 (2.06E-2)	**0.660160** (3.67E-2)	0.657589 (6.10E-2)	0.660002 (2.92E-2)
3	UF10	0.469623 (8.27E-2) *	0.621704 (1.16E-1) *	0.687531 (5.94E-2) *	0.748563 (6.17E-2)	**0.764403** (5.48E-2)

Table 9: Median and IQR values of the WOF-SMPSO for different numbers of t_1. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-SMPSO ($t_1 = 400$)	WOF-SMPSO ($t_1 = 800$)	WOF-SMPSO ($t_1 = 1000$)	WOF-SMPSO ($t_1 = 1500$)	WOF-SMPSO ($t_1 = 2000$)
2	DTLZ1	**0.642949** (—)	0.642949 (1.78E-9)	0.642949 (—)	0.642949 (—)	0.642949 (—)
2	DTLZ2	0.998782 (2.32E-4)	**0.998820** (1.05E-4)	0.998739 (1.24E-4)	0.998665 (3.61E-4) *	0.998538 (4.01E-4) *
2	DTLZ3	**0.520072** (—)	0.520072 (—)	0.520072 (1.49E-9)	0.520072 (—)	0.520072 (—)
2	DTLZ4	0.998755 (1.94E-4)	**0.998807** (1.46E-4)	0.998787 (1.82E-4)	0.998735 (2.52E-4)	0.998520 (3.91E-4) *
2	DTLZ5	**0.999230** (7.09E-5)	0.999222 (1.45E-4)	0.999065 (4.72E-4) *	0.999068 (1.65E-4) *	0.999063 (4.03E-4) *
2	DTLZ6	**0.998895** (3.58E-1)	0.998882 (9.33E-4)	0.998868 (1.88E-1)	0.998713 (3.59E-1)	0.998683 (1.71E-1)
2	DTLZ7	0.798210 (3.56E-5)	**0.798211** (2.49E-5)	0.798209 (3.47E-5)	0.798204 (2.58E-5)	0.798208 (3.31E-5)
2	WFG1	0.645167 (1.53E-2)	0.644094 (2.20E-2)	0.646391 (1.58E-2)	0.643012 (5.04E-3)	**0.646654** (1.63E-2)
2	WFG2	0.972986 (9.16E-3) *	0.979357 (5.87E-3) *	0.981818 (1.40E-2)	**0.988503** (1.16E-2)	0.986145 (1.06E-2)
2	WFG3	0.839836 (6.49E-3) *	0.854434 (6.27E-3)	0.855228 (4.25E-3)	0.856388 (3.07E-3)	**0.856759** (3.26E-3)
2	WFG4	0.966154 (1.66E-2) *	0.977767 (1.33E-2)	0.976668 (1.46E-2)	**0.980056** (1.46E-2)	0.977095 (1.17E-2)
2	WFG5	**0.959149** (2.13E-2)	0.953970 (2.04E-2)	0.949982 (3.34E-2)	0.942738 (2.92E-2)	0.947863 (2.04E-2)
2	WFG6	**0.998551** (3.15E-4)	0.998522 (2.71E-4)	0.998361 (5.38E-4)	0.998461 (4.10E-4)	0.998516 (3.79E-4)
2	WFG7	0.954342 (1.32E-2) *	0.962221 (9.28E-3)	0.960835 (1.75E-2)	**0.963998** (1.41E-2)	0.961216 (1.17E-2)
2	WFG8	0.882333 (3.69E-2)	**0.898493** (4.56E-2)	0.883817 (2.65E-2)	0.888279 (2.03E-2)	0.887525 (2.47E-2)
2	WFG9	**0.968860** (1.24E-2)	0.965314 (1.90E-2)	0.966175 (7.08E-3)	0.964841 (1.24E-2)	0.965677 (1.16E-2)
2	UF1	**0.785980** (2.73E-2)	0.784391 (4.06E-3)	0.783111 (3.48E-3)	0.784766 (4.83E-3)	0.784848 (3.02E-3)
2	UF2	**0.931975** (3.15E-3)	0.931743 (5.01E-3)	0.931243 (6.29E-4)	0.931628 (4.11E-4)	0.931418 (4.96E-4)
2	UF3	0.988367 (1.06E-3) *	0.988904 (1.31E-3)	0.989223 (1.30E-3)	**0.989331** (9.65E-4)	0.989135 (1.36E-3)
2	UF4	0.919877 (1.92E-2)	0.917202 (1.83E-2)	**0.922431** (2.98E-2)	0.909743 (1.70E-2)	0.910421 (2.35E-2)
2	UF5	— (—)	— (—)	— (—)	— (—)	— (—)
2	UF6	0.250476 (1.20E-1)	0.257147 (3.28E-2)	**0.268720** (6.01E-2)	0.256222 (4.20E-2)	0.256854 (1.72E-2)
2	UF7	0.778047 (8.95E-3)	0.776927 (6.50E-3)	0.776867 (9.06E-3)	**0.778563** (5.32E-3)	0.776092 (9.33E-3)
2	ZDT1	0.998209 (3.65E-4)	0.998089 (4.12E-4)	**0.998220** (3.02E-4)	0.998177 (3.76E-4)	0.997988 (2.86E-4)
2	ZDT2	**0.998060** (3.39E-4)	0.997989 (4.23E-4)	0.997963 (4.27E-4)	0.997795 (6.82E-4)	0.997846 (5.31E-4)
2	ZDT3	0.999170 (1.61E-4)	0.999145 (1.67E-4)	0.999128 (1.38E-4)	**0.999174** (1.48E-4)	0.999105 (1.54E-4)
2	ZDT4	0.998648 (4.73E-4)	**0.998660** (3.22E-4)	0.998659 (3.73E-4)	0.998610 (3.58E-4)	0.998476 (4.01E-4)
2	ZDT6	**0.998290** (2.34E-4)	0.998223 (3.15E-4)	0.998081 (3.88E-4)	0.998137 (4.56E-4)	0.998223 (3.35E-4)
3	DTLZ1	— (—) *	— (5.87E-1)	— (5.87E-1)	— (5.87E-1)	**0.587705** (5.87E-1)
3	DTLZ2	0.975423 (4.82E-3) *	0.978236 (3.10E-3)	0.979159 (4.91E-3)	0.979746 (3.12E-3)	**0.979788** (3.92E-3)
3	DTLZ3	— (—)	— (3.90E-1)	— (—)	**0.390379** (3.90E-1)	0.390379 (—)
3	DTLZ4	1.001828 (5.09E-3)	1.001730 (4.54E-3)	1.002188 (4.28E-3)	1.001742 (4.99E-3)	**1.003176** (3.49E-3)
3	DTLZ5	**0.998260** (5.25E-4)	0.998092 (8.25E-4)	0.998213 (7.45E-4)	0.998236 (3.98E-4)	0.998059 (7.32E-4)
3	DTLZ6	0.998827 (4.35E-4)	0.998708 (6.96E-4)	0.998916 (6.78E-4)	**0.999114** (9.36E-4)	0.998914 (7.56E-4)
3	DTLZ7	0.980894 (3.96E-3)	0.978356 (5.78E-3)	0.981271 (5.97E-3)	0.981101 (4.78E-3)	**0.981300** (5.24E-3)
3	WFG1	**0.605098** (8.18E-3)	0.602754 (7.66E-3)	0.599443 (6.68E-3)	0.599190 (5.80E-3)	0.598653 (7.51E-3)
3	WFG2	0.950485 (1.17E-2) *	0.953862 (1.63E-2)	**0.965182** (1.95E-2)	0.961242 (1.35E-2)	0.963595 (6.79E-3)
3	WFG3	0.933855 (2.41E-2) *	0.943159 (3.54E-2) *	0.958334 (3.41E-2)	**0.961983** (2.33E-2)	0.944663 (3.26E-2)
3	WFG4	0.892150 (2.32E-2)	**0.901133** (3.02E-2)	0.889625 (2.06E-2)	0.886064 (1.66E-2)	0.893642 (4.48E-2)
3	WFG5	**0.854477** (3.74E-2)	0.838531 (4.55E-2)	0.849303 (3.74E-2)	0.838210 (4.37E-2)	0.825427 (3.49E-2) *
3	WFG6	**0.972182** (1.81E-2)	0.965149 (1.23E-2)	0.966428 (1.84E-2)	0.965347 (1.78E-2)	0.963800 (1.49E-2)
3	WFG7	**0.864691** (2.20E-2)	0.849416 (1.49E-2)	0.857360 (2.50E-2)	0.853222 (2.88E-2)	0.856561 (2.37E-2)
3	WFG8	0.816116 (3.31E-2)	0.807692 (2.92E-2)	0.813789 (3.36E-2)	**0.816477** (2.47E-2)	0.813632 (3.00E-2)
3	WFG9	**0.913365** (2.52E-2)	0.912642 (2.71E-2)	0.900832 (2.61E-2)	0.893330 (2.28E-2) *	0.898379 (2.81E-2)
3	UF8	0.857925 (4.48E-4)	**0.857960** (3.80E-4)	0.857940 (4.56E-4)	0.857950 (2.24E-3)	0.857803 (7.10E-4)
3	UF9	0.640312 (1.56E-2)	0.641026 (7.19E-3)	0.642081 (1.18E-2)	**0.644007** (1.05E-2)	0.643385 (4.24E-2)
3	UF10	**0.856401** (2.39E-3)	0.855014 (2.60E-3)	0.854942 (2.79E-3)	0.854899 (3.43E-3)	0.855289 (1.55E-3)

Table 10: Median and IQR values of the WOF-NSGA-II for different numbers of t_1. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-NSGA-II ($t_1 = 400$)	WOF-NSGA-II ($t_1 = 800$)	WOF-NSGA-II ($t_1 = 1000$)	WOF-NSGA-II ($t_1 = 1500$)	WOF-NSGA-II ($t_1 = 2000$)
2	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ2	**0.997787** (4.37E-4)	0.997699 (4.60E-4)	0.997586 (4.60E-4)	0.997558 (4.42E-4)	0.997493 (1.71E-4) *
2	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ4	0.994679 (1.83E-3) *	0.996115 (1.17E-3)	0.996353 (1.29E-3)	**0.996563** (9.51E-4)	0.996553 (7.58E-4)
2	DTLZ5	**0.998138** (7.39E-4)	0.997918 (4.43E-4)	0.997899 (5.90E-4)	0.997914 (4.97E-4)	0.997979 (2.70E-4)
2	DTLZ6	0.999135 (7.99E-5)	**0.999138** (1.20E-4)	0.999120 (8.89E-5)	0.999121 (9.27E-5)	0.999114 (9.36E-5)
2	DTLZ7	0.798232 (7.66E-2)	0.798233 (1.75E-5)	**0.798238** (2.19E-5)	0.798237 (2.16E-5)	0.798235 (1.70E-5)
2	WFG1	0.646354 (6.53E-3)	0.647749 (7.06E-3)	0.644980 (8.47E-3)	0.647394 (5.30E-3)	**0.648246** (8.36E-3)
2	WFG2	0.802290 (1.53E-2)	0.802131 (2.23E-2)	**0.804394** (1.08E-2)	0.800505 (7.77E-3)	0.801779 (6.25E-3)
2	WFG3	**0.813589** (7.74E-3)	0.810337 (7.03E-3)	0.806035 (6.75E-3) *	0.803886 (9.52E-3) *	0.801577 (8.25E-3) *
2	WFG4	**0.877519** (3.31E-2)	0.864133 (1.71E-2)	0.863415 (1.65E-2)	0.854500 (1.82E-2)	0.844784 (1.15E-2) *
2	WFG5	0.879005 (8.11E-2)	0.874525 (7.38E-2)	0.868729 (9.47E-2)	0.870247 (9.16E-2)	**0.879121** (5.67E-2)
2	WFG6	**0.892840** (4.35E-2)	0.844481 (6.55E-2)	0.831705 (5.97E-2) *	0.834365 (5.90E-2) *	0.830587 (2.18E-2) *
2	WFG7	0.860894 (5.44E-3)	**0.862607** (3.64E-3)	0.860997 (6.48E-3)	0.859772 (6.91E-3)	0.861482 (6.98E-3)
2	WFG8	0.817108 (1.96E-2)	**0.820246** (1.27E-2)	0.818165 (1.26E-2)	0.809397 (1.60E-2) *	0.803291 (1.69E-2) *
2	WFG9	**0.938712** (1.69E-2)	0.936978 (2.17E-2)	0.933668 (1.68E-2)	0.916900 (3.86E-2) *	0.938615 (2.45E-2)
2	UF1	0.887084 (5.13E-2) *	0.878911 (4.66E-2)	**0.901691** (4.88E-2)	0.898933 (7.00E-2)	0.860396 (6.91E-2)
2	UF2	0.911392 (1.57E-3)	0.911687 (2.27E-3)	0.911830 (1.50E-3)	**0.911940** (1.55E-3)	0.911866 (2.56E-3)
2	UF3	0.981637 (3.18E-3)	**0.982569** (2.46E-3)	0.981256 (4.03E-3)	0.979742 (4.41E-3)	0.978771 (3.70E-3) *
2	UF4	**0.907383** (1.05E-2)	0.905169 (1.15E-2)	0.905179 (2.15E-2)	0.889966 (2.41E-2) *	0.906880 (2.23E-2)
2	UF5	0.140262 (6.97E-2) *	0.188895 (4.52E-2)	0.208617 (8.23E-2)	0.202555 (1.48E-1)	**0.225024** (1.34E-1)
2	UF6	0.655568 (5.22E-2) *	0.681887 (6.79E-2)	0.686155 (2.04E-1)	0.689249 (2.38E-1)	**0.702801** (5.10E-2)
2	UF7	0.890876 (3.40E-2) *	0.889396 (5.40E-2) *	0.905731 (3.07E-2)	0.898482 (2.76E-2)	**0.909422** (2.51E-2)
2	ZDT1	0.998461 (1.07E-4)	0.998429 (1.29E-4)	0.998429 (1.63E-4)	**0.998466** (1.31E-4)	0.998418 (1.64E-4)
2	ZDT2	0.998211 (1.80E-4)	0.998261 (1.01E-4)	0.998283 (1.65E-4)	**0.998342** (1.47E-4)	0.998266 (1.30E-4)
2	ZDT3	**0.999319** (4.51E-5)	0.999308 (5.45E-5)	0.999299 (6.12E-5)	0.999308 (8.88E-5)	0.999299 (7.35E-5)
2	ZDT4	— (—)	— (—)	— (—)	— (—)	— (—)
2	ZDT6	**0.998640** (4.37E-5)	0.998625 (1.05E-4)	0.998600 (1.00E-4)	0.998634 (6.04E-5)	0.998573 (1.23E-4) *
3	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ2	0.970740 (6.60E-3) *	0.976107 (4.70E-3)	0.976257 (3.84E-3)	0.976269 (3.06E-3)	**0.977925** (4.49E-3)
3	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ4	0.956953 (2.11E-2) *	0.973469 (1.35E-2) *	0.975495 (1.28E-2) *	0.980395 (8.37E-3)	**0.984954** (8.98E-3)
3	DTLZ5	0.946741 (2.56E-2) *	0.981269 (4.47E-3) *	0.982569 (6.98E-3) *	0.985972 (4.65E-3)	**0.986454** (3.60E-3)
3	DTLZ6	0.999325 (1.17E-4)	0.999357 (1.66E-4)	0.999363 (1.33E-4)	**0.999372** (1.65E-4)	0.999332 (1.71E-4)
3	DTLZ7	0.987945 (4.38E-2) *	0.989219 (3.84E-3) *	0.990599 (2.42E-3)	0.990492 (3.07E-3)	**0.990837** (2.15E-3)
3	WFG1	**0.591116** (8.46E-3)	0.588944 (6.96E-3)	0.588790 (5.69E-3)	0.590617 (7.65E-3)	0.587666 (5.79E-3)
3	WFG2	**0.873938** (1.48E-2)	0.871840 (1.36E-2)	0.862696 (2.94E-2) *	0.848066 (5.03E-2) *	0.827471 (6.63E-2) *
3	WFG3	0.816307 (1.86E-2)	**0.820059** (2.11E-2)	0.801267 (1.92E-2) *	0.805636 (2.34E-2) *	0.789678 (3.34E-2) *
3	WFG4	**0.823463** (2.31E-2)	0.814765 (1.94E-2)	0.802794 (2.64E-2) *	0.810515 (3.51E-2) *	0.784792 (2.48E-2) *
3	WFG5	0.803714 (2.75E-2)	**0.804159** (2.48E-2)	0.801844 (6.40E-2)	0.788132 (3.22E-2)	0.779043 (5.62E-2)
3	WFG6	**0.775831** (8.40E-2)	0.751077 (1.14E-1)	0.740190 (9.63E-2)	0.739915 (9.22E-2)	0.753801 (8.83E-2)
3	WFG7	0.805766 (1.94E-2)	**0.807128** (2.50E-2)	0.798382 (1.85E-2)	0.796843 (9.16E-3)	0.793321 (1.51E-2) *
3	WFG8	**0.757215** (1.36E-2)	0.755871 (2.51E-2)	0.751879 (1.49E-2)	0.750656 (2.82E-2)	0.744023 (2.54E-2)
3	WFG9	**0.858508** (1.79E-2)	0.851211 (2.05E-2)	0.842097 (3.14E-2)	0.832771 (2.47E-2) *	0.842882 (2.12E-2) *
3	UF8	**0.857649** (9.78E-4)	0.857528 (1.02E-3)	0.857543 (7.23E-4)	0.857382 (1.17E-3)	0.857463 (4.79E-4)
3	UF9	0.644630 (2.70E-2)	**0.655444** (6.65E-2)	0.643011 (7.26E-2)	0.649731 (6.92E-2)	0.654694 (6.16E-2)
3	UF10	**0.736837** (7.25E-2)	0.727450 (1.00E-1)	0.706797 (6.65E-2)	0.686247 (5.69E-2) *	0.696944 (4.14E-2) *

Table 11: Median and IQR values of the WOF-SMPSO for different numbers of t_2. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		WOF-SMPSO ($t_2 = 100$)	WOF-SMPSO ($t_2 = 300$)	WOF-SMPSO ($t_2 = 500$)	WOF-SMPSO ($t_2 = 700$)	WOF-SMPSO ($t_2 = 900$)
2	DTLZ1	**0.642949** (—)	0.642949 (—)	0.642949 (1.04E-9)	0.642949 (2.77E-9)	0.642949 (3.53E-9)
2	DTLZ2	**0.998797** (7.79E-5)	0.998729 (1.92E-4)	0.998794 (2.55E-4)	0.998784 (1.67E-4)	0.998796 (6.92E-5)
2	DTLZ3	**0.520072** (—)	0.520072 (—)	0.520072 (3.74E-10)	0.520072 (—)	0.520072 (—)
2	DTLZ4	**0.998830** (1.02E-4)	0.998765 (1.97E-4)	0.998802 (9.39E-5)	0.998799 (8.30E-5)	0.998795 (1.08E-4)
2	DTLZ5	0.999175 (1.58E-4)	0.999116 (4.27E-4)	0.999199 (2.27E-4)	0.999167 (2.54E-4)	**0.999210** (1.31E-4)
2	DTLZ6	0.998794 (9.09E-4)	0.998730 (5.70E-4) *	0.998795 (1.20E-3)	0.998843 (3.75E-1)	**0.999386** (6.25E-4)
2	DTLZ7	0.798195 (3.39E-5)	0.798207 (1.56E-5)	**0.798219** (4.10E-5)	0.798199 (3.17E-5)	0.798215 (2.03E-5)
2	WFG1	0.642159 (1.03E-2)	0.650270 (1.31E-2)	**0.652118** (7.70E-3)	0.646725 (1.40E-2)	0.646320 (1.00E-2)
2	WFG2	0.981836 (1.09E-2)	0.983203 (6.77E-3)	0.983099 (1.26E-2)	0.985033 (1.18E-2)	**0.985336** (9.52E-3)
2	WFG3	0.854702 (5.97E-3) *	0.855147 (3.71E-3)	0.856464 (3.91E-3)	0.856889 (3.76E-3)	**0.857286** (4.23E-3)
2	WFG4	**0.980005** (1.27E-2)	0.978601 (1.04E-2)	0.977442 (9.86E-3)	0.978251 (1.45E-2)	0.977473 (1.47E-2)
2	WFG5	0.949261 (2.88E-2)	**0.951310** (3.16E-2)	0.942824 (2.06E-2)	0.942160 (2.06E-2)	0.943000 (2.86E-2)
2	WFG6	**0.998470** (2.41E-4)	0.998440 (5.15E-4)	0.998325 (3.68E-4)	0.998402 (3.64E-4)	0.998437 (7.45E-4)
2	WFG7	0.961484 (1.34E-2)	**0.963588** (9.45E-3)	0.962551 (1.10E-2)	0.959227 (9.90E-3)	0.962359 (1.28E-2)
2	WFG8	0.881427 (3.05E-2)	0.891382 (2.06E-2)	0.891053 (2.93E-2)	**0.895764** (3.32E-2)	0.883949 (1.68E-2)
2	WFG9	**0.969658** (1.25E-2)	0.968900 (1.79E-2)	0.966885 (1.64E-2)	0.961481 (1.62E-2)	0.963662 (1.74E-2)
2	UF1	0.785193 (3.54E-3)	**0.787049** (1.80E-2)	0.783835 (5.41E-3)	0.783976 (7.10E-3)	0.783750 (3.42E-3)
2	UF2	**0.931995** (1.10E-2)	0.931797 (3.12E-3)	0.931644 (2.01E-3)	0.931544 (9.49E-4)	0.931398 (8.58E-4)
2	UF3	0.989168 (7.45E-4)	**0.989227** (9.05E-4)	0.989089 (7.88E-4)	0.989224 (8.13E-4)	0.988324 (1.98E-3)
2	UF4	**0.921462** (2.42E-2)	0.921264 (2.53E-2)	0.914598 (1.70E-2)	0.913392 (2.02E-2)	0.906162 (1.49E-2) *
2	UF5	— (—)	— (—)	— (—)	— (—)	— (—)
2	UF6	0.254975 (5.87E-2)	**0.264830** (5.05E-2)	0.250936 (4.73E-2)	0.253758 (2.38E-2)	0.253103 (5.23E-2)
2	UF7	0.777952 (6.21E-3)	**0.779738** (1.86E-2)	0.776870 (7.67E-3)	0.778085 (2.85E-2)	0.776203 (1.68E-2)
2	ZDT1	0.998148 (4.33E-4)	0.998103 (3.95E-4)	0.998142 (2.78E-4)	**0.998218** (2.30E-4)	0.998151 (3.33E-4)
2	ZDT2	0.998011 (5.19E-4)	0.997968 (4.12E-4)	0.998014 (4.62E-4)	0.997874 (4.34E-4)	**0.998066** (3.88E-4)
2	ZDT3	0.999100 (1.54E-4)	0.999128 (1.88E-4)	0.999128 (1.74E-4)	**0.999151** (1.10E-4)	0.999138 (1.07E-4)
2	ZDT4	0.998433 (5.68E-4)	0.998657 (5.82E-4)	0.998470 (6.23E-4)	0.998655 (2.21E-4)	**0.998689** (1.35E-4)
2	ZDT6	0.998211 (2.82E-4)	0.998165 (3.58E-4)	**0.998242** (2.79E-4)	0.998188 (3.27E-4)	0.998200 (2.66E-4)
3	DTLZ1	0.587705 (5.87E-1)	**0.587705** (5.87E-1)	0.058838 (5.87E-1)	— (5.87E-1)	— (5.72E-1)
3	DTLZ2	**0.979640** (3.18E-3)	0.978532 (5.38E-3)	0.979017 (3.13E-3)	0.976801 (4.94E-3) *	0.976360 (2.58E-3) *
3	DTLZ3	**0.390379** (3.90E-1)	— (3.90E-1)	— (1.95E-1) *	— (3.90E-1)	— (3.90E-1)
3	DTLZ4	**1.003512** (3.12E-3)	1.002110 (4.25E-3)	1.002265 (2.59E-3)	1.002488 (2.91E-3)	0.999324 (7.10E-3)
3	DTLZ5	**0.998399** (5.55E-4)	0.998343 (5.30E-4)	0.998245 (3.98E-4)	0.998236 (2.97E-4)	0.998209 (4.43E-4)
3	DTLZ6	0.998715 (4.15E-4)	0.998837 (7.56E-4) *	0.998905 (9.29E-4) *	0.998994 (7.59E-4)	**0.999757** (1.15E-3)
3	DTLZ7	0.984050 (5.10E-3)	**0.984147** (4.53E-3)	0.980983 (5.03E-3)	0.980241 (4.13E-3) *	0.979372 (4.78E-3) *
3	WFG1	0.602574 (5.60E-3)	0.602005 (8.09E-3)	0.600404 (9.90E-3)	**0.603585** (9.31E-3)	0.597931 (7.00E-3)
3	WFG2	0.957491 (8.22E-3)	**0.961921** (1.32E-2)	0.956968 (9.84E-3)	0.961405 (1.48E-2)	0.952951 (1.39E-2)
3	WFG3	0.945929 (2.48E-2)	0.942690 (3.24E-2)	**0.953774** (2.50E-2)	0.952324 (2.20E-2)	0.947640 (2.44E-2)
3	WFG4	**0.914095** (2.07E-2)	0.897688 (2.23E-2) *	0.897776 (3.56E-2)	0.873828 (3.50E-2) *	0.880461 (2.88E-2) *
3	WFG5	0.834121 (4.64E-2)	**0.856489** (3.72E-2)	0.840842 (2.71E-2)	0.833304 (3.28E-2) *	0.834295 (3.84E-2)
3	WFG6	**0.966511** (1.30E-2)	0.964431 (1.78E-2)	0.963059 (1.68E-2)	0.957878 (1.94E-2)	0.959747 (1.78E-2)
3	WFG7	**0.882702** (2.51E-2)	0.863109 (1.90E-2) *	0.868082 (3.39E-2)	0.862441 (2.76E-2) *	0.845836 (1.22E-2) *
3	WFG8	**0.827369** (3.64E-2)	0.813313 (2.54E-2)	0.826049 (2.79E-2)	0.805527 (2.93E-2)	0.810798 (2.79E-2)
3	WFG9	0.900355 (2.36E-2)	0.909942 (2.84E-2)	**0.911551** (2.39E-2)	0.905422 (2.40E-2)	0.908522 (2.91E-2)
3	UF8	0.857989 (2.85E-3)	**0.858010** (5.85E-4)	0.857762 (8.13E-4)	0.857967 (4.74E-4)	0.857799 (4.30E-4)
3	UF9	**0.649340** (4.46E-2)	0.643563 (1.11E-2)	0.643347 (1.02E-2)	0.639506 (8.27E-3)	0.640873 (1.06E-2)
3	UF10	0.855583 (2.08E-3)	0.855121 (2.99E-3)	0.855312 (4.68E-3)	**0.856027** (3.18E-3)	0.855117 (1.70E-3)

Table 12: Median and IQR values of the WOF-NSGA-II for different numbers of t_2. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

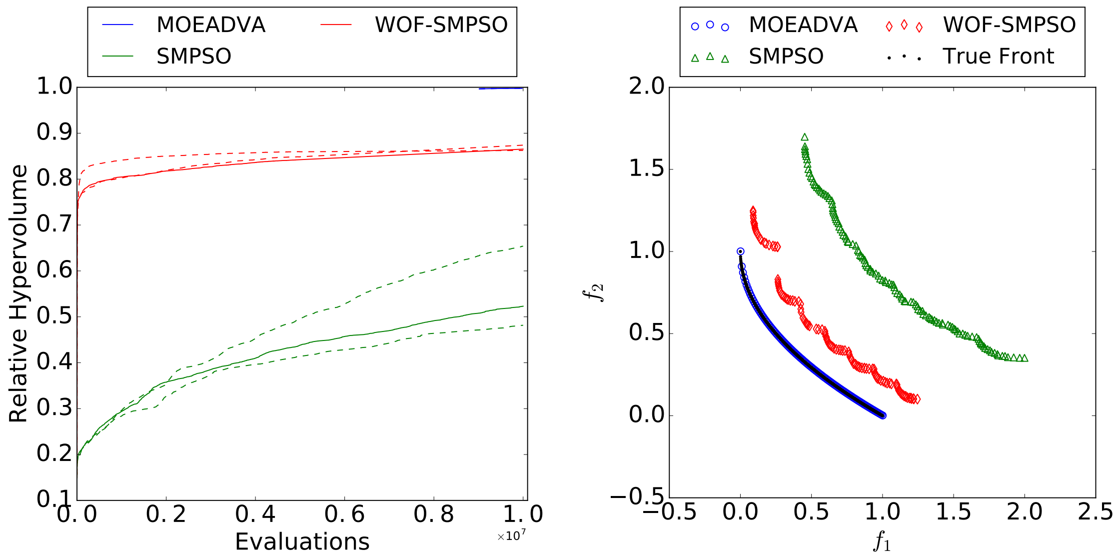
		WOF-NSGA-II ($t_2 = 100$)	WOF-NSGA-II ($t_2 = 300$)	WOF-NSGA-II ($t_2 = 500$)	WOF-NSGA-II ($t_2 = 700$)	WOF-NSGA-II ($t_2 = 900$)
2	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ2	0.990076 (5.00E-3) *	0.997172 (4.42E-4) *	0.997572 (2.75E-4)	**0.997599** (4.14E-4)	0.997230 (9.99E-4)
2	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
2	DTLZ4	0.978392 (9.74E-3) *	0.996411 (1.34E-3)	**0.996467** (9.43E-4)	0.995430 (2.09E-3) *	0.994755 (1.35E-3) *
2	DTLZ5	0.988818 (6.85E-3) *	0.997791 (5.81E-4) *	**0.997990** (3.48E-4)	0.997938 (7.12E-4)	0.997969 (1.08E-3)
2	DTLZ6	0.999126 (1.25E-4)	**0.999141** (5.52E-5)	0.999093 (1.14E-4)	0.999104 (1.12E-4)	0.999098 (9.91E-5)
2	DTLZ7	0.798231 (1.53E-5)	0.798238 (1.66E-5)	0.798233 (9.24E-6)	0.798238 (1.41E-5)	**0.798239** (1.53E-5)
2	WFG1	0.637471 (3.10E-3) *	0.644714 (7.85E-3) *	0.645916 (7.75E-3)	0.643237 (2.90E-2)	**0.648908** (1.13E-2)
2	WFG2	0.802680 (8.69E-3)	**0.806074** (7.07E-2)	0.800711 (5.81E-3)	0.803572 (1.99E-2)	0.792957 (1.77E-2) *
2	WFG3	0.805312 (4.23E-3) *	**0.808999** (6.81E-3)	0.808149 (7.82E-3)	0.805013 (7.51E-3) *	0.803861 (4.64E-3) *
2	WFG4	0.860073 (1.79E-2)	**0.860956** (1.62E-2)	0.860731 (1.21E-2)	0.859669 (1.72E-2)	0.850658 (1.12E-2) *
2	WFG5	0.879844 (9.17E-2)	0.909683 (9.58E-2)	**0.931980** (9.75E-2)	0.873077 (7.31E-2)	0.857564 (2.02E-2) *
2	WFG6	0.840057 (4.88E-2)	**0.844843** (6.75E-2)	0.836743 (4.31E-2)	0.835301 (6.49E-2)	0.824705 (6.03E-2) *
2	WFG7	**0.865474** (5.56E-3)	0.864653 (5.38E-3)	0.861538 (5.14E-3) *	0.863148 (7.44E-3) *	0.857951 (6.31E-3) *
2	WFG8	**0.826979** (2.83E-2)	0.822022 (1.67E-2)	0.811990 (1.56E-2)	0.811236 (1.32E-2) *	0.802832 (3.02E-2) *
2	WFG9	0.931167 (2.19E-2)	**0.934687** (1.84E-2)	0.929446 (1.80E-2)	0.925832 (2.68E-2)	0.930425 (2.37E-2)
2	UF1	**0.915149** (7.19E-2)	0.909207 (2.75E-2)	0.901166 (5.88E-2)	0.882157 (1.22E-1) *	0.885924 (3.47E-2)
2	UF2	0.911507 (2.20E-3)	**0.912048** (1.07E-3)	0.911463 (2.05E-3)	0.910916 (2.62E-3)	0.910018 (4.52E-3) *
2	UF3	**0.984666** (2.27E-3)	0.983540 (1.15E-3)	0.981546 (4.74E-3) *	0.979829 (3.08E-3) *	0.978755 (5.93E-3) *
2	UF4	**0.907593** (1.29E-2)	0.906686 (2.26E-2)	0.906210 (1.96E-2)	0.906030 (2.23E-2)	0.887150 (2.14E-2) *
2	UF5	**0.252267** (9.91E-2)	0.225505 (4.68E-2)	0.203290 (5.85E-2)	0.180445 (8.35E-2)	0.170107 (5.75E-2) *
2	UF6	**0.750763** (7.35E-2)	0.716532 (6.74E-2)	0.690489 (4.22E-2) *	0.663255 (7.48E-2) *	0.651417 (5.48E-2) *
2	UF7	**0.918789** (2.34E-2)	0.914678 (3.01E-2)	0.903532 (3.54E-2) *	0.898739 (2.87E-2) *	0.885210 (3.33E-2) *
2	ZDT1	0.998419 (1.44E-4)	0.998403 (1.24E-4)	**0.998464** (1.05E-4)	0.998446 (1.18E-4)	0.998442 (1.06E-4)
2	ZDT2	0.998271 (1.85E-4)	0.998285 (9.38E-5)	**0.998313** (1.96E-4)	0.998297 (1.56E-4)	0.998268 (1.21E-4)
2	ZDT3	0.999285 (5.56E-5)	0.999306 (3.96E-5)	0.999295 (8.41E-5)	0.999315 (5.39E-5)	**0.999328** (4.57E-5)
2	ZDT4	— (—)	— (—)	— (—)	— (—)	— (—)
2	ZDT6	0.998628 (7.78E-5)	**0.998654** (9.13E-5)	0.998635 (8.19E-5)	0.998626 (7.10E-5)	0.998589 (1.64E-4) *
3	DTLZ1	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ2	0.967037 (5.28E-3) *	**0.978039** (2.08E-3)	0.975643 (5.03E-3) *	0.972621 (8.00E-3) *	0.963279 (6.74E-3) *
3	DTLZ3	— (—)	— (—)	— (—)	— (—)	— (—)
3	DTLZ4	0.963850 (1.05E-2) *	**0.985192** (6.36E-3)	0.975950 (7.92E-3) *	0.961829 (2.41E-2) *	0.935480 (2.78E-2) *
3	DTLZ5	0.961713 (2.11E-2) *	**0.986805** (4.09E-3)	0.981223 (6.71E-3) *	0.968246 (1.84E-2) *	0.950542 (1.70E-2) *
3	DTLZ6	**0.999369** (1.53E-4)	0.999322 (1.52E-4)	0.999324 (1.73E-4)	0.999290 (1.08E-4)	0.999322 (1.78E-4)
3	DTLZ7	0.989791 (3.51E-3)	0.989868 (2.47E-3)	**0.990874** (1.60E-3)	0.989136 (5.55E-3)	0.990869 (2.50E-3)
3	WFG1	0.586932 (1.02E-2)	0.589281 (6.09E-3)	0.589274 (6.69E-3)	**0.592191** (8.37E-3)	0.591681 (9.24E-3)
3	WFG2	**0.869535** (1.93E-2)	0.865498 (1.72E-2)	0.865165 (3.62E-2)	0.861515 (2.05E-2)	0.850786 (5.79E-2)
3	WFG3	0.804638 (3.72E-2)	**0.818397** (1.38E-2)	0.799939 (3.13E-2)	0.791043 (1.58E-2) *	0.789323 (4.43E-2) *
3	WFG4	0.817023 (2.74E-2)	**0.818961** (1.79E-2)	0.807018 (2.74E-2)	0.803121 (2.64E-2) *	0.795698 (1.63E-2) *
3	WFG5	**0.797314** (5.38E-2)	0.792476 (6.34E-2)	0.780878 (2.73E-2)	0.770604 (4.04E-2) *	0.783332 (5.24E-2)
3	WFG6	**0.793630** (1.02E-1)	0.741700 (9.75E-2)	0.739641 (3.47E-2) *	0.739877 (6.17E-2) *	0.748292 (1.03E-1)
3	WFG7	**0.806943** (1.20E-2)	0.800394 (2.24E-2)	0.798850 (1.93E-2) *	0.793862 (1.59E-2) *	0.790967 (1.87E-2) *
3	WFG8	**0.765047** (1.97E-2)	0.762469 (3.22E-2)	0.752178 (2.36E-2)	0.744856 (1.96E-2) *	0.738921 (2.72E-2) *
3	WFG9	0.851328 (1.76E-2)	**0.855995** (2.96E-2)	0.842645 (2.93E-2)	0.838030 (2.63E-2) *	0.838400 (4.23E-2)
3	UF8	0.854949 (2.07E-3) *	**0.857717** (7.20E-4)	0.857263 (9.61E-4)	0.857136 (5.11E-4) *	0.857289 (1.36E-3)
3	UF9	0.663395 (6.34E-2)	**0.671340** (2.87E-2)	0.658553 (3.72E-2)	0.640260 (6.77E-2) *	0.642718 (5.72E-2) *
3	UF10	0.504693 (1.11E-1) *	0.658974 (1.15E-1) *	**0.727199** (8.96E-2)	0.718786 (5.26E-2)	0.706777 (6.73E-2)

Table 13: Median and IQR values of the MOEA/DVA, SMPSO and WOF-SMPSO algorithms. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		MOEADVA	SMPSO	WOF-SMPSO
2	UF1	**0.997674** (4.23E-5)	0.523064 (1.72E-1) *	0.865301 (1.15E-2) *
2	UF2	**0.998379** (4.73E-5)	0.900924 (5.61E-3) *	0.953731 (1.45E-3) *
2	UF3	**0.993348** (2.61E-4)	0.890209 (3.76E-2) *	0.991422 (3.26E-4) *
2	UF4	**0.975712** (3.87E-4)	0.900361 (7.05E-3) *	0.955756 (1.24E-2) *
2	UF5	**0.903066** (2.92E-3)	— (—) *	0.036240 (1.67E-1) *
2	UF6	**0.983886** (4.01E-4)	0.552088 (4.34E-1) *	0.609818 (1.79E-1) *
2	UF7	**0.976348** (9.96E-5)	0.890488 (4.05E-3) *	0.891790 (3.06E-2) *
3	UF8	0.856557 (1.13E-1)	0.853943 (2.87E-3) *	**0.905787** (6.33E-2)
3	UF9	**0.982560** (4.33E-4)	0.628937 (5.62E-3) *	0.773874 (2.29E-2) *
3	UF10	0.555236 (1.48E-1) *	0.828312 (1.47E-2)	**0.835854** (8.91E-3)

Table 14: Median and IQR values of the MOEA/DVA, SMPSO and WOF-SMPSO algorithms. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

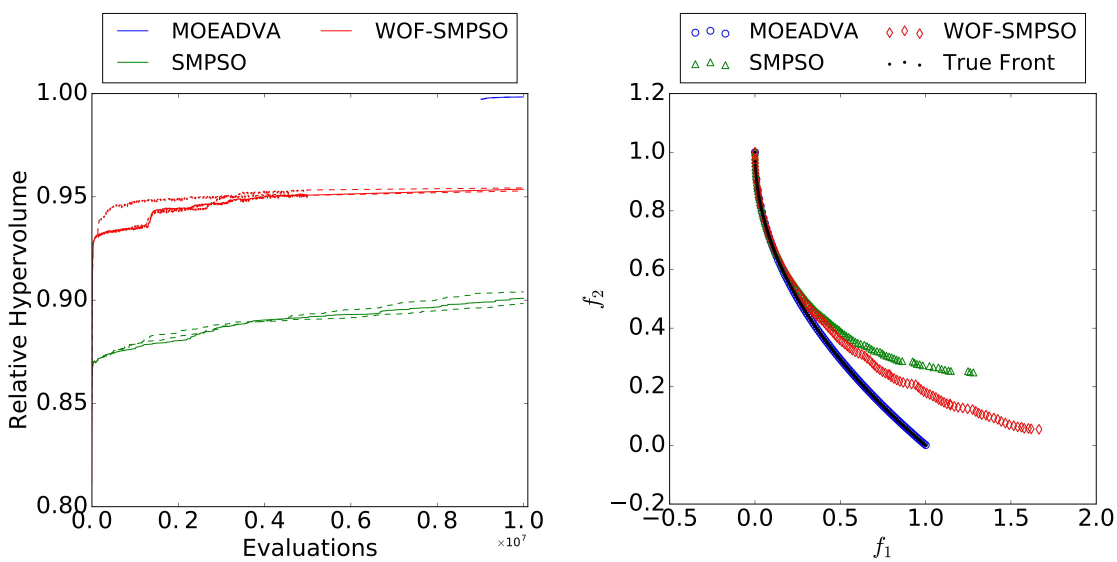
		MOEADVA	SMPSO	WOF-SMPSO
2	WFG1	0.323227 (1.52E-3) *	0.621543 (3.62E-3) *	**0.673297** (3.38E-2)
2	WFG2	0.725802 (8.46E-2) *	0.862627 (6.86E-3) *	**0.998624** (7.89E-4)
2	WFG3	0.614965 (5.35E-3) *	0.755552 (2.16E-2) *	**0.861032** (1.09E-3)
2	WFG4	0.571593 (1.18E-1) *	0.877598 (6.74E-3) *	**0.993336** (4.38E-3)
2	WFG5	0.562093 (1.00E-2) *	0.851155 (8.78E-3) *	**0.986566** (3.26E-3)
2	WFG7	0.659123 (1.22E-2) *	0.806824 (1.49E-2) *	**0.995933** (6.33E-3)
3	WFG1	0.285450 (1.60E-3) *	0.599994 (1.21E-3) *	**0.659671** (6.33E-2)
3	WFG2	0.769211 (2.46E-2) *	0.809230 (9.52E-3) *	**0.984807** (3.97E-3)
3	WFG3	0.601182 (6.71E-3) *	0.841009 (1.05E-2) *	**0.989117** (3.61E-3)
3	WFG4	0.460089 (6.86E-3) *	0.762498 (1.04E-2) *	**0.931041** (1.95E-2)
3	WFG5	0.439999 (1.11E-2) *	0.718014 (1.42E-2) *	**0.915040** (4.24E-2)



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

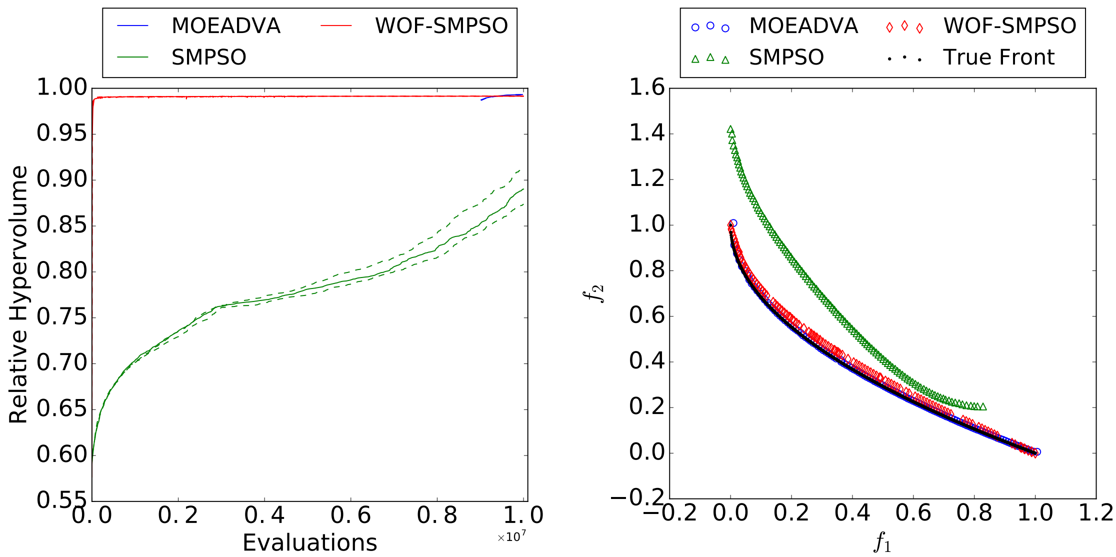
Figure 1: Convergence and obtained solution sets on the UF1 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

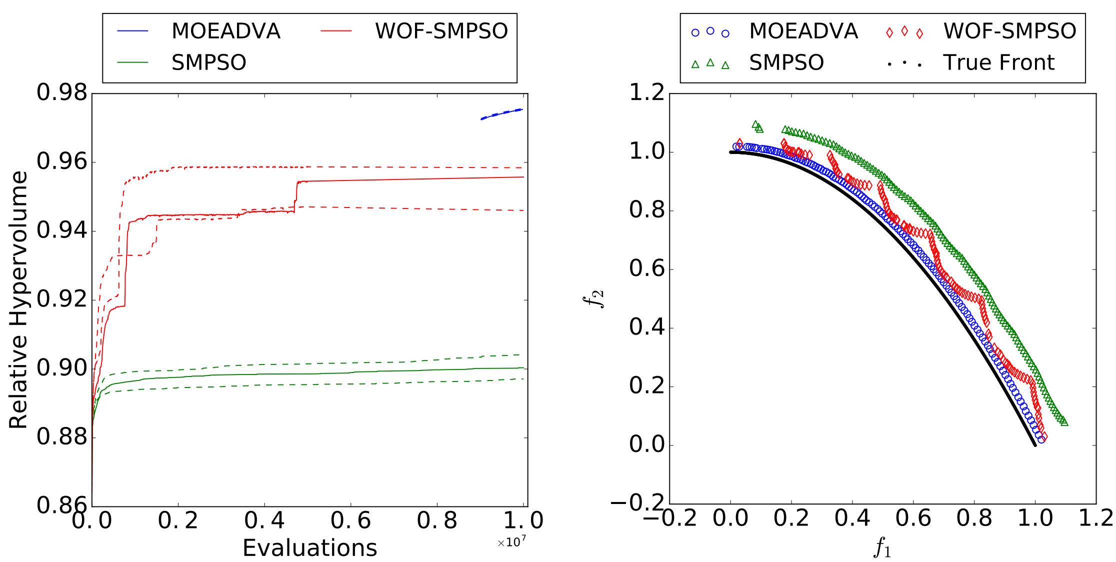
Figure 2: Convergence and obtained solution sets on the UF2 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

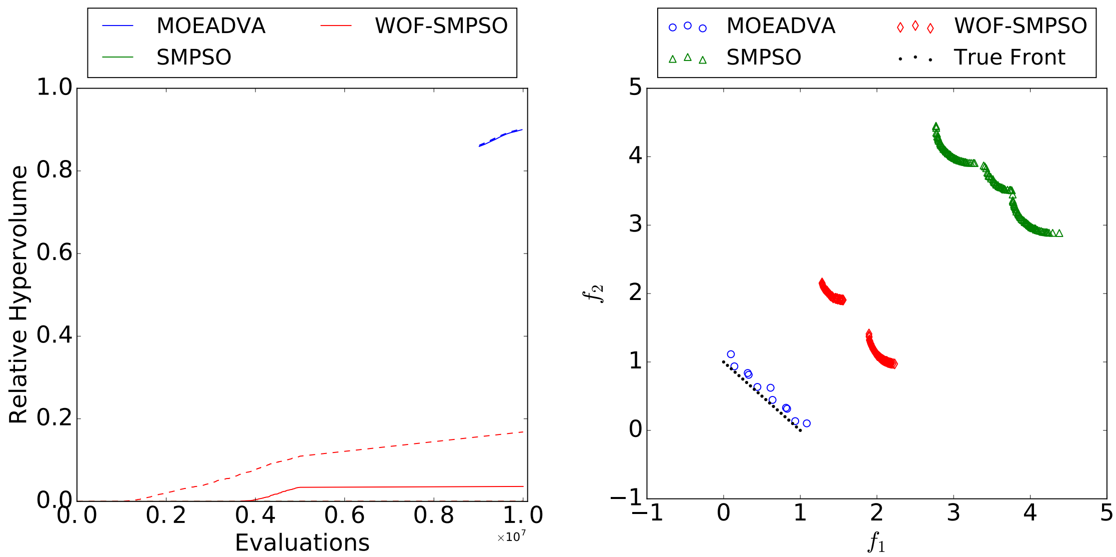
Figure 3: Convergence and obtained solution sets on the UF3 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

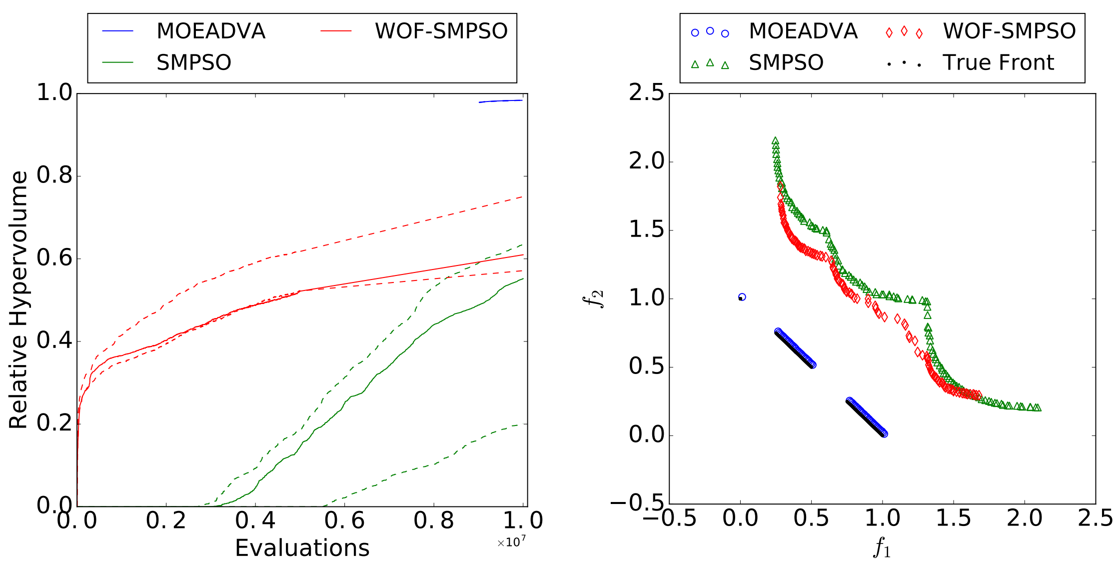
Figure 4: Convergence and obtained solution sets on the UF4 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

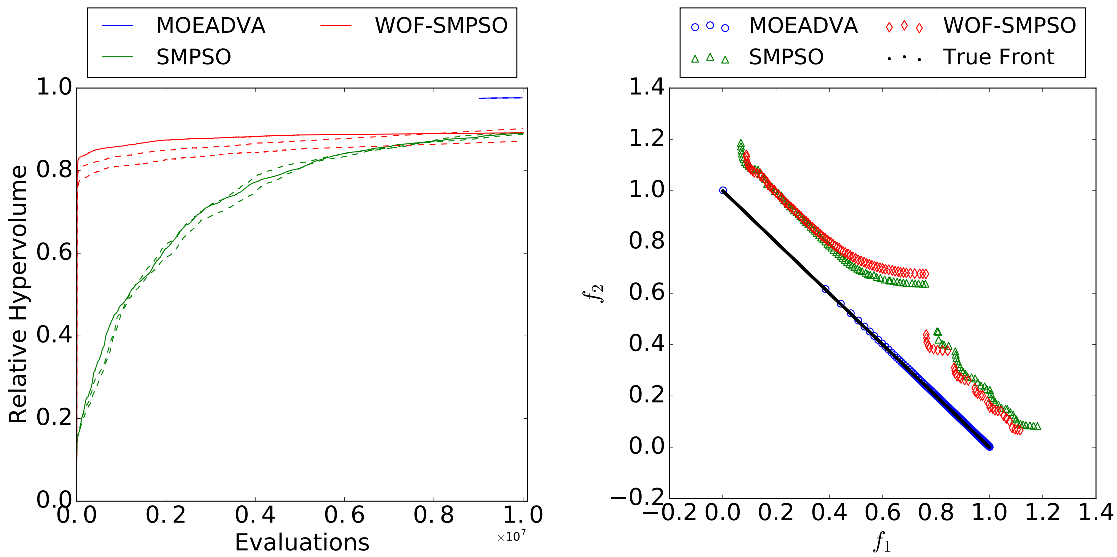
Figure 5: Convergence and obtained solution sets on the UF5 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

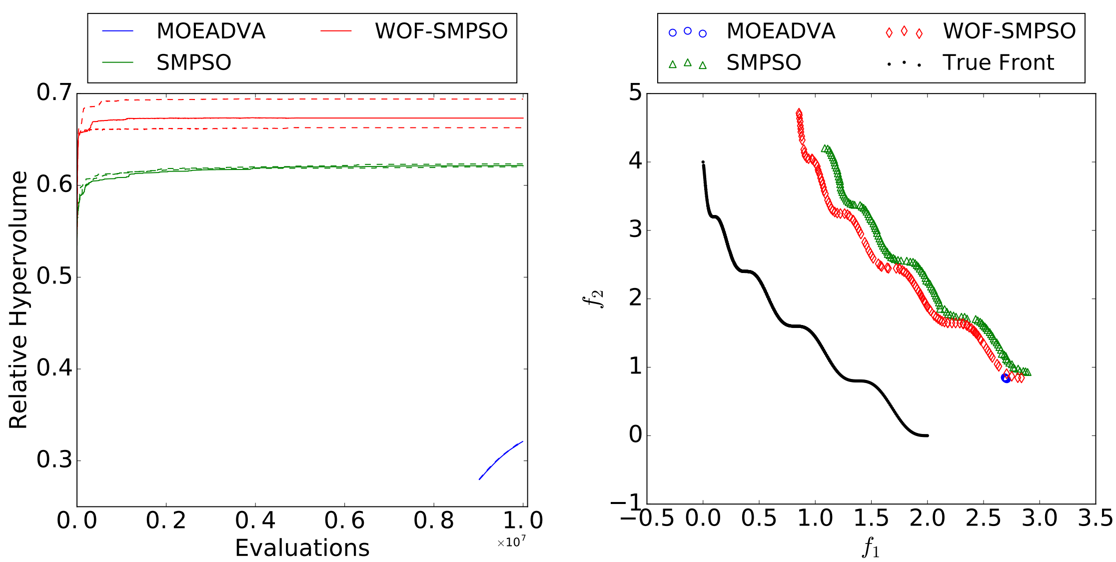
Figure 6: Convergence and obtained solution sets on the UF6 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

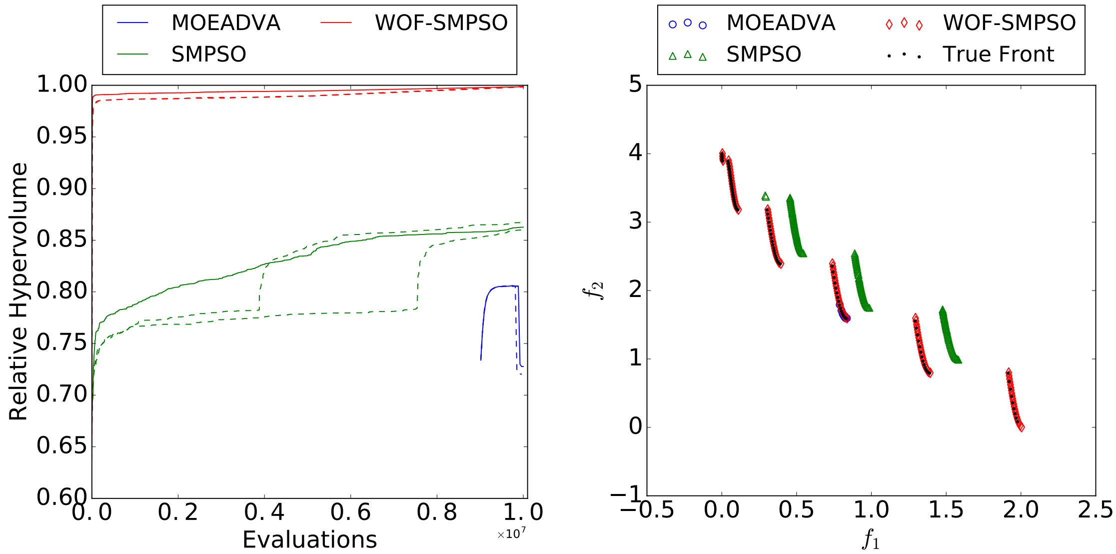
Figure 7: Convergence and obtained solution sets on the UF7 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

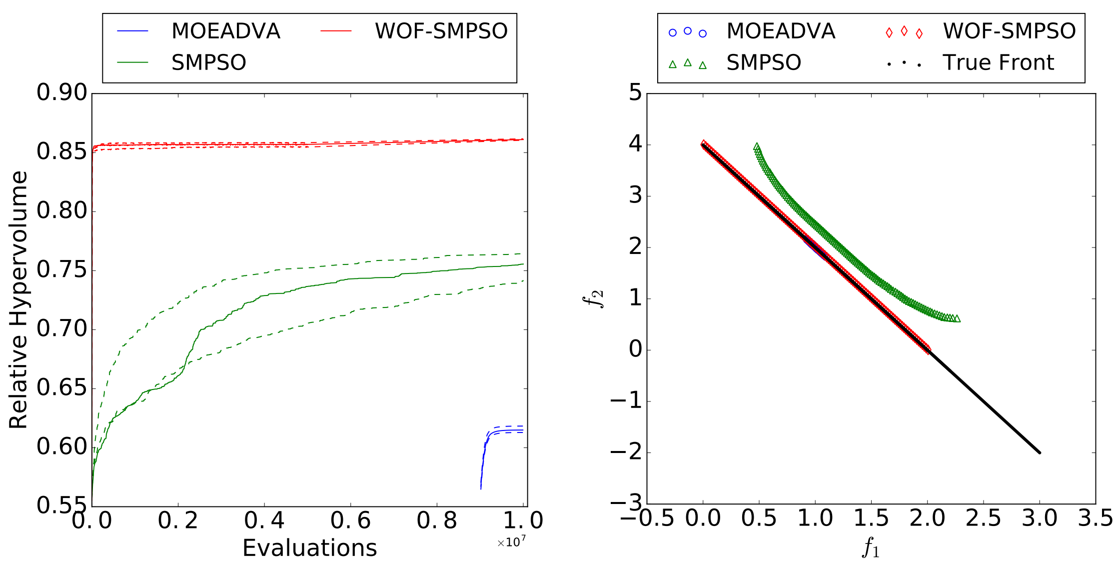
Figure 8: Convergence and obtained solution sets on the WFG1 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

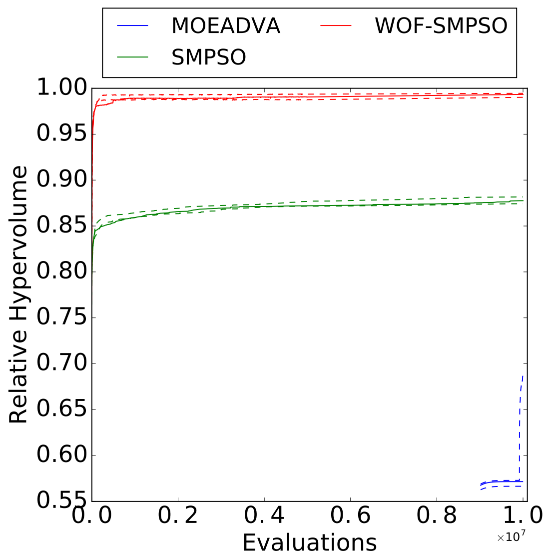
Figure 9: Convergence and obtained solution sets on the WFG2 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



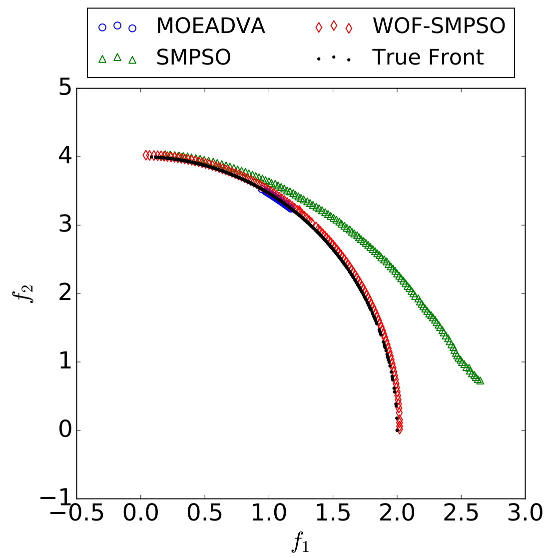
(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

Figure 10: Convergence and obtained solution sets on the WFG3 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).

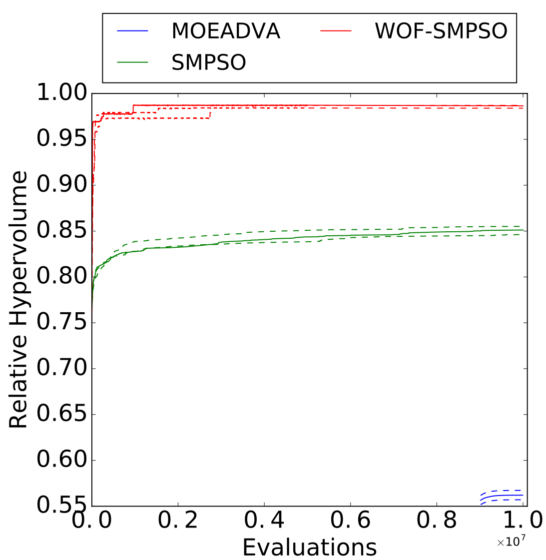


(a) Convergence of rel. hypervolume

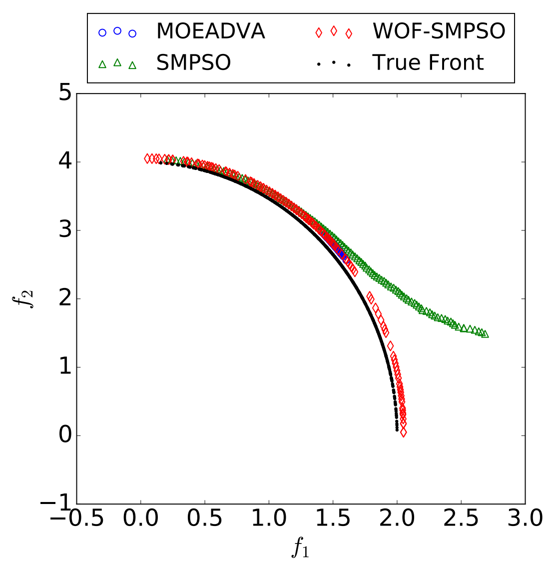


(b) Solution sets and Pareto-front

Figure 11: Convergence and obtained solution sets on the WFG4 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).

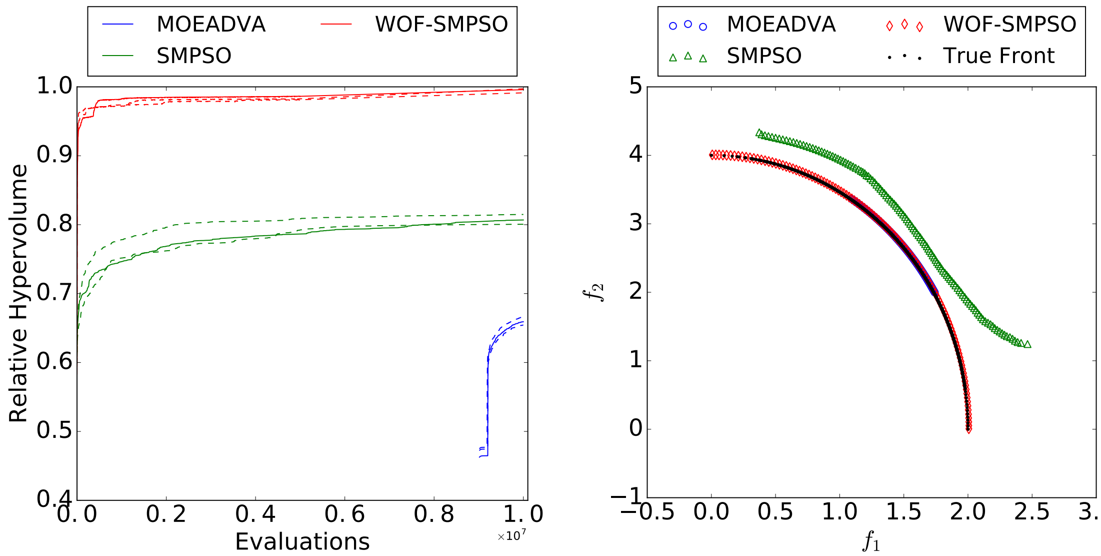


(a) Convergence of rel. hypervolume



(b) Solution sets and Pareto-front

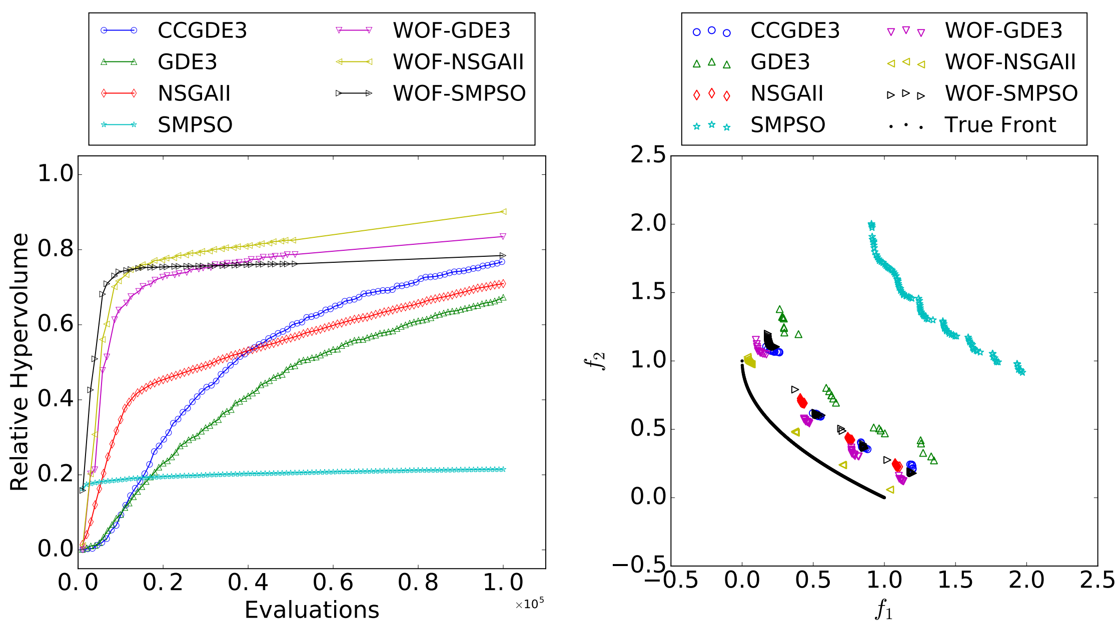
Figure 12: Convergence and obtained solution sets on the WFG5 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

Figure 13: Convergence and obtained solution sets on the WFG7 problem with $n = 1000$ variables. dashed lines show the 1st and 3rd quartile runs, solid lines show the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

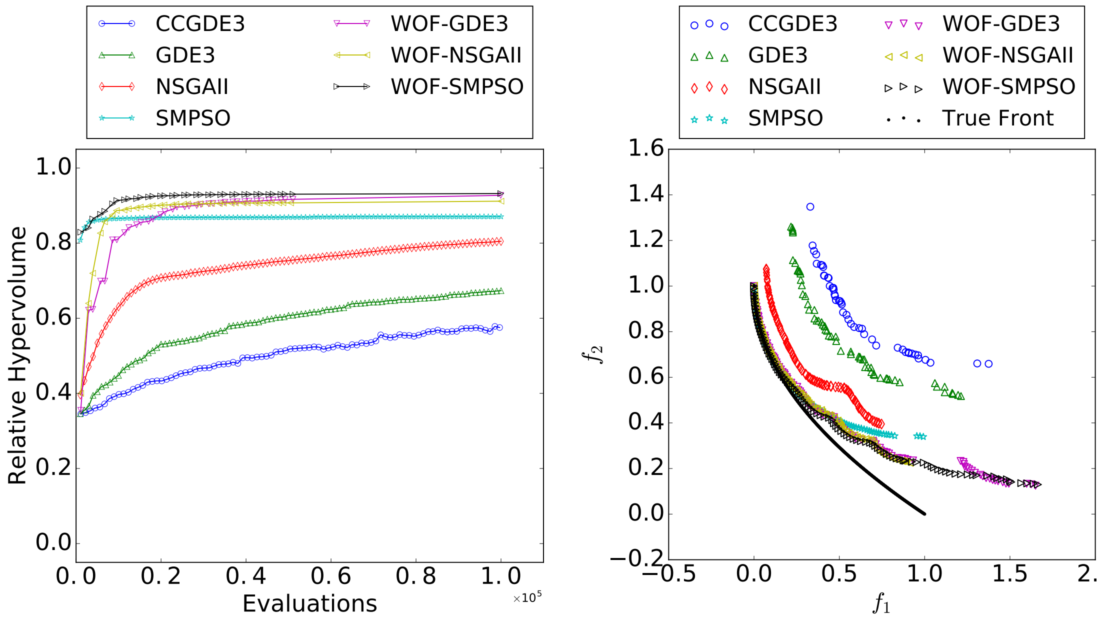
Figure 14: Convergence and obtained solution sets on the UF1 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).

Table 15: Best and second best algorithms based on median relative hypervolume for $m = 2$ objectives and $n = 40$ to $n = 2000$ variables. Bold indicates a WOF-based algorithm. And asterisk indicates statistical significance ($p < 0.01$) compared to the respective next-best method.

m = 2	n = 40	n = 80	n = 200	n = 600	n = 1000	n = 2000
DTLZ1	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO (WOF-SMPSO *)	**WOF-SMPSO** * / **SMPSO** *	**WOF-SMPSO** * / **SMPSO** *
DTLZ2	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)
DTLZ3	**WOF-SMPSO** (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO (WOF-SMPSO *)	**WOF-SMPSO** * / **SMPSO** *	**WOF-SMPSO** * / **SMPSO** *
DTLZ4	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)
DTLZ5	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)
DTLZ6	SMPSO (GDE3 *)	GDE3 * (WOF-SMPSO *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** (WOF-NSGAII *)	**WOF-NSGAII** (WOF-SMPSO)	**WOF-NSGAII** * (WOF-GDE3)
DTLZ7	GDE3 * (SMPSO *)	GDE3 (SMPSO *)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)
WFG1	**WOF-NSGAII** * (GDE3 *)	**WOF-NSGAII** * (WOF-GDE3 *)	**WOF-GDE3** * (WOF-SMPSO)	**WOF-GDE3** * (WOF-SMPSO)	**WOF-GDE3** * (WOF-NSGAII)	**WOF-GDE3** (WOF-SMPSO)
WFG2	GDE3 * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)
WFG3	GDE3 * (WOF-SMPSO)	**WOF-SMPSO** * (GDE3)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3)
WFG4	**WOF-NSGAII** (NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (SMPSO)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)
WFG5	**WOF-NSGAII** * (NSGAII *)	**WOF-NSGAII** * (NSGAII)	**WOF-SMPSO** (WOF-GDE3)	**WOF-SMPSO** (WOF-GDE3 *)	**WOF-SMPSO** (WOF-GDE3)	**WOF-SMPSO** (WOF-GDE3 *)
WFG6	**WOF-SMPSO** * (GDE3 *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)
WFG7	**WOF-NSGAII** * (GDE3 *)	**WOF-SMPSO** * (GDE3 *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII)
WFG8	**WOF-NSGAII** (WOF-SMPSO *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)
WFG9	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3)
UF1	GDE3 (WOF-GDE3 *)	GDE3 * (WOF-GDE3 *)	**WOF-GDE3** (GDE3)	**WOF-NSGAII** * (WOF-GDE3 *)	**WOF-NSGAII** * (WOF-GDE3 *)	**WOF-NSGAII** * (WOF-GDE3 *)
UF2	**WOF-GDE3** (WOF-NSGAII)	**WOF-GDE3** * (WOF-NSGAII)	**WOF-GDE3** * (WOF-SMPSO *)	**WOF-SMPSO** (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)
UF3	**WOF-NSGAII** * (SMPSO)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3)
UF4	GDE3 * (WOF-GDE3 *)	**WOF-GDE3** (GDE3 *)	**WOF-GDE3** (GDE3 *)	**WOF-GDE3** (GDE3)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3 *)
UF5	GDE3 * (WOF-GDE3 *)	CCGDE3 * (WOF-GDE3 *)	**WOF-NSGAII** * (NSGAII)	**WOF-NSGAII** * (NSGAII *)	**WOF-NSGAII** * (CCGDE3)	**WOF-NSGAII** * (CCGDE3)
UF6	GDE3 (WOF-GDE3 *)	GDE3 (WOF-GDE3 *)	**WOF-GDE3** * (NSGAII)	**WOF-NSGAII** * (CCGDE3)	**WOF-NSGAII** * (WOF-GDE3 *)	**WOF-NSGAII** * (WOF-GDE3 *)
UF7	GDE3 * (WOF-GDE3 *)	GDE3 * (WOF-GDE3 *)	GDE3 * (WOF-GDE3)	**WOF-NSGAII** * (WOF-GDE3)	**WOF-NSGAII** * (WOF-GDE3 *)	**WOF-NSGAII** * (WOF-GDE3 *)
ZDT1	SMPSO * (GDE3 *)	SMPSO * (GDE3 *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII *)
ZDT2	SMPSO * (GDE3 *)	SMPSO * (GDE3 *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)
ZDT3	GDE3 * (SMPSO *)	GDE3 * (WOF-SMPSO *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)
ZDT4	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)
ZDT6	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)

Table 16: Best and second best algorithms based on median relative hypervolume for $m = 3$ objectives and $n = 40$ to $n = 2000$ variables. Bold indicates a WOF-based algorithm. And asterisk indicates statistical significance ($p < 0.01$) compared to the respective next-best method.

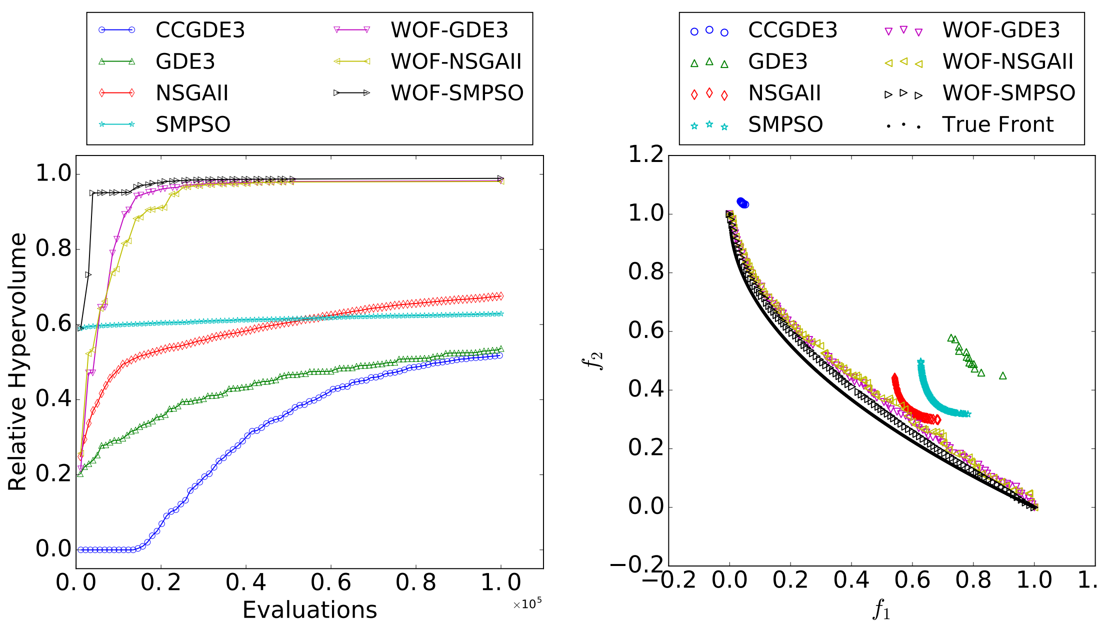
m = 3	n = 40	n = 80	n = 200	n = 600	n = 1000	n = 2000
DTLZ1	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (CCGDE3)
DTLZ2	**WOF-NSGAII** * (NSGAII *)	**WOF-NSGAII** * (NSGAII *)	**WOF-NSGAII** * (SMPSO *)	SMPSO (WOF-NSGAII *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)
DTLZ3	**WOF-SMPSO** (SMPSO *)	SMPSO (WOF-SMPSO *)	SMPSO (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (CCGDE3)	SMPSO * (CCGDE3)
DTLZ4	**WOF-NSGAII** (NSGAII *)	NSGAII (WOF-NSGAII *)	SMPSO (WOF-NSGAII *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)
DTLZ5	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)	SMPSO * (WOF-SMPSO *)
DTLZ6	GDE3 * (WOF-SMPSO *)	GDE3 * (WOF-SMPSO *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)
DTLZ7	GDE3 * (WOF-GDE3)	**WOF-GDE3** (WOF-NSGAII *)	**WOF-GDE3** (WOF-NSGAII *)	**WOF-NSGAII** (WOF-GDE3 *)	**WOF-GDE3** (WOF-NSGAII *)	**WOF-NSGAII** * (WOF-GDE3 *)
WFG1	**WOF-NSGAII** * (WOF-GDE3 *)	**WOF-NSGAII** * (WOF-SMPSO)	**WOF-GDE3** (WOF-SMPSO *)	**WOF-GDE3** * (WOF-SMPSO *)	**WOF-GDE3** (WOF-SMPSO *)	**WOF-SMPSO** * (WOF-GDE3 *)
WFG2	**WOF-SMPSO** * (GDE3 *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3 *)
WFG3	**WOF-NSGAII** (NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)
WFG4	**WOF-NSGAII** * (NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)
WFG5	**WOF-NSGAII** * (NSGAII *)	**WOF-NSGAII** * (NSGAII)	**WOF-NSGAII** * (WOF-SMPSO *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3 *)
WFG6	SMPSO (WOF-SMPSO *)	**WOF-SMPSO** (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)
WFG7	NSGAII * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII)	**WOF-SMPSO** * (WOF-NSGAII)
WFG8	NSGAII * (WOF-NSGAII *)	**WOF-NSGAII** * (NSGAII)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII)
WFG9	NSGAII * (WOF-NSGAII *)	NSGAII (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-NSGAII *)	**WOF-SMPSO** * (WOF-GDE3)	**WOF-SMPSO** * (WOF-GDE3 *)	**WOF-SMPSO** * (WOF-GDE3 *)
UF8	NSGAII (WOF-SMPSO)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** (SMPSO *)	**WOF-SMPSO** (SMPSO *)	SMPSO * (WOF-SMPSO *)
UF9	GDE3 (WOF-GDE3)	**WOF-GDE3** * (WOF-NSGAII)	**WOF-GDE3** * (WOF-NSGAII *)	**WOF-GDE3** * (WOF-NSGAII *)	**WOF-GDE3** * (WOF-NSGAII)	**WOF-GDE3** * (WOF-SMPSO)
UF10	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** (SMPSO *)	**WOF-SMPSO** (SMPSO *)	**WOF-SMPSO** * (SMPSO *)	**WOF-SMPSO** (SMPSO *)	SMPSO (WOF-SMPSO *)



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

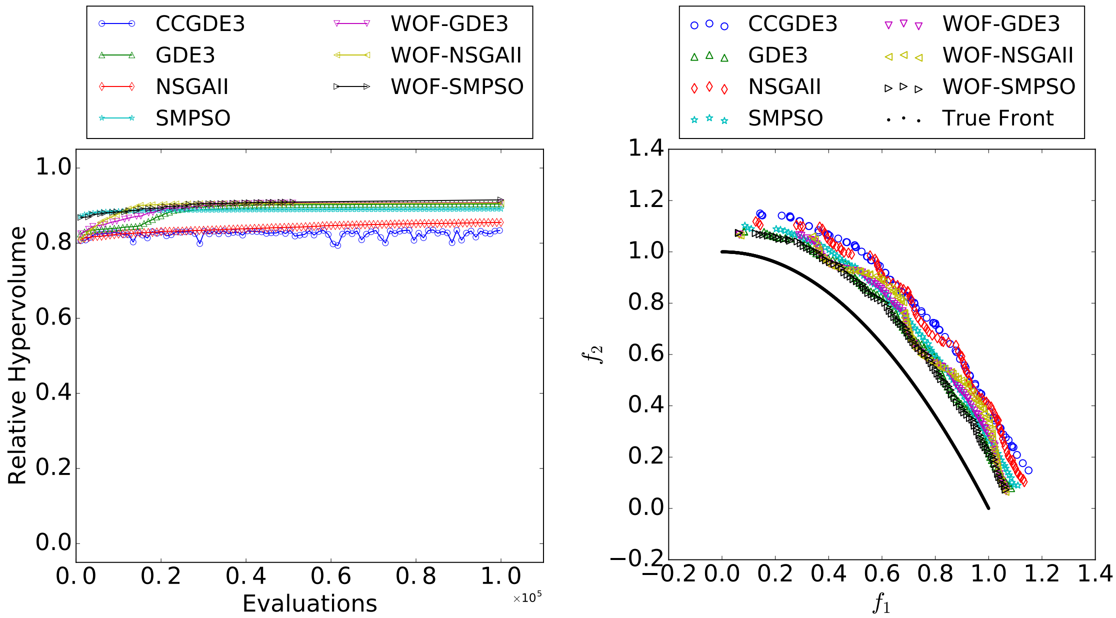
Figure 15: Convergence and obtained solution sets on the UF2 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

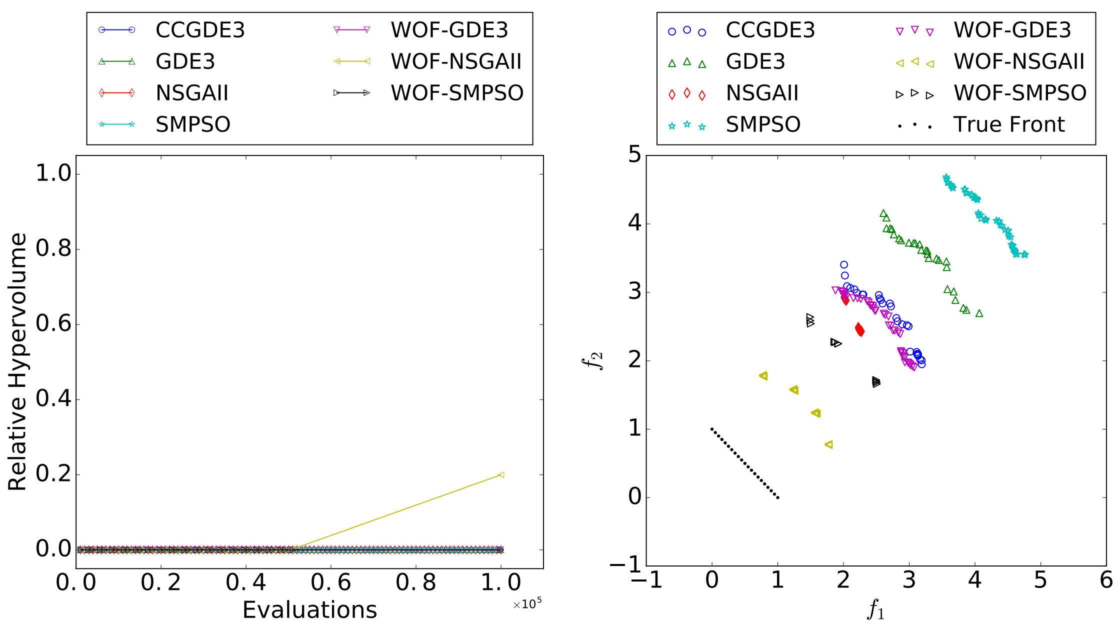
Figure 16: Convergence and obtained solution sets on the UF3 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

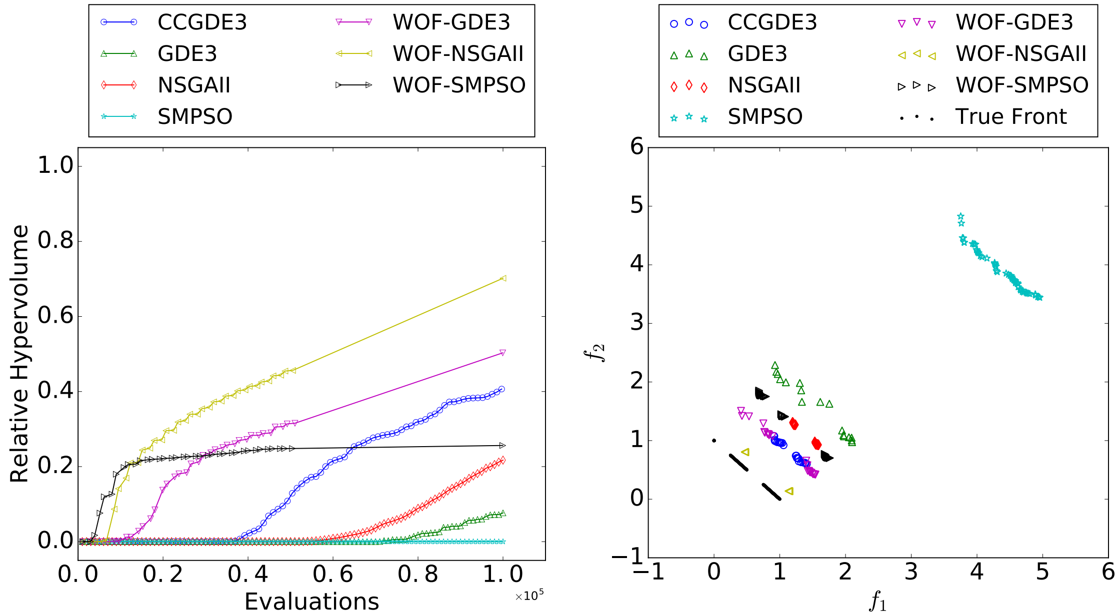
Figure 17: Convergence and obtained solution sets on the UF4 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

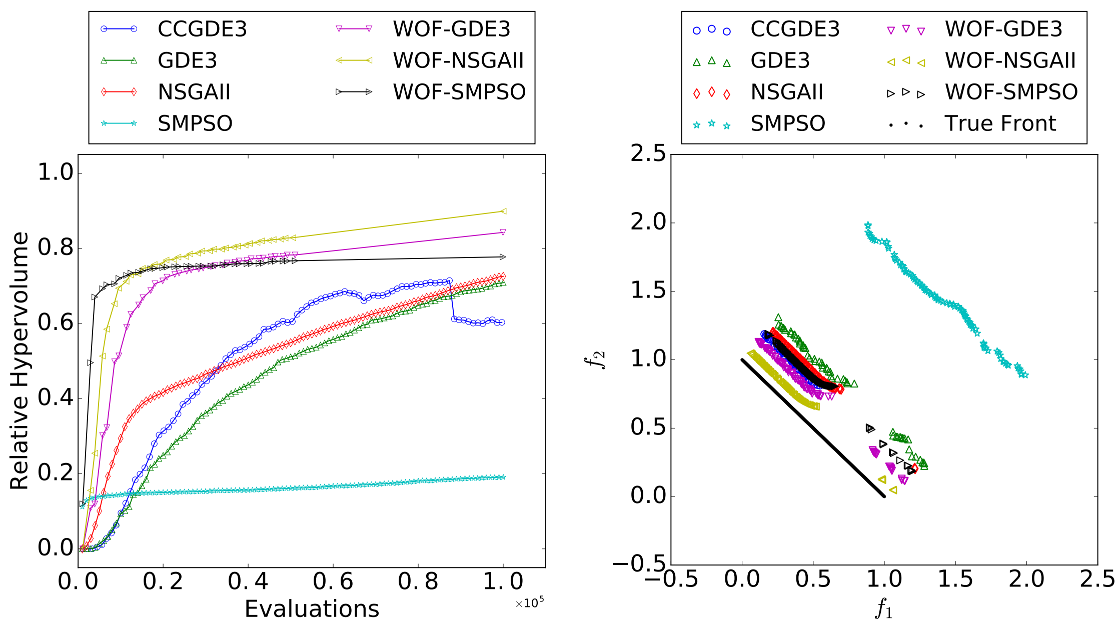
Figure 18: Convergence and obtained solution sets on the UF5 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

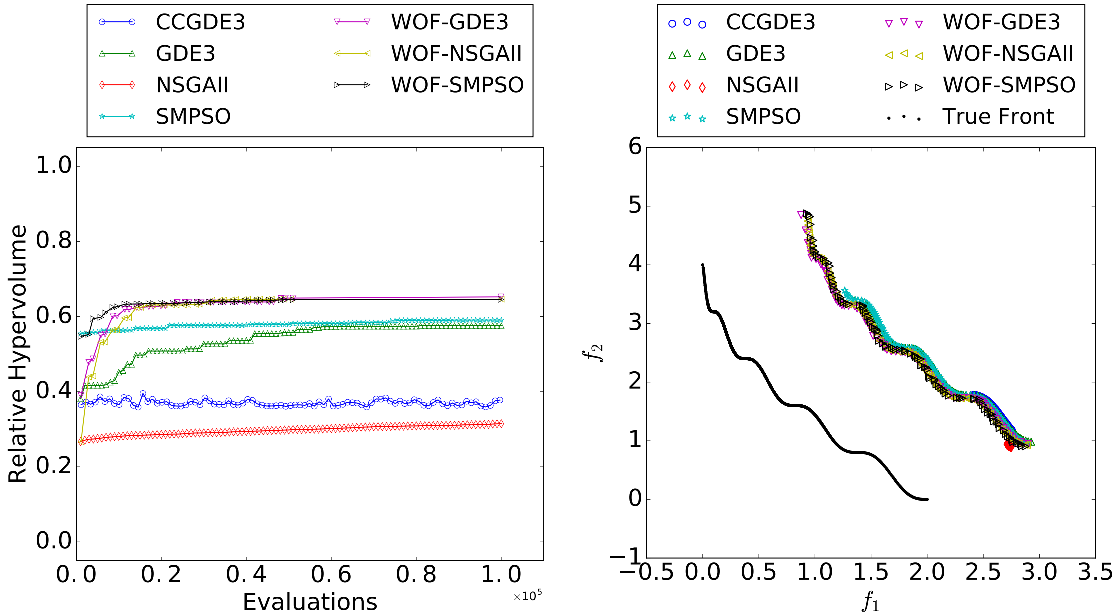
Figure 19: Convergence and obtained solution sets on the UF6 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

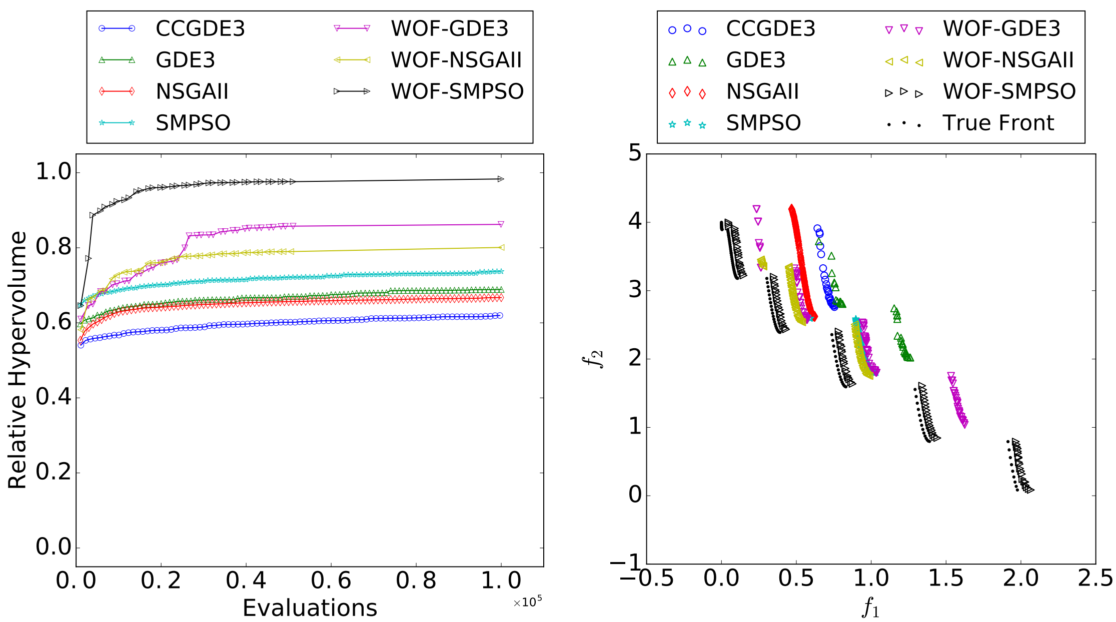
Figure 20: Convergence and obtained solution sets on the UF7 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

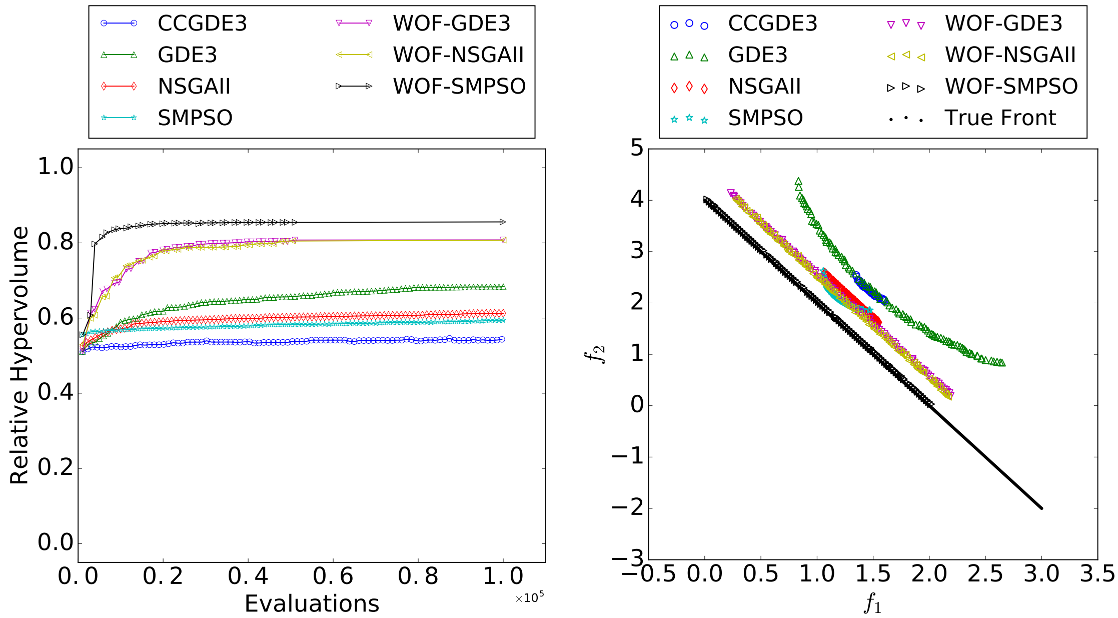
Figure 21: Convergence and obtained solution sets on the WFG1 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

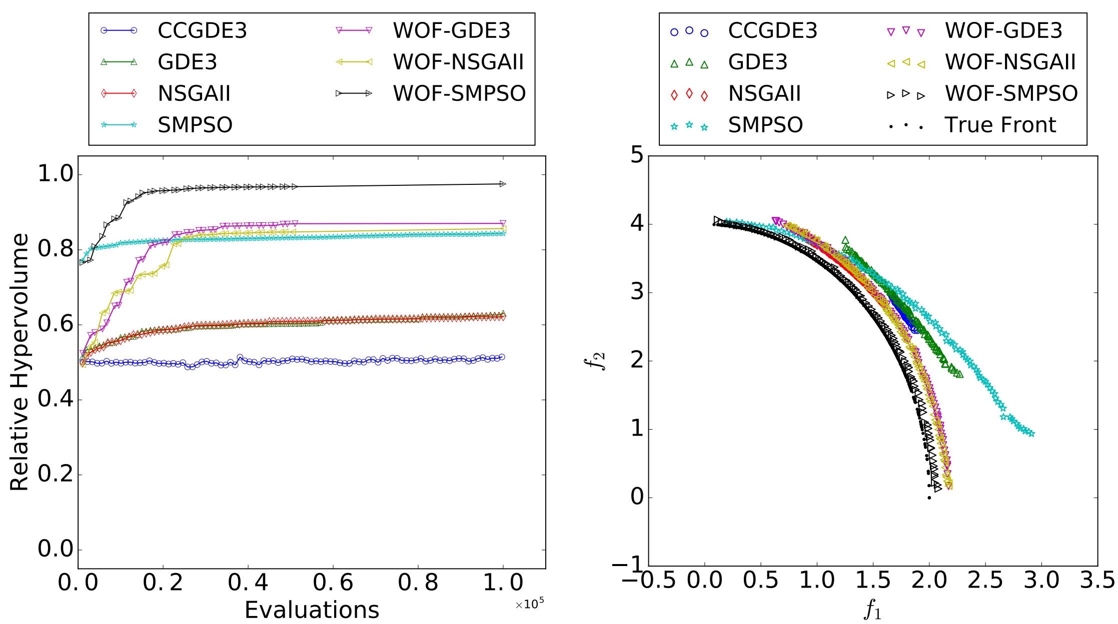
Figure 22: Convergence and obtained solution sets on the WFG2 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

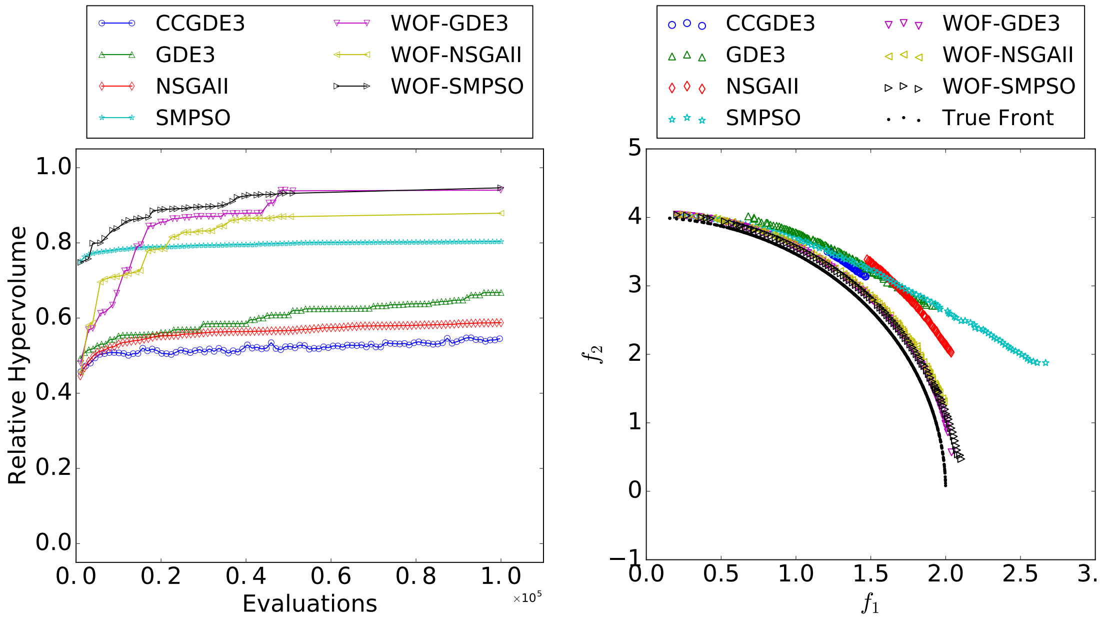
Figure 23: Convergence and obtained solution sets on the WFG3 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

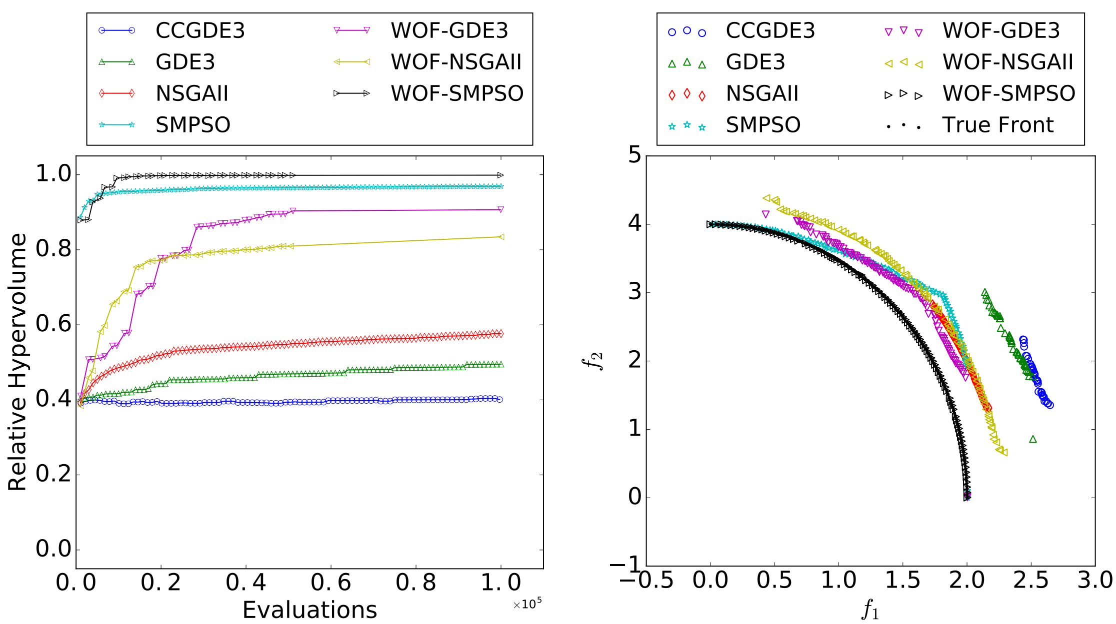
Figure 24: Convergence and obtained solution sets on the WFG4 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

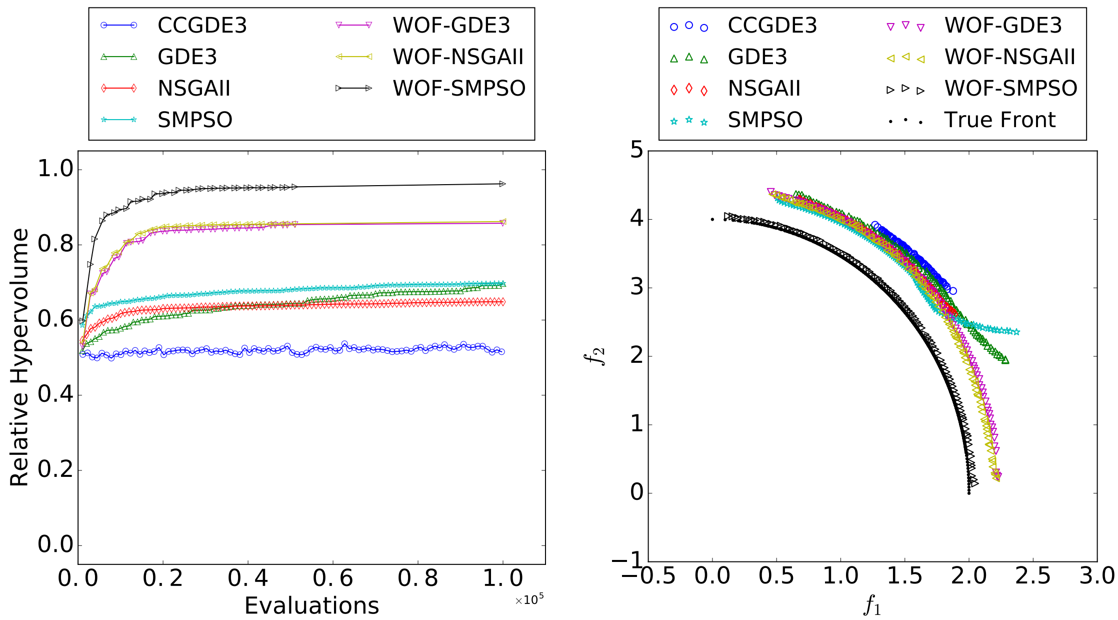
Figure 25: Convergence and obtained solution sets on the WFG5 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

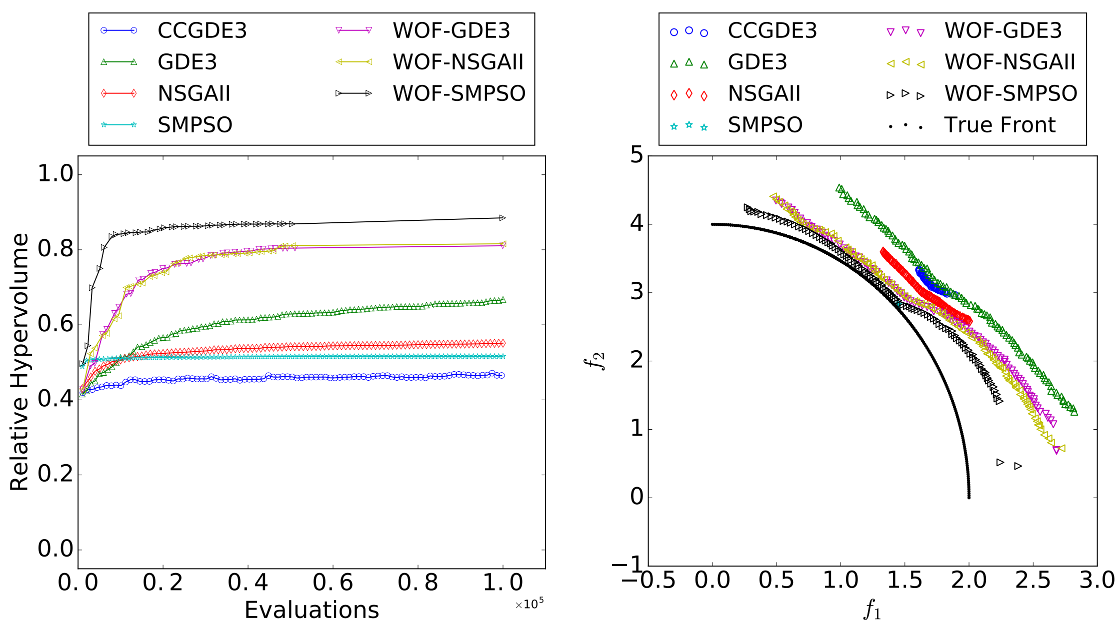
Figure 26: Convergence and obtained solution sets on the WFG6 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

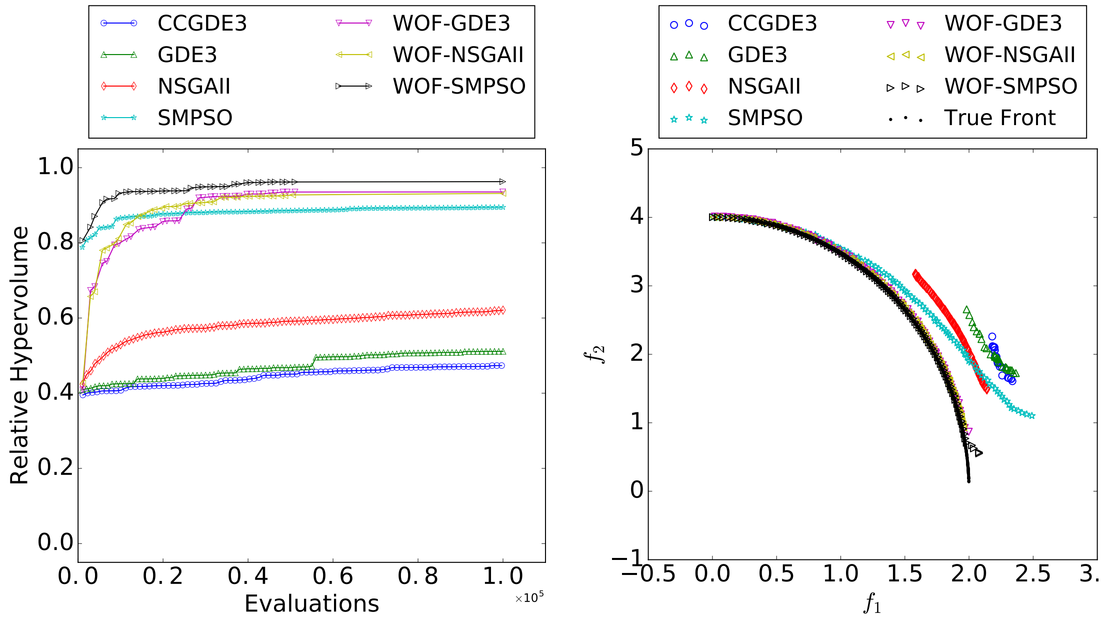
Figure 27: Convergence and obtained solution sets on the WFG7 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

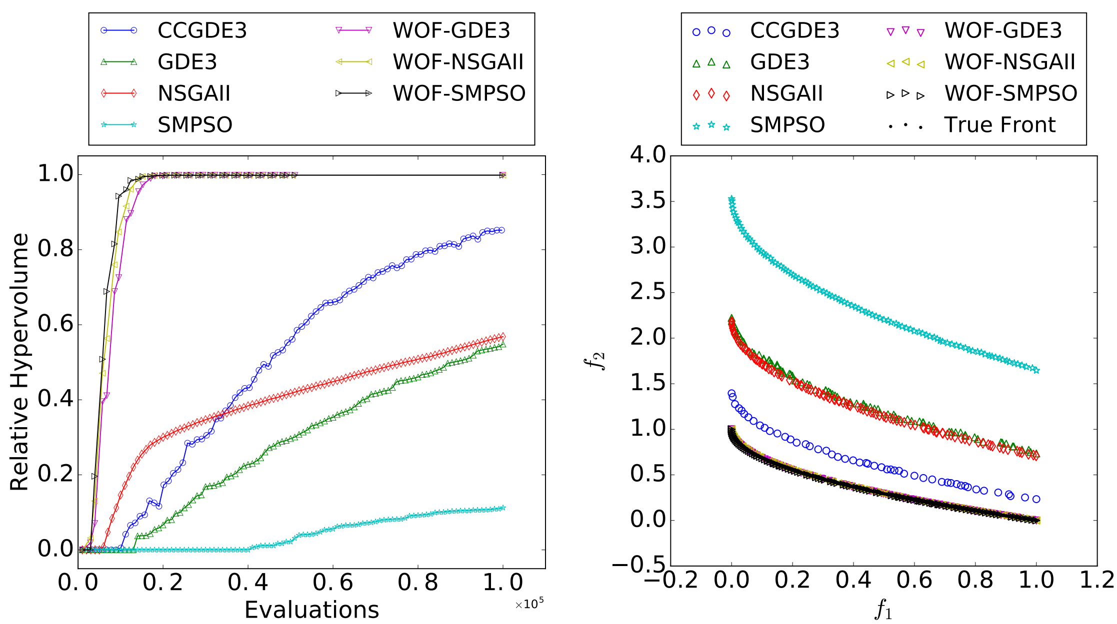
Figure 28: Convergence and obtained solution sets on the WFG8 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

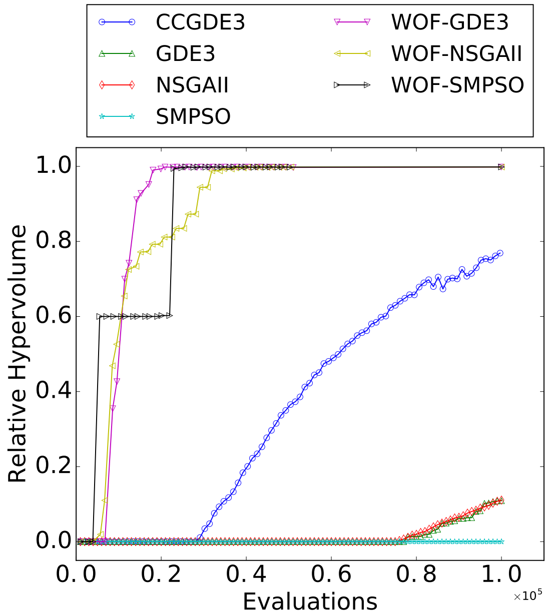
Figure 29: Convergence and obtained solution sets on the WFG9 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



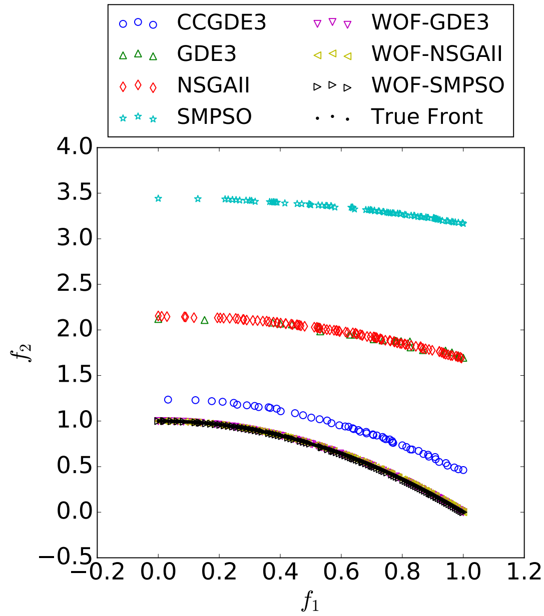
(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

Figure 30: Convergence and obtained solution sets on the ZDT1 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).

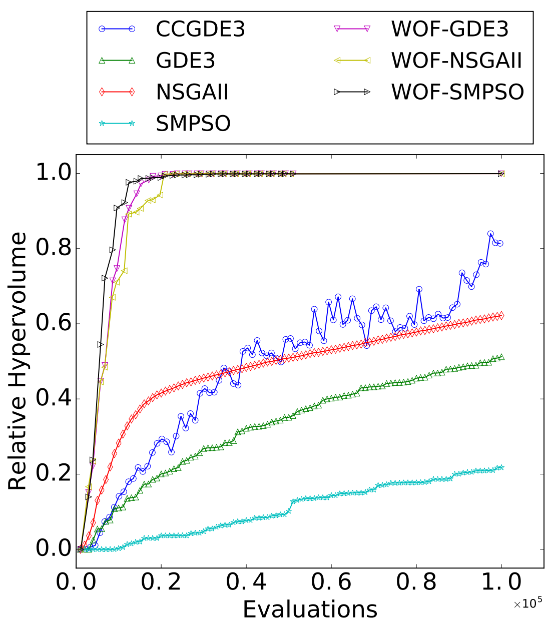


(a) Convergence of rel. hypervolume

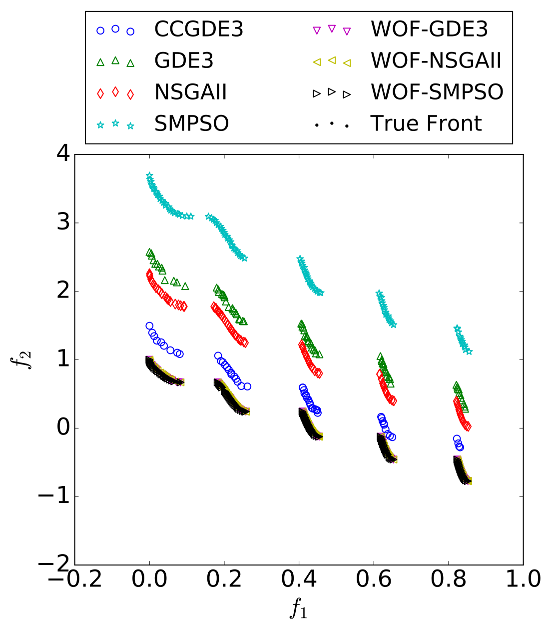


(b) Solution sets and Pareto-front

Figure 31: Convergence and obtained solution sets on the ZDT2 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).

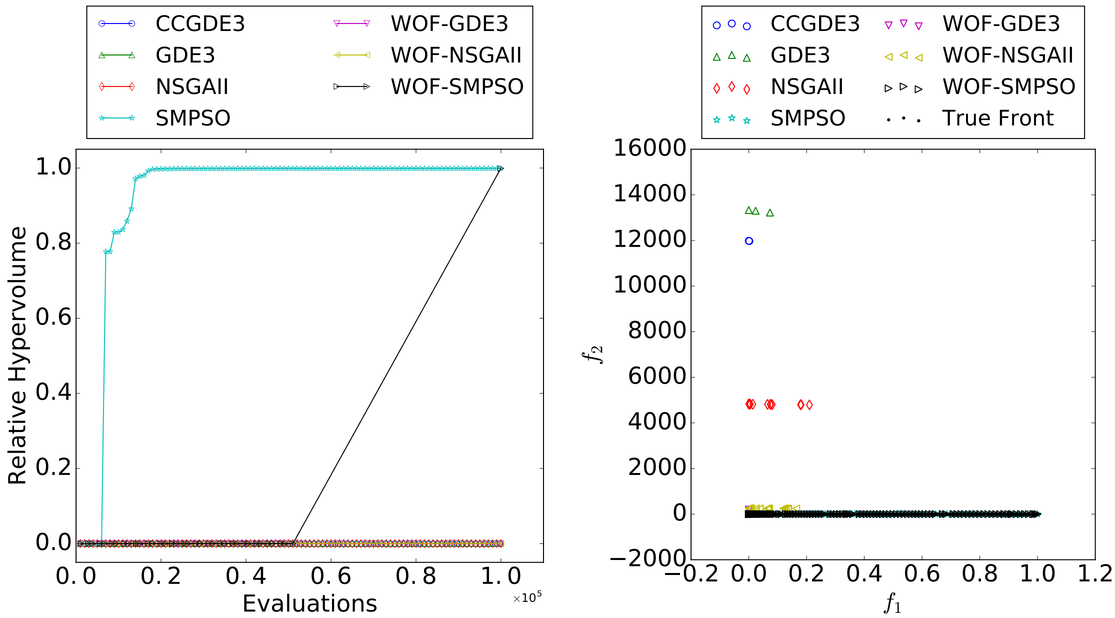


(a) Convergence of rel. hypervolume



(b) Solution sets and Pareto-front

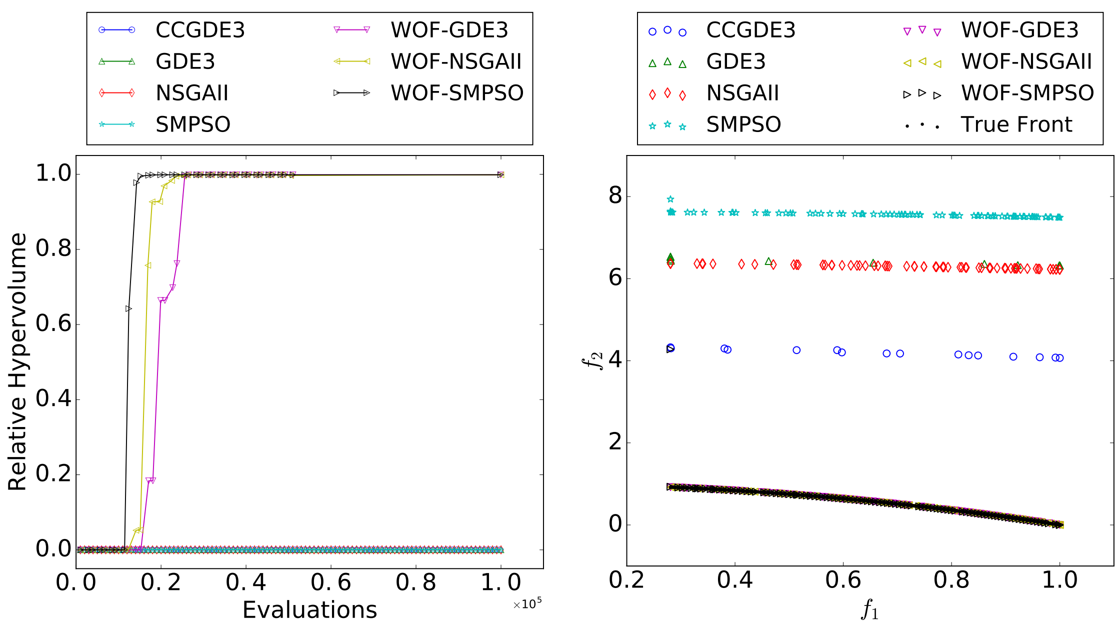
Figure 32: Convergence and obtained solution sets on the ZDT3 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

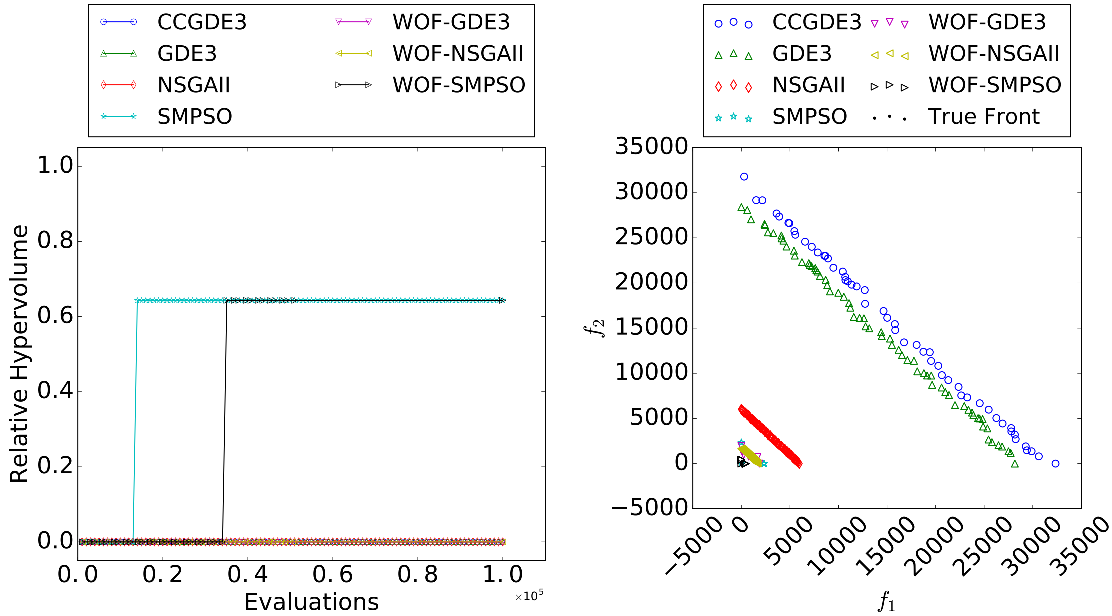
Figure 33: Convergence and obtained solution sets on the ZDT4 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

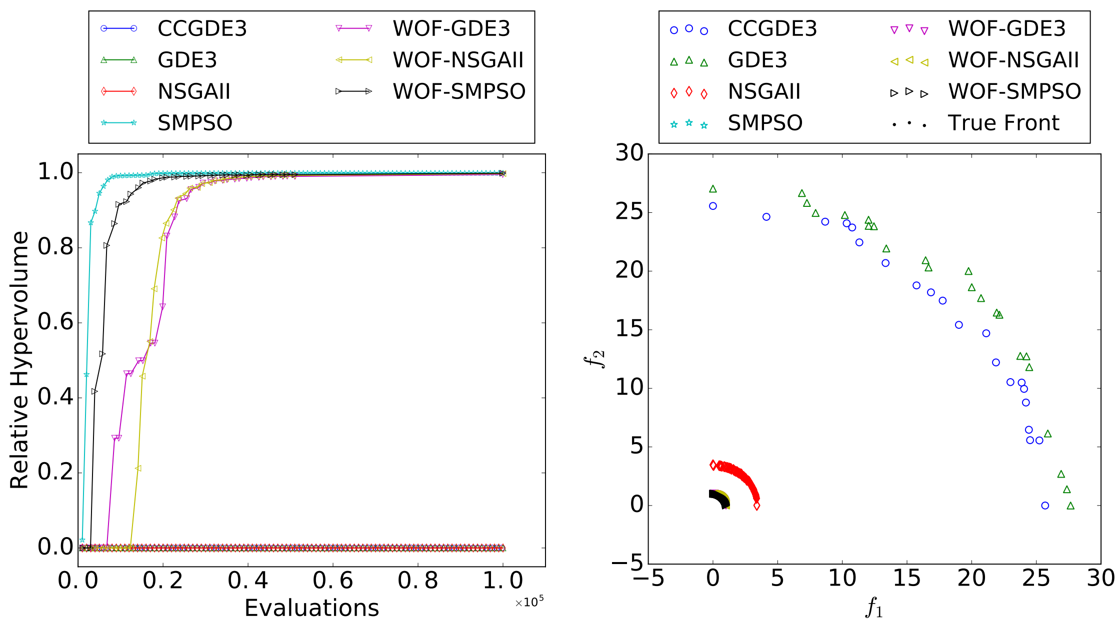
Figure 34: Convergence and obtained solution sets on the ZDT6 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

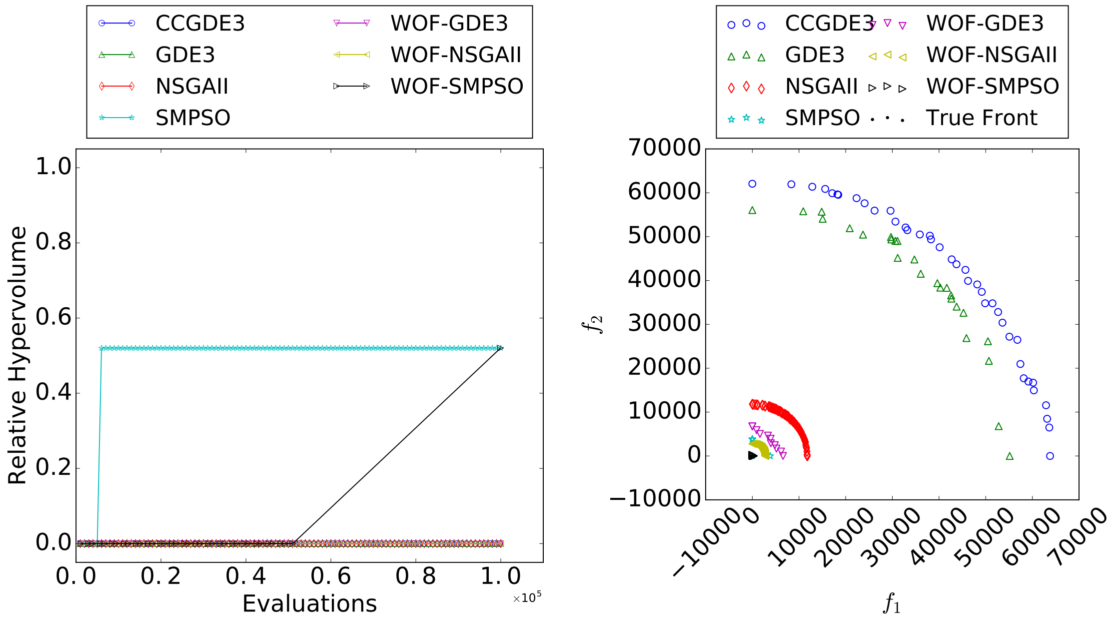
Figure 35: Convergence and obtained solution sets on the DTLZ1 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

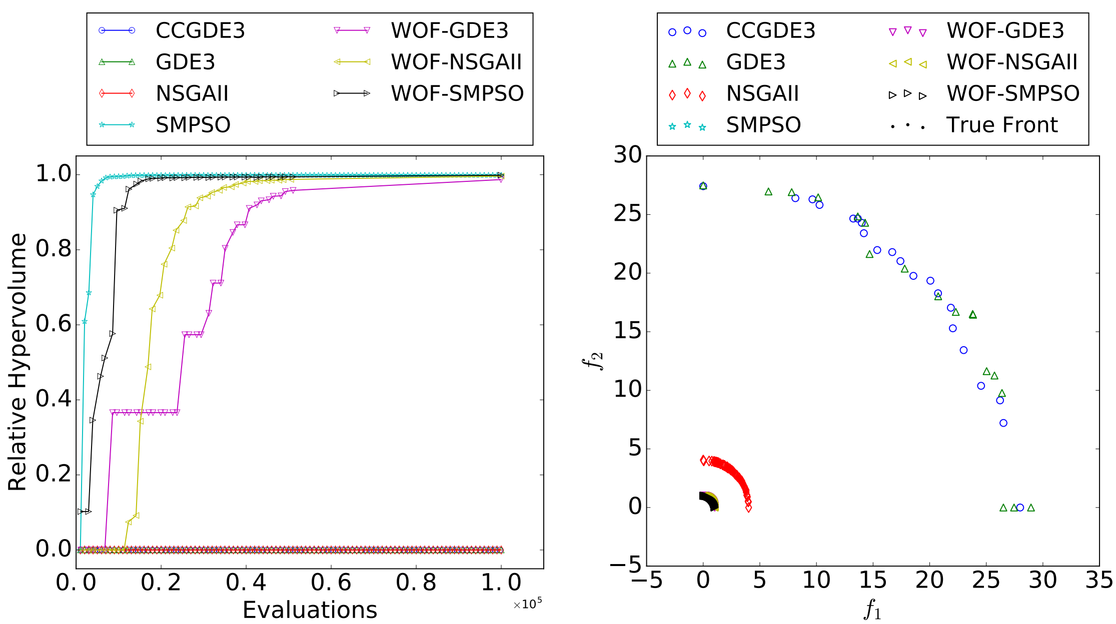
Figure 36: Convergence and obtained solution sets on the DTLZ2 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

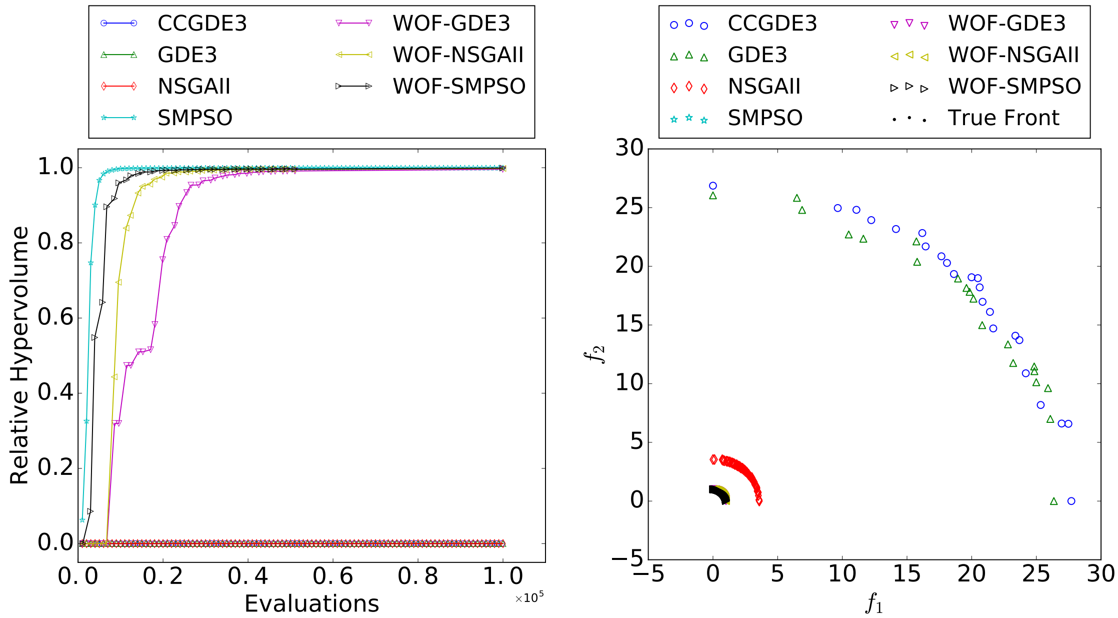
Figure 37: Convergence and obtained solution sets on the DTLZ3 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

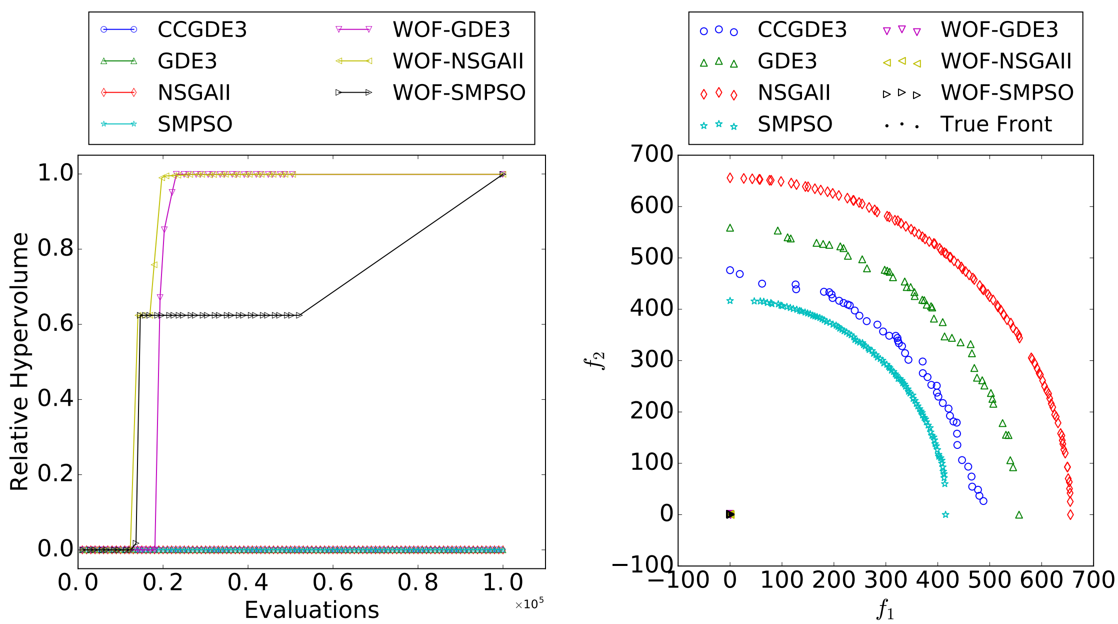
Figure 38: Convergence and obtained solution sets on the DTLZ4 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

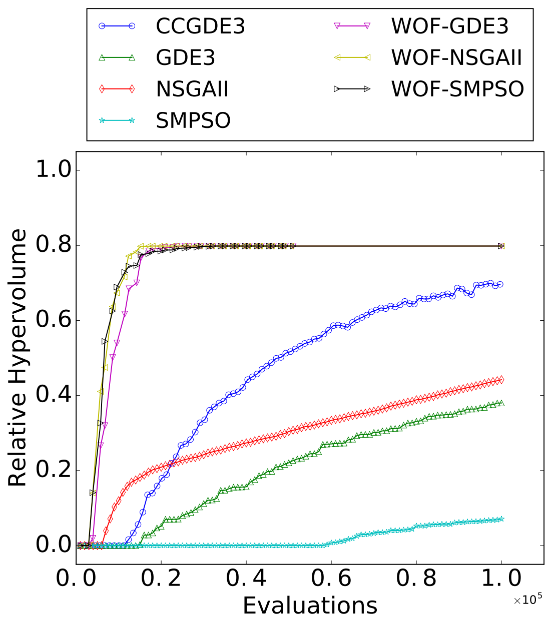
Figure 39: Convergence and obtained solution sets on the DTLZ5 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



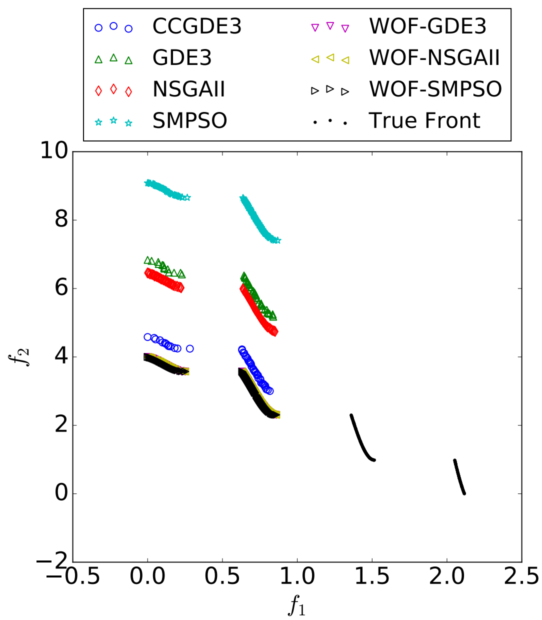
(a) Convergence of rel. hypervolume

(b) Solution sets and Pareto-front

Figure 40: Convergence and obtained solution sets on the DTLZ6 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) Convergence of rel. hypervolume



(b) Solution sets and Pareto-front

Figure 41: Convergence and obtained solution sets on the DTLZ7 problem with $n = 1000$ variables. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).

Table 17: Median and IQR hypervolume values for the SMPSO and its WOF-based versions. CD = Crowding Distance, RAN = Random selection. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

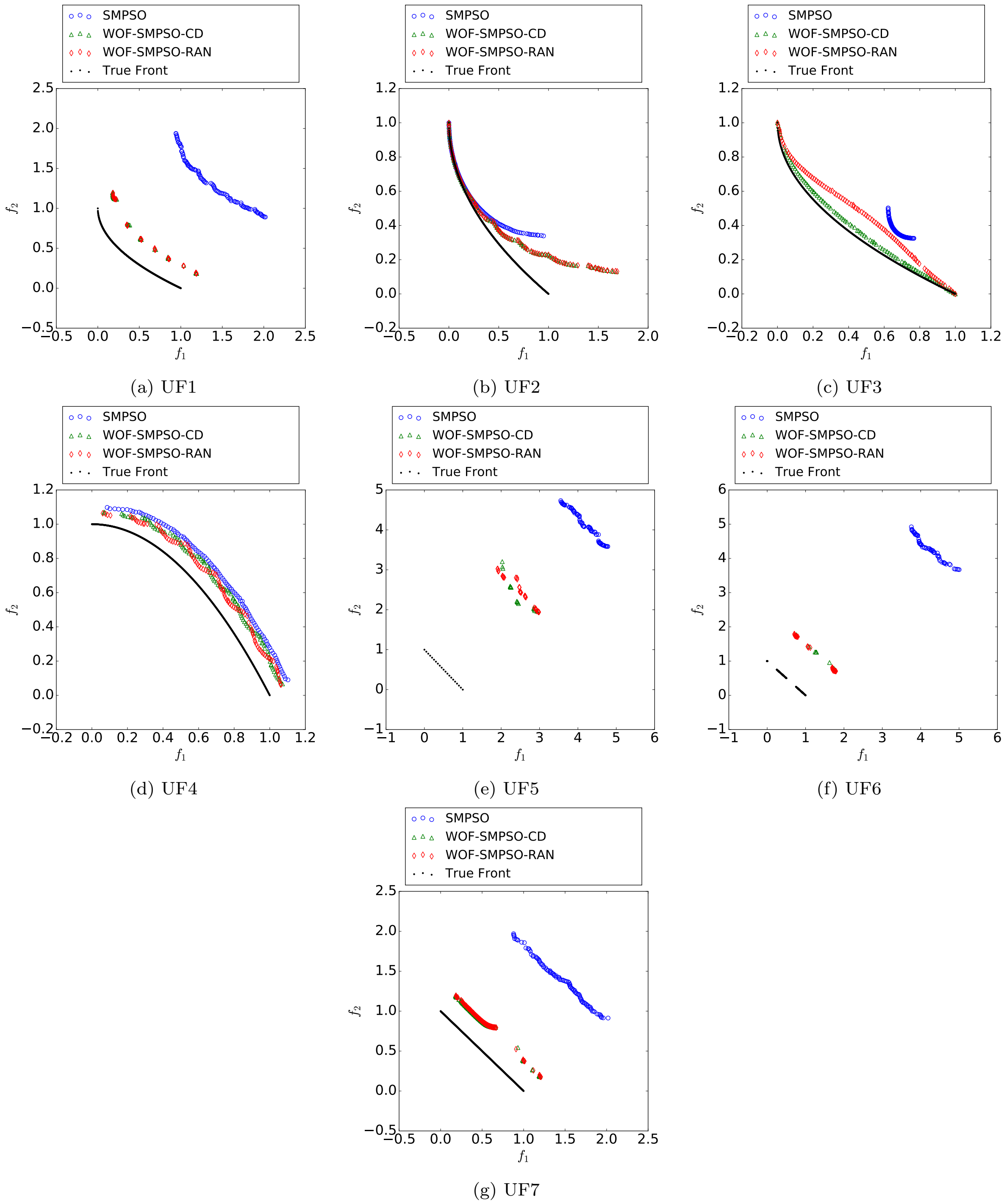
		SMPSO	WOF-SMPSO-CD	WOF-SMPSO-RAN
2	DTLZ1	**0.642949** (—)	0.642949 (—)	0.642949 (—)
2	DTLZ2	**0.998918** (4.48E-5)	0.998759 (2.17E-4) *	0.998766 (1.74E-4) *
2	DTLZ3	**0.520072** (—)	0.520072 (—)	0.520072 (—)
2	DTLZ4	**0.998895** (5.06E-5)	0.998781 (9.85E-5) *	0.998781 (1.90E-4) *
2	DTLZ5	**0.999339** (4.14E-5)	0.999193 (2.34E-4) *	0.999192 (1.57E-4) *
2	DTLZ6	— (—) *	0.998709 (1.01E-3)	**0.998860** (3.42E-1)
2	DTLZ7	0.065415 (3.81E-2) *	**0.798210** (2.86E-5)	0.798201 (3.19E-5)
2	WFG1	0.591510 (8.31E-3) *	**0.648191** (1.52E-2)	0.637240 (2.80E-3) *
2	WFG2	0.735514 (2.65E-2) *	**0.982822** (8.93E-3)	0.977075 (7.56E-3) *
2	WFG3	0.595415 (2.67E-2) *	0.855258 (4.87E-3)	**0.855309** (4.88E-3)
2	WFG4	0.844021 (3.66E-3) *	**0.977752** (1.07E-2)	0.974669 (8.98E-3)
2	WFG5	0.804384 (4.07E-3) *	**0.949251** (1.81E-2)	0.933080 (2.04E-2) *
2	WFG6	0.968425 (4.28E-3) *	**0.998508** (3.79E-4)	0.998042 (3.86E-4) *
2	WFG7	0.699467 (1.43E-2) *	0.960331 (9.21E-3)	**0.965292** (1.41E-2)
2	WFG8	0.515913 (9.61E-4) *	**0.888568** (2.74E-2)	0.885164 (1.86E-2)
2	WFG9	0.895389 (1.12E-2) *	**0.966547** (1.45E-2)	0.964929 (8.58E-3)
2	UF1	0.213552 (6.27E-3) *	0.784514 (5.75E-3)	**0.784923** (3.19E-3)
2	UF2	0.870441 (1.51E-3) *	**0.931617** (1.28E-3)	0.931518 (2.22E-3)
2	UF3	0.628261 (3.57E-3) *	**0.989180** (1.09E-3)	0.968995 (2.83E-2) *
2	UF4	0.891254 (1.61E-3) *	0.916527 (1.60E-2) *	**0.923472** (1.33E-2)
2	UF5	**—** (—)	— (—)	— (—)
2	UF6	— (—) *	**0.261973** (1.36E-1)	0.255834 (9.91E-2)
2	UF7	0.190185 (1.07E-2) *	**0.778761** (1.66E-2)	0.777729 (9.98E-3)
2	ZDT1	0.114787 (3.57E-2) *	**0.998164** (3.11E-4)	0.998124 (3.76E-4)
2	ZDT2	— (—) *	0.997875 (4.49E-4)	**0.997912** (4.35E-4)
2	ZDT3	0.222386 (3.55E-2) *	0.999152 (9.10E-5)	**0.999153** (1.47E-4)
2	ZDT4	**0.998786** (1.21E-4)	0.998680 (1.38E-4) *	0.998654 (3.07E-4) *
2	ZDT6	— (—) *	0.998132 (3.58E-4)	**0.998164** (3.08E-4)
3	DTLZ1	**0.587705** (1.51E-1)	— (5.87E-1) *	— (—) *
3	DTLZ2	**0.980981** (2.73E-3)	0.977884 (4.65E-3) *	0.980071 (1.97E-3) *
3	DTLZ3	**0.390379** (—)	— (3.90E-1) *	— (—) *
3	DTLZ4	**1.004826** (2.50E-3)	1.001984 (3.48E-3) *	1.003026 (2.31E-3) *
3	DTLZ5	**0.998542** (2.29E-4)	0.998195 (5.77E-4) *	0.998215 (4.85E-4) *
3	DTLZ6	— (—) *	**0.998851** (4.51E-4)	0.998641 (7.18E-4)
3	DTLZ7	0.000288 (5.39E-3) *	**0.981917** (6.37E-3)	0.980064 (5.60E-3)
3	WFG1	0.579493 (5.29E-3) *	**0.600865** (8.82E-3)	0.597586 (7.22E-3) *
3	WFG2	0.739441 (3.44E-3) *	**0.957771** (1.38E-2)	0.953639 (1.30E-2)
3	WFG3	0.560882 (1.31E-2) *	**0.949504** (2.54E-2)	0.948570 (2.05E-2)
3	WFG4	0.718487 (7.98E-3) *	0.896870 (2.99E-2) *	**0.907802** (1.46E-2)
3	WFG5	0.677864 (5.18E-3) *	**0.839591** (2.99E-2)	0.836890 (1.85E-2)
3	WFG6	0.875053 (5.24E-3) *	0.963806 (1.31E-2) *	**0.971365** (1.23E-2)
3	WFG7	0.662136 (1.05E-2) *	0.861478 (2.11E-2)	**0.862228** (2.75E-2)
3	WFG8	0.551158 (2.06E-2) *	**0.813844** (2.64E-2)	0.808057 (2.16E-2)
3	WFG9	0.822970 (2.58E-2) *	0.905615 (2.51E-2)	**0.906084** (1.76E-2)
3	UF8	**0.858007** (5.01E-4)	0.857912 (4.54E-4)	0.857628 (5.42E-4) *
3	UF9	0.556027 (3.34E-3) *	**0.641570** (1.63E-2)	0.639402 (8.39E-3)
3	UF10	0.855091 (2.63E-3)	**0.855724** (2.20E-3)	0.855401 (2.19E-3)

Table 18: Median and IQR hypervolume values for the NSGA-II and its WOF-based versions. CD = Crowding Distance, RAN = Random selection. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		NSGAII	WOF-NSGAII-CD	WOF-NSGAII-RAN
2	DTLZ1	— (—)	— (—)	— (—)
2	DTLZ2	— (—) *	**0.997599** (4.96E-4)	0.997286 (4.13E-4) *
2	DTLZ3	— (—)	— (—)	— (—)
2	DTLZ4	— (—) *	**0.996286** (6.40E-4)	0.978856 (2.00E-2) *
2	DTLZ5	— (—) *	**0.997982** (4.39E-4)	0.997660 (5.28E-4) *
2	DTLZ6	— (—) *	**0.999135** (1.00E-4)	0.999104 (7.02E-5)
2	DTLZ7	0.438463 (2.75E-2) *	**0.798237** (1.66E-5)	0.798233 (1.99E-5)
2	WFG1	0.315130 (2.30E-3) *	**0.645937** (7.58E-3)	0.630589 (9.79E-3) *
2	WFG2	0.668336 (5.48E-2) *	**0.802678** (1.06E-2)	0.790956 (1.15E-2) *
2	WFG3	0.613178 (7.29E-3) *	**0.807746** (5.00E-3)	0.790732 (4.46E-3) *
2	WFG4	0.623073 (9.05E-3) *	**0.858527** (2.14E-2)	0.828085 (3.03E-2) *
2	WFG5	0.588708 (1.37E-2) *	**0.876561** (8.39E-2)	0.856455 (4.14E-2) *
2	WFG6	0.574802 (9.15E-3) *	**0.836853** (6.07E-2)	0.808181 (2.04E-2) *
2	WFG7	0.649376 (9.69E-3) *	**0.862676** (5.65E-3)	0.848651 (7.28E-3) *
2	WFG8	0.552847 (1.15E-2) *	**0.816987** (1.95E-2)	0.765758 (1.58E-2) *
2	WFG9	0.621896 (3.33E-2) *	**0.933816** (2.51E-2)	0.921233 (2.61E-2) *
2	UF1	0.704838 (6.39E-2) *	**0.901289** (8.91E-3)	0.896158 (5.29E-2) *
2	UF2	0.804426 (1.16E-2) *	**0.911806** (1.89E-3)	0.911654 (1.89E-3)
2	UF3	0.675996 (5.26E-3) *	**0.981318** (3.36E-3)	0.959466 (1.21E-2) *
2	UF4	0.855315 (3.23E-3) *	0.905081 (2.35E-2)	**0.910213** (4.83E-2)
2	UF5	— (—) *	**0.202609** (7.19E-2)	0.186294 (1.00E-1)
2	UF6	0.224582 (9.66E-2) *	**0.686332** (5.62E-2)	0.682334 (3.72E-2)
2	UF7	0.714118 (1.17E-1) *	**0.904670** (2.65E-2)	0.886384 (3.45E-2) *
2	ZDT1	0.563881 (4.14E-2) *	**0.998437** (1.44E-4)	0.998437 (1.04E-4)
2	ZDT2	0.102213 (4.65E-2) *	**0.998305** (1.65E-4)	0.998264 (1.00E-4)
2	ZDT3	0.614058 (2.79E-2) *	**0.999308** (6.89E-5)	0.999264 (8.37E-5) *
2	ZDT4	— (—)	— (—)	— (—)
2	ZDT6	— (—) *	**0.998640** (8.52E-5)	0.998603 (1.34E-4) *
3	DTLZ1	— (—)	— (—)	— (—)
3	DTLZ2	— (—) *	0.976192 (3.15E-3) *	**0.980329** (2.01E-3)
3	DTLZ3	— (—)	— (—)	— (—)
3	DTLZ4	— (—) *	**0.978010** (1.07E-2)	0.929277 (8.41E-2) *
3	DTLZ5	— (—) *	0.982235 (5.28E-3) *	**0.987972** (2.60E-3)
3	DTLZ6	— (—) *	0.999340 (1.22E-4)	**0.999353** (1.98E-4)
3	DTLZ7	0.483623 (2.21E-2) *	**0.989765** (3.50E-3)	0.989539 (4.03E-3)
3	WFG1	0.288955 (2.78E-3) *	**0.589909** (5.98E-3)	0.579851 (6.19E-3) *
3	WFG2	0.656630 (4.76E-3) *	**0.866116** (2.18E-2)	0.793431 (7.23E-3) *
3	WFG3	0.505391 (1.75E-2) *	**0.806780** (2.16E-2)	0.765170 (1.42E-2) *
3	WFG4	0.511231 (1.28E-2) *	**0.806552** (2.39E-2)	0.763915 (2.04E-2) *
3	WFG5	0.458463 (1.02E-2) *	**0.788175** (2.78E-2)	0.786718 (2.70E-2)
3	WFG6	0.450742 (1.34E-2) *	**0.740160** (7.20E-2)	0.715558 (1.79E-2) *
3	WFG7	0.552895 (8.90E-3) *	**0.802916** (1.60E-2)	0.775686 (1.69E-2) *
3	WFG8	0.453790 (1.04E-2) *	**0.745608** (2.24E-2)	0.725282 (1.74E-2) *
3	WFG9	0.489449 (1.56E-2) *	0.845735 (3.29E-2)	**0.846156** (1.95E-2)
3	UF8	0.294208 (3.76E-2) *	**0.857385** (9.06E-4)	0.855048 (1.69E-3) *
3	UF9	0.378028 (5.32E-2) *	0.650223 (5.03E-2)	**0.655735** (2.44E-2)
3	UF10	— (—) *	**0.704378** (8.19E-2)	0.700340 (7.04E-2)

Table 19: Median and IQR hypervolume values for the GDE3 and its WOF-based versions. CD = Crowding Distance, RAN = Random selection. Best performance is indicated by dark blue color as well as bold font. Second best performance is indicated by light blue color. The worst performance is indicated by red color. Statistical significance ($p < 0.01$) compared to the respective best performance (in each line) is indicated by an asterisk.

		GDE3	WOF-GDE3-CD	WOF-GDE3-RAN
2	DTLZ1	— (—)	— (—)	— (—)
2	DTLZ2	— (—) *	**0.995275** (2.80E-3)	0.993925 (1.78E-3) *
2	DTLZ3	— (—)	— (—)	— (—)
2	DTLZ4	— (—) *	**0.990729** (1.58E-2)	0.964002 (3.95E-2) *
2	DTLZ5	— (—) *	**0.995498** (3.65E-3)	0.993806 (1.73E-3) *
2	DTLZ6	— (—) *	**0.999013** (1.79E-4)	0.998973 (2.20E-4)
2	DTLZ7	0.379681 (1.37E-2) *	**0.798235** (1.48E-5)	0.798231 (1.95E-5)
2	WFG1	0.574548 (8.07E-3) *	**0.654669** (1.41E-2)	0.633416 (8.58E-3) *
2	WFG2	0.688493 (8.20E-3) *	**0.856565** (6.30E-2)	0.849850 (1.81E-2)
2	WFG3	0.684992 (4.04E-3) *	**0.806385** (6.68E-3)	0.780408 (9.32E-3) *
2	WFG4	0.633162 (1.59E-2) *	**0.872904** (2.42E-2)	0.817386 (4.13E-2) *
2	WFG5	0.665094 (2.70E-2) *	**0.954091** (5.63E-2)	0.837636 (4.11E-2) *
2	WFG6	0.492081 (1.34E-2) *	**0.912567** (5.45E-2)	0.775413 (8.92E-2) *
2	WFG7	0.696461 (1.29E-2) *	**0.857403** (4.59E-3)	0.833907 (1.14E-2) *
2	WFG8	0.664091 (9.73E-3) *	**0.811729** (1.28E-2)	0.762199 (2.02E-2) *
2	WFG9	0.510170 (1.03E-2) *	**0.937116** (2.73E-2)	0.936575 (2.55E-2)
2	UF1	0.668001 (2.88E-2) *	**0.834067** (7.03E-3)	0.829617 (5.26E-3) *
2	UF2	0.670571 (1.39E-2) *	**0.926938** (1.99E-3)	0.920269 (4.18E-3) *
2	UF3	0.535839 (1.03E-2) *	**0.982725** (1.34E-3)	0.973971 (8.13E-3) *
2	UF4	0.904251 (5.93E-3) *	0.909221 (3.18E-2) *	**0.940753** (1.61E-2)
2	UF5	— (—)	— (—)	— (—)
2	UF6	0.072378 (6.45E-2) *	**0.499220** (2.82E-2)	0.487258 (1.59E-2) *
2	UF7	0.711165 (1.33E-2) *	**0.844134** (5.57E-3)	0.828890 (8.96E-3) *
2	ZDT1	0.543085 (2.15E-2) *	0.998381 (1.76E-4)	**0.998406** (1.80E-4)
2	ZDT2	0.111199 (3.22E-2) *	**0.998288** (1.71E-4)	0.998286 (1.60E-4)
2	ZDT3	0.517681 (1.70E-2) *	**0.999275** (8.87E-5)	0.999251 (1.00E-4)
2	ZDT4	— (—)	— (—)	— (—)
2	ZDT6	— (—) *	0.998489 (1.83E-4)	**0.998510** (1.20E-4)
3	DTLZ1	— (—)	— (—)	— (—)
3	DTLZ2	— (—) *	0.768941 (5.86E-2) *	**0.795812** (1.11E-1)
3	DTLZ3	— (—)	— (—)	— (—)
3	DTLZ4	— (—) *	0.503802 (2.39E-1)	**0.535764** (3.27E-1)
3	DTLZ5	— (—) *	**0.021397** (9.95E-2)	0.011849 (8.03E-2)
3	DTLZ6	— (—) *	**0.999122** (2.93E-4)	0.999080 (3.54E-4)
3	DTLZ7	0.220150 (2.05E-2) *	**0.989566** (3.58E-3)	0.988160 (3.10E-3) *
3	WFG1	0.570552 (3.92E-3) *	**0.602257** (1.41E-2)	0.590719 (7.75E-3) *
3	WFG2	0.616240 (5.56E-3) *	**0.867054** (1.15E-2)	0.817216 (3.31E-2) *
3	WFG3	0.473648 (7.22E-3) *	**0.765682** (4.09E-2)	0.666703 (3.88E-2) *
3	WFG4	0.538147 (1.26E-2) *	**0.807465** (2.01E-2)	0.738029 (4.02E-2) *
3	WFG5	0.604148 (1.11E-2) *	**0.801722** (4.63E-2)	0.786334 (2.36E-2)
3	WFG6	0.444464 (1.08E-2) *	**0.824538** (8.18E-2)	0.815037 (1.22E-1)
3	WFG7	0.568938 (9.08E-3) *	**0.793751** (1.82E-2)	0.761459 (1.11E-2) *
3	WFG8	0.535693 (7.76E-3) *	**0.739919** (2.91E-2)	0.708133 (2.24E-2) *
3	WFG9	0.368427 (1.07E-2) *	**0.855264** (2.91E-2)	0.848153 (2.84E-2)
3	UF8	0.093896 (1.65E-2) *	**0.855437** (8.03E-4)	0.853410 (3.26E-3) *
3	UF9	0.061738 (1.48E-2) *	0.694232 (1.55E-2)	**0.694808** (1.40E-2)
3	UF10	— (—) *	**0.781952** (1.49E-1)	0.769003 (1.47E-1)

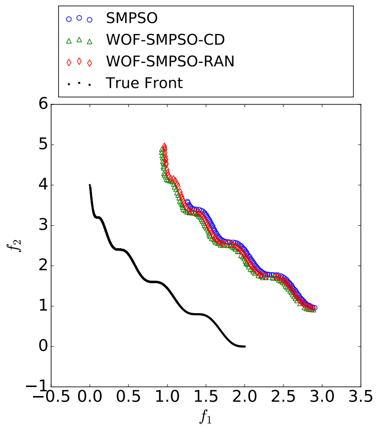


(a) UF1 (b) UF2 (c) UF3

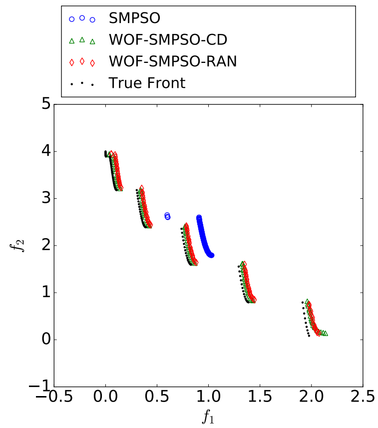
(d) UF4 (e) UF5 (f) UF6

(g) UF7

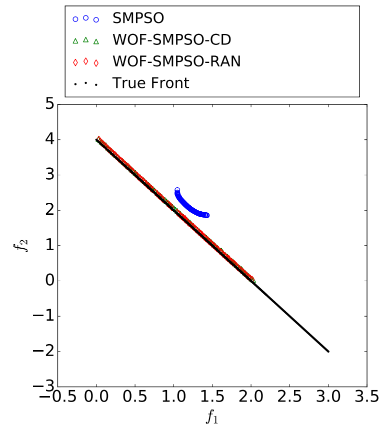
Figure 42: SMPSO solution sets on the UF problems comparing different selection mechanisms of the x'_k in WOF. *CD* = Crowding Distance, *RAN* = Random selection. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



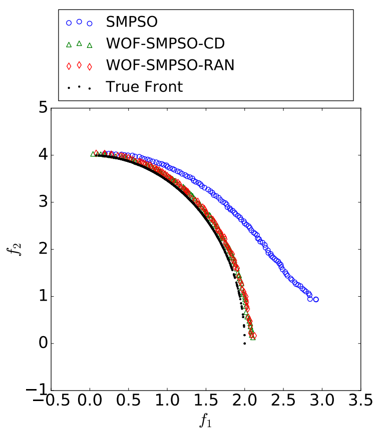
(a) WFG1



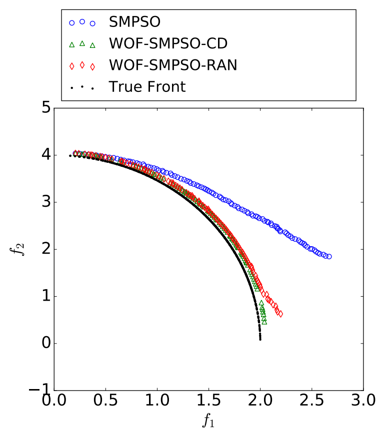
(b) WFG2



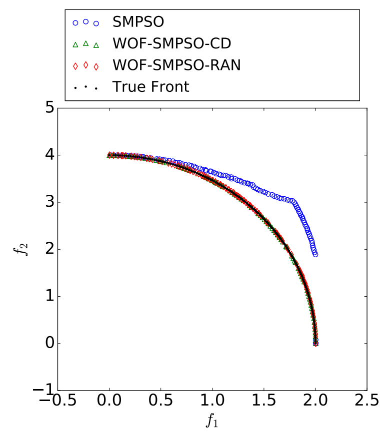
(c) WFG3



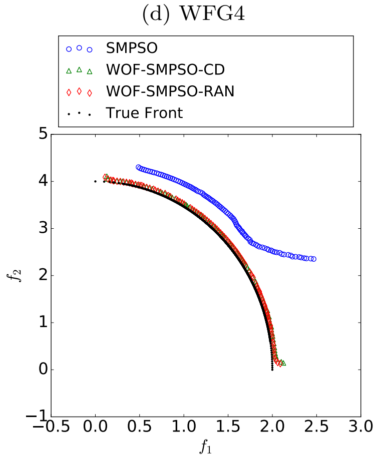
(d) WFG4



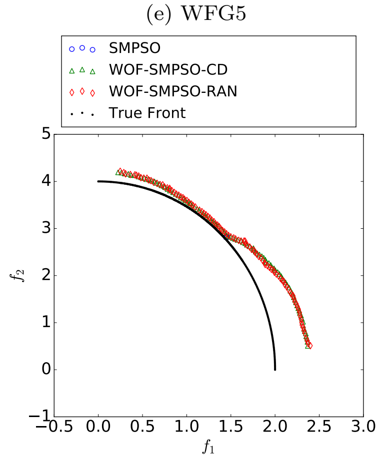
(e) WFG5



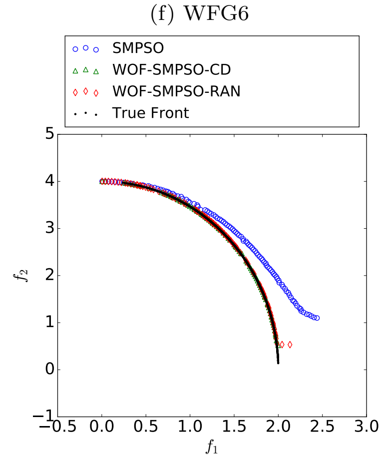
(f) WFG6



(g) WFG7

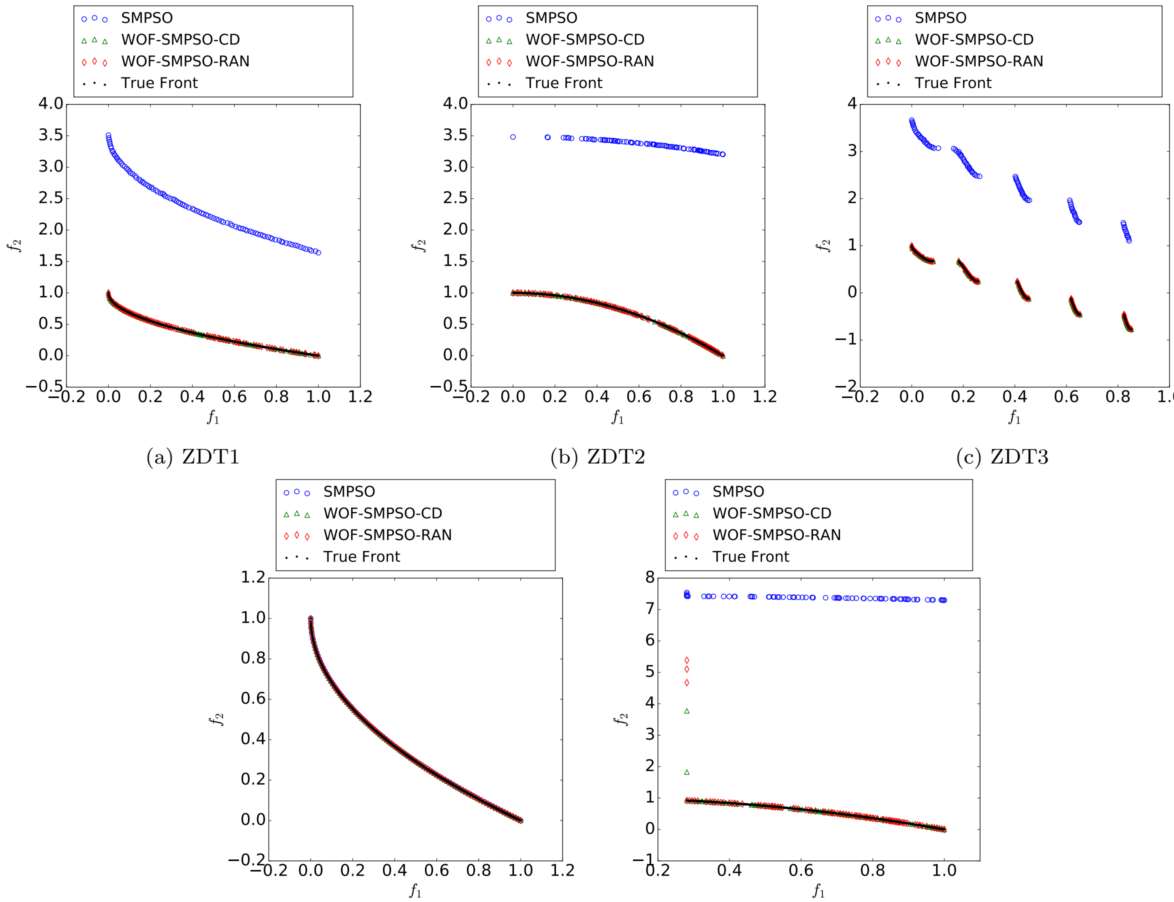


(h) WFG8



(i) WFG9

Figure 43: SMPSO solution sets on the WFG problems comparing different selection mechanisms of the x'_k in WOF. CD = Crowding Distance, RAN = Random selection. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).



(a) ZDT1 (b) ZDT2 (c) ZDT3

(d) ZDT4 (e) ZDT6

Figure 44: SMPSO solution sets on the ZDT problems comparing different selection mechanisms of the x'_k in WOF. CD = Crowding Distance, RAN = Random selection. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).

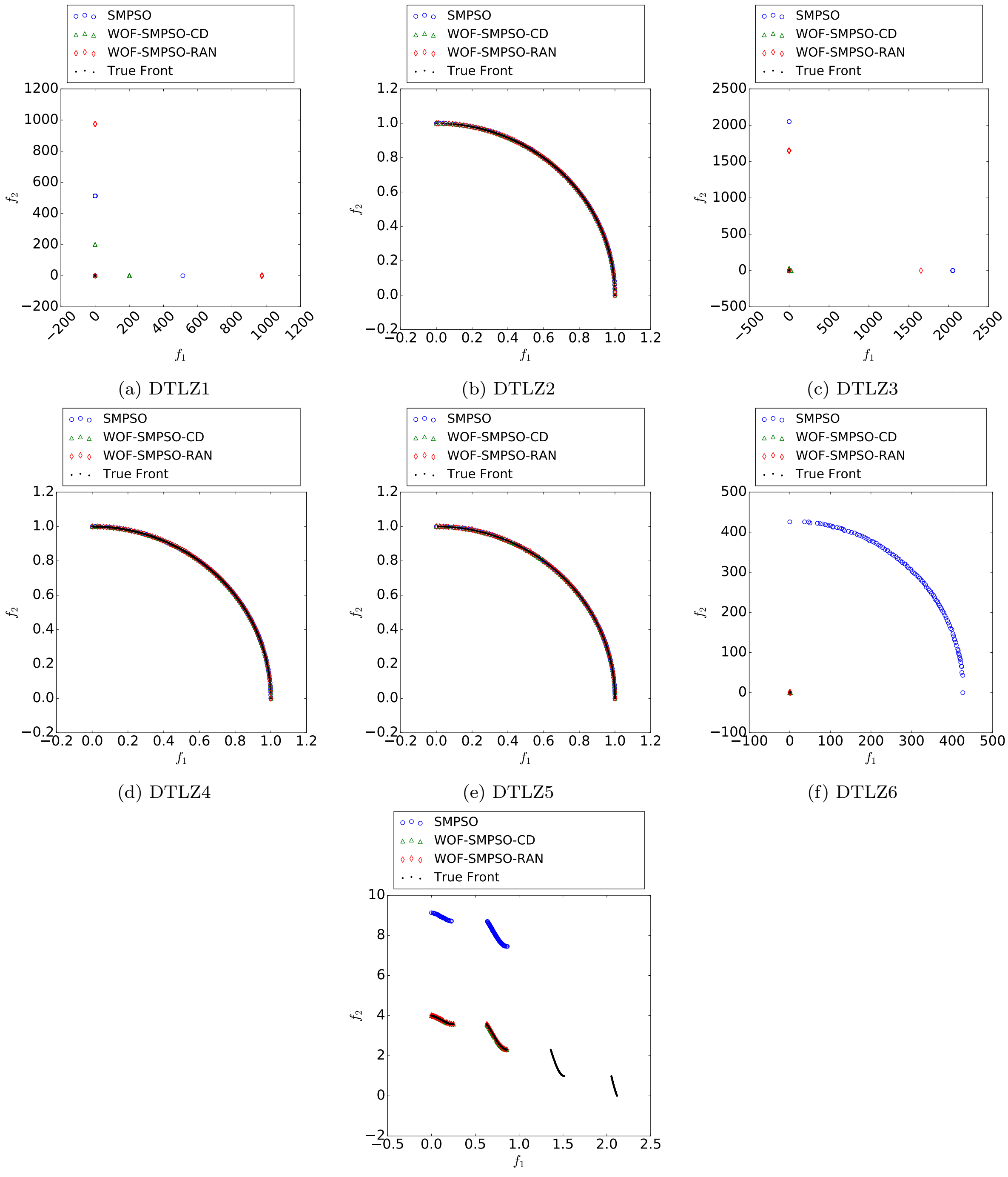


Figure 45: SMPSO solution sets on the DTLZ problems comparing different selection mechanisms of the x'_k in WOF. *CD* = Crowding Distance, *RAN* = Random selection. Shown are the median runs (i.e. the runs which obtained the median hypervolume value by the end of the optimization).